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Savings and the Social Rate of Discount

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〈Abstract〉

This paper examines the relation between the problem of optimal social rate of discount and the market savings. I introduce the effects of individual and collective saving behavior on the discount rate, using the non-zero sum game theory. Then I review an alternative set of discount rate for the public sector that has been suggested and construct the criteria of discount rate in a developing country where the inoptimal market savings are characterized. Finally although the choice of the social rate of discount is an unavoidable indeterminacy, I propose that the choice of that rate depend on one country's savings condition, and that the social rate of time preference(call it the economy's real growth rate) be appropriate discount rate in Korea.

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〈요 약〉

사회편익비용분석에 이용되는 변수들 중에서 학자들간에 가장 많이 논쟁이 되고 있는 것은 할인율 결정문제이다. 얼핏 생각하면 시장이자율이 할인율의 대리변수가 될 수 있을 것 같지만 자본시장의 불완전성, 외부효과등 많은 현실적인 왜곡요인 때문에 시장이자율 사용은 불가능하다. 결국, 할인율 결정은 임의성을 배제할 수 없다.

본 논문에서는 사회저축수준과 할인율 결정문제에 대하여 논하였다. 게임이론을 이용하여 사회구성원의 저축행위가 적정사회저축이 될 수 없음을 보이고 여기서 사회할인율이 시장이자율과 같지 않음을 살펴보았다. 그리고, 할인율 결정문제에 있어서는, 저축수준이나 자본집약도가 상대적으로 낮은 나라의 경우에는 간접적으로 저축이나 자본의 축적을 증대시키는 방향에서 할인율은 시장이자율보다 낮게 결정되어야 함을 보였다. 비록, 정확한 할인율을 선택할 수 없더라도 그 나라의 저축수준, 자본집약도 수준에 따라 그것의 기준이 설정되어야 한다. 그리고 현재 우리나라에서 제시되고 있는 12—14%의 자본의 잠재비용은 너무 높은 수치임을 밝혔다.

I. Introduction

Economists thoroughly understand that the social rate of discount should be measured. They also agree on the components that should be considered in making up this figure. Furthermore, economists are quite generally in accord on the view that a very serious misallocation of resources can result from the

use of an incorrect estimate of the value of this variable in a cost-benefit calculation.

There is, however, substantial divergence of views in the determination of the social rate of discount. We can conveniently divide the recent literature on discount rates for the public sector in two parts. The first group of papers construct and elaborate criteria applicable in the public sector when the social opportunity cost(SOC) diverges from the social rate of time preference(STP). In fact, such papers take the divergence factor as their starting point. (1) In contrast, the contribution of the second group of papers is that they develop models with property so that the SOC/STP divergence can be shown to characterize the (second-best) social optimum. Of course such divergence does not necessarily imply that the return to invest in the public and private sector should differ. (2)

It will be argued that most of the literature of constraints and assumptions generating such divergences are not realistically descriptive of a developing country where distorted capital markets prevail. Thus by considering the reality of the Korean economy, I hope to construct criteria that are applicable in the public sector when SOC diverges from STP.

Section II theoretically introduces the effect of individual and collective saving behavior on the discount rate, using the non-zero sum game theory. Section III practically reviews an alternative set of discount rates for the public sector that has been suggested and draws attention to the social rate of discount in a developing country where the inoptimal market savings are characterized. Finally, although the choice of the social rate of discount is an unavoidable indeterminacy, I shall propose that the choice of that rate depends on one's saving condition.

II. Saving Behavior

In a market economy where there are no distortions there will be just one rate of interest which will be optimal both from a private and social point of view. However, even if we suppose the perfect capital market rate of interest to be uniquely determined, there may be other reasions for rejecting the rate of private discount, i.e. the market rate of interest as an expression of the social rate of time preference. (3) In reality there are many distortions in a market. Thus we can consider the divergence of these rate as a general phenomenon. But the pre-acknowledgement of the those divergence means that the criteria proposed involve recommending policies which do not go to the heart of the matter! In this chapter I briefly review the alternative method (two person non-zero sum game known as the "prisoners' dillemma") to show the divergence between the social rate of discount and the private rate of discount.

1. The Isolation Paradox

Consider a community of N individuals, each of whom must do one and only one of two alternatives, X or Y. Let the preference ordering of each individual satisfy the two following conditions: (i) given the set of actions of the others, the indiuidual is better off X than Y; and (ii) given the choice between everyone doing X or everyone doing Y, each individual prefers the latter to the former.

