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DETERMINATION OF THE LEWIS FORM FACTOR FOR SPUR GEAR TEETH

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<Abstract>

This paper presents a simple numerical method for computing the tooth form factors. It simulates directly the tooth form generating process with a hob. The approach is so general that the program developed for the Lewis method can be extended for the 30° tangential line method with few modifications. Moreover, the withdrawal of the hob from the gear blank can be easily expressed. It is confirmed that the computed tooth form factors are identical with the published ones.

평치차에 대한 루이스 치형계수의 계산

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<요 약>

치차의 굽힘강도와 관련된 치형계수를 계산하는 간단한 수치적 방법을 개발하였다. 이 방법은 절삭공구인 홉으로 치형을 생성하는 과정을 직접 모사한다. 본 논문의 접근방법은 일반적이어서 치차에 내접하는 포물선을 이용하는 루이스 방법에 대하여 개발된 프로그램도 약간만 고치면 30° 접선법에 대한 프로그램으로 확장된다. 그리고, 랙 커터를 후퇴하여 전위치차를 절삭하는 것도 쉽게 표현된다. 계산된 치형계수가 기존 결과와 같음을 확인하였다.

1. Introduction

It is customary to idealize the gear tooth profile as a constant stress parabola for gear tooth bending stress analysis. The parabola inscribed within the tooth profile is tangent to the tooth fillets and passes through the intersection point of the line of action and the tooth centerline. The section which includes the points of tangency is called as the weakest section. The maximum bending stress is derived from basic beam theory [1, 2]. The Lewis form factor reflects the effect of the width of the weakest section and the distance of it from the point of loading. Besides the Lewis method, other method such as the 30° tangential line method is also used.

The tooth form factor depends only on tooth profile. In other words, it is a function of the number of teeth, the pressure angle, addendum, dedendum and the hob tip radius. The methods by which the tooth form factors were determined are either graphical or analytical. Recently, the numerical methods[3, 4] which are convenient to use and accurate are presented. In these methods, the condition of tangency is expressed as a nonlinear equation and Newton's method is used to solve it. The derived equation is, however, still complicated.

In this paper, simpler numerical method with sufficient accuracy is developed. It is a direct approach and easy to understand but the efficiency is sacrificed a little bit. Since it is a general approach, both the Lewis method and the 30° tangential line method can be treated with few modifications.

2. Development of Method

The module m is assumed as $m = 1$ for convenience throughout the paper since the form factor does not depend on the module itself.

The tooth profile is generated as the pitch surface of a hob rolls on the pitch surface of the gear blank. The hob profile is determined by the pressure angle φ, addendum a, dedendum b, the hob tip radius r_f while the radius of the pitch circle is determined by the number of teeth N .

The radius of pitch circle r_p, the base circle r_b , and the addendum circle r_a can be expressed in terms of φ, N and ξ.

$$r_p = N/2, \quad r_b = r_p \cos\varphi, \quad r_a = r_p + a + \xi \qquad (1)$$

where ξ is the addendum modification coefficient by which a hob withdraws from the gear blank.

The initial relative position of a hob and a gear blank is as shown in Fig. 1. The pitch surface of the hob is tangent to the pitch surface of the pitch circle on the vertical axis. The generated gear tooth will be as in Fig. 2. Let's focus on the center tooth.

The orientation of the tooth centerline, θ_c, will be

$$\theta_c = p/(4r_p) = \pi/(2N) \tag{2}$$

where θ is measured counterclockwise from the vertical axis and p is the circular pitch.

Assume the gear tooth comes into contact with another gear tooth at point A on the addendum circle in Fig. 2. The line of action intersects with the tooth centerline at point C. The angle between the line of action and the tangential direction at each point A and C are, φ_a and φ_c, respectively.

$$\varphi_a = \cos^{-1}(r_b/r_a) \qquad \varphi = \tan\varphi_a - (\tan\varphi - \varphi) - (\pi/2 + 2\xi\tan\varphi)/N \tag{3}$$

The coordinates of the point C is

$$x_0^* = 0, \;\; y_0^* = r_b/\cos\varphi_c \tag{4}$$

where the coordinate system (x^*, y^*) is as shown in Fig. 2. The origin is located at the center of the gear blank and the y^* axis is aligned with the tooth centerline.

For the hob profile in Fig. 1, the distance Δ, which is the half width of the hop tip land, is

$$\Delta = \pi/4 - (b - r_f)\tan\varphi - r_f/\cos\varphi \tag{5}$$

Let's find the position of a point on the hob tip profile after the hob rolls on the pitch surface of the gear blank by θ. Assume that the point P in Fig. 1 will come into contact with the base circle. From the point P to the point Q, the horizontal distance, l_{11}, and the vertical distance, l_{12}, are

$$l_{11} = r_p\theta \qquad l_{12} = 0 \tag{6}$$

From the point Q to the point R,

$$l_{21} = \pi/4 - \Delta - r_f\sin\Psi \qquad l_{22} = b - r_f(1 - \cos\Psi) - \xi \tag{7}$$

where Ψ is an angular position of the point R on the hop tip profile in Fig. 1.

After the hob rolls by θ on the pitch surface of the gear blank , the coordinates of the point R will be

$$x_1 = -r_p\sin\theta + l_{11}\cos\theta + l_{21}\cos\theta + l_{22}\sin\theta \tag{8}$$

$$y_1 = r_p \cos\theta + l_{11}\sin\theta + l_{21}\sin\theta - l_{22}\cos\theta$$

where the coordinate system (x, y) is as shown in Fig. 2. The origin is at the center of the gear blank and the axis is aligned with the vertical axis. The following transformation yields the coordinates in (x^*, y^*) coordinate system.

$$x_1^* = x_1\cos\theta_c + y_1\sin\theta_c \qquad y_1^* = -x_1\sin\theta_c + y_1\cos\theta_c \tag{9}$$

The temporary Lewis form factor Y is

$$Y = \frac{[2(x_1^* - x_0^*)]^2}{6(y_1^* - y_0^*)} \tag{10}$$

Both θ and Ψ are varied over a possible range, $\theta_1 < \theta < \theta_2$ and $\Psi_1 < \Psi < \Psi_2$ with finite increments, the minimum temporary Lewis form factor is picked out as shown in Fig. 3. This is the true Lewis form factor.

For the 30° tangential line method in Fig. 4, the problem changes slightly to find a minimum of y_{30}^*, instead of a minimum of Y,

$$y_{30}^* = y_1^* + x_1^*/\tan 30° \tag{11}$$

JSME defines the tooth form factor as the reciprocal of the Lewis form factor.

$$Y_{30} = \frac{6(y_1^* - y_0^*)}{[2(x_1^* - x_0^*)]^2}\cos\varphi \tag{12}$$

Except eq.(11) and eq. (12), the same solution procedures can be applied in the present approach.

To increase the accuracy, the ranges of and are decreased gradually as follows.

$$\theta_{\min}{}^i - f|\theta_1{}^i - \theta_{\min}{}^i| < \theta^{i+1} < \theta_{\min}{}^i + f|\theta_2{}^i - \theta_{\min}{}^i| \tag{13}$$

$$\Psi_{\min}{}^i - f|\Psi_1{}^i - \Psi_{\min}{}^i| < \Psi^{i+1} < \Psi_{\min}{}^i + f|\Psi_2{}^i - \Psi_{\min}{}^i|$$

where the superscript i, $i+1$ indicate the iteration steps and the subscript *min* indicates the angular position corresponding to the minimum temporary form factor and f is a value around 0.5.

The iteration ends when both errors fall within a tolerance ε=0.00001.

$$|\theta_{\min}{}^{i+1} - \theta_{\min}{}^i| \le \varepsilon \quad \text{and} \quad |\Psi_{\min}{}^{i+1} - \Psi_{\min}{}^i| \le \varepsilon \tag{14}$$

3. Results

The computed form factors Y for the weakest sections located by the Lewis method are shown in Table 1 and Y_{30} for the weakest sections located by the 30° tangential line method are shown in Table 2. For every combinations of N, φ, a, b, ξ and r_f in the literature (3,4), the present results are identical with the published ones.