Given the two features noted for the

(1) This literature is associated particularly with Marglin (1963) and Feldstein (1974) but does not originate with them. This approach has been extensively expounded and reviewed in the burgeoning literature of cost-benefit analysis and its applications e.g. Mishan (1982), Dasgupta and Pearce (1972), Layard (1972).
(2) Arrow (1966) and Sando and Drèze (1971) have developed models in which, at the optimum, returns are not equated in the public and private sectors.
(3) See Mishan(1982), Baumol(1968), and Maglin(1963).

preference pattern, certain results follow immediately.

(a) Pareto-inferior Outcome:

Each individual prefers to do X rather than Y. So the outcome will be regarded as worse by each than the alternative by all. Thus the outcome is pareto-inferior.

(b) Strict dominance of individual strategy:

Irrespective of each person's expectations of the other's action, each prefer to do X, i.e. the strategy of doing X strictly dominates over the alternative.

(c) Need for enforcement:

Everyone would like the others to do Y, while he himself does X, so that even if a contract (doing Y) is arrived at, it will be in the interest of each to break it, i.e. the problem of free-rider occurs.

In the special case when there are only two individuals, the above corresponds exactly to the game of the prisoners' dilemma. But we shall stick to the N-person version, and call it the "isolation paradox."[4]

The saving problem is only a special application of this. Suppose Y stands for the policy of saving one more unit for the sake of the future of the community, and X for not doing it. Given the action of all others, each individual is better off not doing the additional unit of saving himself. Hence nobody will, but every one would have preferred one more unit of saving by each rather than by none.[5]

2. Optimum savings and the rate of discount

Consider now the following ordering of individual i. He attaches a weight of unity to his consumption today(set the numeraire), β per unit to the consumption of his contemporaries, α per unit to the consumption of the future generation.[6] The marginal rate of return on one unit of saving(i.e. one unit less of consumption) today is an increase in the consumption in the future by k units, where $k>1$.

The individual figures given for the actions of other people today, the net gain $G(i)$ from one unit more of personal saving is as follows. If $G(i)>0$, the individual i will clearly save the extra unit, i.e. do Y. However, when we start with the amount of savings on which each has already made a decision, and then consider the extra unit to be a tiny bit more, G clearly cannot be positive or they would not have been in atomistic equilibrium:

$$G(i)=\alpha k-1 \tag{1}$$

Making the usual assumptions about well-behaved and continuously differentiable functions, we shall indeed find that in the atomistic equilibrium, $G=0$ for every individual, which amounts to:

$$\alpha k=1 \tag{2}$$

Now, consider the possibility of a social contract of everyone (N in all) saving one more tiny unit, so that as evaluated by any individual the consumption of oneself and one's immediate loss is $1+(N-1)\beta$, attaching the appropriate weights from the contemporaries. This has to be set against the gain in the future. The net gain $G(s)$ from the social contract, will be of the general from:

$$G(s)=\alpha kN-1-(N-1)\beta \tag{3}$$

We can now examine what conditions have to be satisfied for the isolation paradox to hold. Note that (2) indicates the weak from of the preference relations (a) i.e. people in isolation from the others do not want to save more than they are already saving. For

(4) Due to A.W. Tucker; also see, M. Shubik(1964) and A.K. Sen(1967).

(5) It will argued that this collective and individual saving behavior is based on the assumption of a situation of the type of the isolation paradox. But my argument is based on this situation. See A.K. Sen(1967) and Maglin (1963).

(6) We can divide the future generation in two parts, i.e., one's heir and others in the future generation. But when N is large, it may be unnecessary to divide them.

condition (b) to hold, we need $G(s)>0$, i.e. everyone prefers Y (saving) by each rather than X (not saving) by each:

$$\alpha kN>1+(N-1)\beta \qquad (4)$$

When (4) is consistent with (2), we have the isolation paradox holding.

The consequence of these conditions can be checked immediately. Then form $k=\frac{1}{\alpha}$ and condition(4) reduces to $\beta<1$. The result means that I value my consumption more than that of my other contemporaries.

To derive the formula for the social rate of discount and the private rate of discount, we have to specify the symmetrical sharing of the general pool. The symmetrical assumption is the following: the results of this generation's savings equally goes to each set of the future generation, out of the total of which each set again gets $1/N$ portion.

With this symmetrical sharing of the general pool the social rate of discount(ρ) is given by exactly the same formula as that of Sen

$$\rho=\frac{1+(N-1)\beta}{\alpha N}-1$$

And the private rate of discount(r) is[7]

$$r=\frac{1}{\alpha}-1$$

And $\rho \gtreqless r$ according as

$$\frac{1+(N-1)\beta}{\alpha N} \gtreqless \frac{1}{\alpha}\ [8] \qquad (5)$$

However, there is nothing at all in the market mechanism to guarantee that we shall indeed have one of these critical pairs holding. Of course, it can happen, but it will be an accidental outcome. And this is dual for the problem of the optimum rate of saving, and $\beta<1$ can be seen to be sufficient for the social rate to be below the private raet of discount.[9]

Ⅲ. The Appropriate Discount Rate.