It is interesting to compare the weakest sections located by the Lewis method and the 30° tangential line method. For the example case of $\varphi = 20°$, $a = m$, $b = 1.25 m$, $\xi = 0$ and $r_f = 0.375 m$, the weakest sections located by the 30° (tangential line method are more distant from the apex of the parabola by 4~10 % and wider by 3~9 % than the ones located by the Lewis method. For the both weakest sections, the Lewis form factors Y are computed and compared in Fig. 5. The weakest section located by the 30° tangential line method yields larger form factors by 2~7 % than the weakest section located by the Lewis method.

4. Conclusion

Using the simple numerical method presented herein, one could, locate the weakest section either by the Lewis method or by the 30° tangential line method and compute the tooth form factor.

It is confirmed that the present results are identical with the published ones for every combinations of the number of teeth N, pressure angle φ, addendum a, dedendum b, the addendum modification coefficient ξ and the hob tip radius r_f in the literature (3,4).

References

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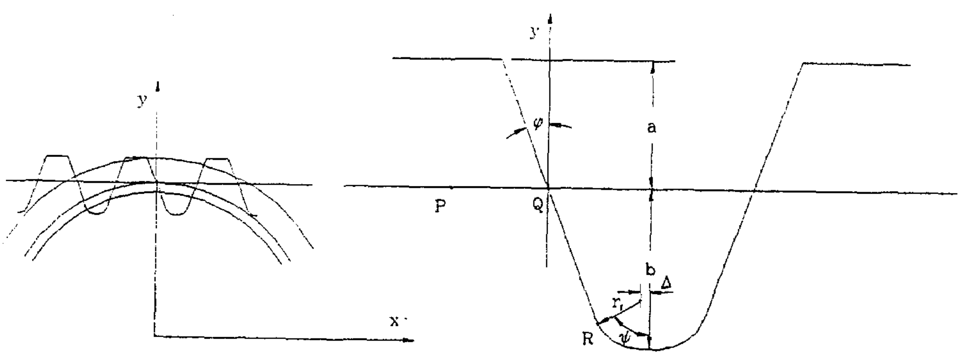

Fig. 1 Initial position of the hob and the gear blank and enlarged view of the hob

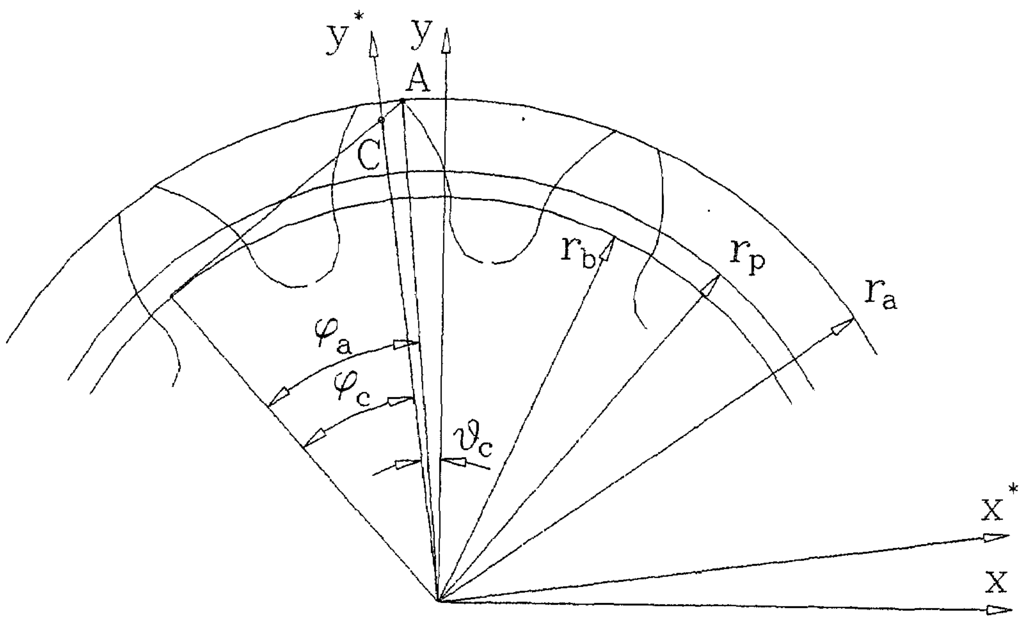

Fig. 2 The tooth profile and the coordinate systems

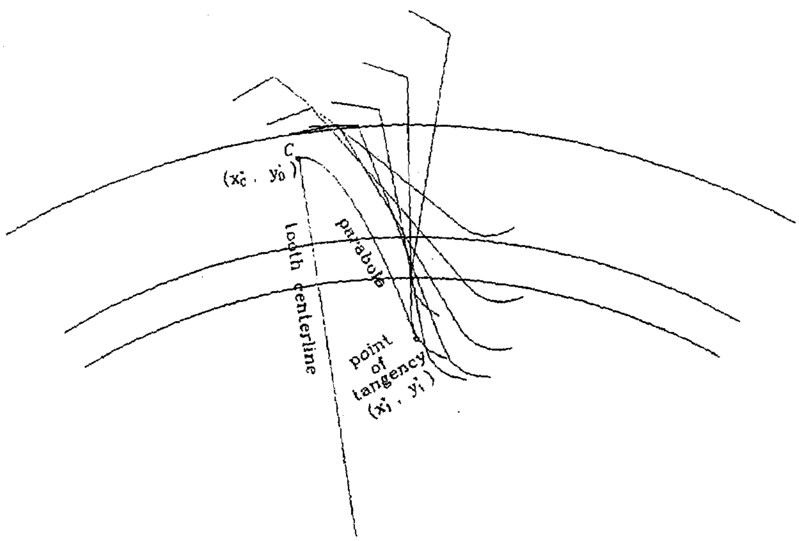

Fig. 3 The constant stress parabola and the Lewis method

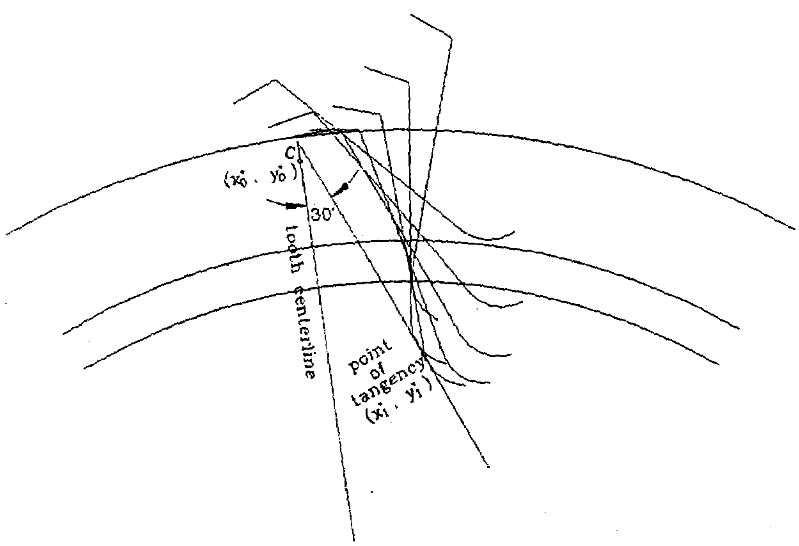

Fig. 4 The 30° tangential line method

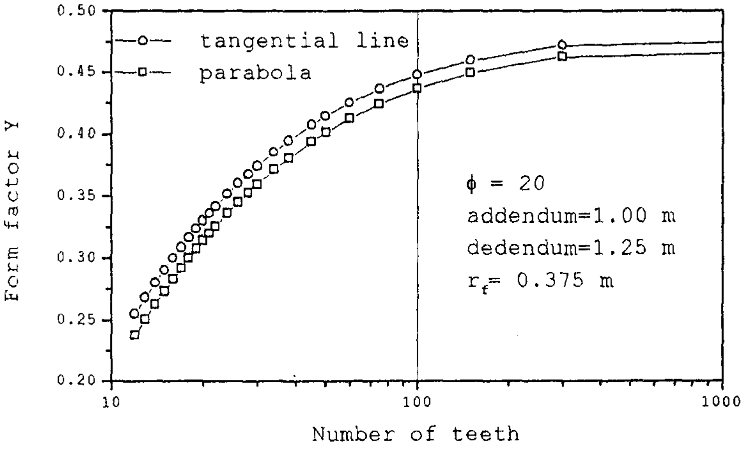

Fig. 5 Comparison of the Lewis from factors for the weakest section located either by the Lewis method or the 30° tangential line method