Previous chapter I reviewed the social rate in relation to the market rate of interest. In this chapter I introduce the appropriate discount rate which has attracted the attention of literally hundreds of economists over the past 30 years. I will review the pro and con of each discount rate, attempting to show how to measure the social optimum discount rate.

1. The Before-tax Rate of Return on Private Investment.

The logic for using the gross rate of return on a private investment is straightforward. Assume that the goverment investment project does not affect the overall portion of national output devoted to investment and cosumption. If this is so, one Won of public investment exactly displaces one Won of private investment and the benefit-cost problem is simply that of achieving the optimal allocation of this fixed pool of resources devoted to investment.

Since the before-tax rate of return measures the marginal productivity of private capital, it also represents the opportunity cost of having these same funds devoted to public investment.[10] The problem with this argument is that it is often hard to view public investment projects as mechanically displacing the corresponding Won volume in private investment projects. In the first place, the same investment opportunities may not be available for the public and private investors. Second, over time public and private projects may yield different streams of secondary private investment. Third, many public investment projects may ultimately be financed by

(7) Due to the condition $\alpha k=1$.

(8) When N is large, then condition (5) can reduce: $\frac{\beta}{\alpha} \gtreqless \frac{1}{\alpha}$.

(9) There are some controversies regarding the social rate being below the private rate of discount. See Baumol (1968), Mishan(1982), and Marglin(1963). But a proportion in the values of α would leave the fulfilment of the inequality entirely unchanged. See also Sen(1967).

(10) The best statement of this position is by W.J. Baumol. See W.J. Baumol(1968).

taxes that reduce private consumption. Given all these difficulties, the before-tax private market rate can not be viewed as the oppotunity cost of public sector capital.

2. The Weighted-average Rate of Return

A second approach to discounting argues that since funds come from both private investment and private consumption, a weighted-average approach must be used.(11) For that portion of the public investment financed by reducing private investment, the before-tax return on private investment gives the appropriate marginal opportunity cost. But for that portion financed out of private consumption the after-tax return on consumers saving gives the relevant marginal cost. This is, after all, the rate actually faced by consumers as they allocate their own budgets between cosumption now and consumption later. The logical defense of this standard is due to the different shadow price on the marginal output in the capital market.

But taxes and deposit-rate restriction would be strong enough to ensure that savers would not even be compensated for the effect of inflation on their savings, and they would receive and after-tax rate of return for below the before-tax investment interest.(12) Thus although it sounds sensible to use a weight-average rate, the presence of market imperfections such as financial regulations and taxes greatly diminishes the attractiveness of after-tax rate of return to savers, or any average based on it, as a standard for discounting.

3. The Optimum Social Discount Rate

One way to find the social rate of discount follows a suggestion of Stephen Marglin(13) that the discount rate be inferred from a country's optimal combinations of investment and saving. It might seem that this approach is not very practical, but in fact economic growth theorists such as Edmund Phelps have developed very simple models that determine these optimal saving and investment policies. The model can also be used to find the optimal discount rate to be used for public investment. Even though the models are very simple, the implied discounting rule turns out to give a reasonably sensible estimate of what a social discount rate ought to be.

Imagine an economy that conforms to all the neoclassical assumptions. In particular output per work, y, is a well behaved function, $y=f(k)$, of capital per work, k, the labor force, L, grows at a constant exogeneous rate n; the saving(and investment) rate is constant, and for simplicity, we assume that there is no technical progress or depreciation of the capital stock. This is shown in the Figure 1. Here, the vertical axis represent the level of per capita output, denoted by y, and the horizontal axis the stock of capital per capita, k(sometimes known as the capital/labor ratio or rate of capital intensity for a society), We can find the equilibrium capital intensity for any economy by comparing the two bottom curves. The first, called the national saving curve, shows the amount of output devoted to new capital formation, this curve can be found simply by multiplying the output curve y by the proposition s, as shown in the diagram.

The difference between total output and the requirement curve is devoted to current consumption or the living standard of the population. The optimum saving rate can be found to be the one that maximizes the distance between output(y) and capital

(11) This approach was first suggested by J. Krutilla and Otto Eckstein(1958), and R. A. Musgrave(1969)
(12) Indeed, in Korea there are many restrictions on private capital market for investment, implying that the observed before-tax rate will also not be determined in the market.
(13) See Phelps(1961) and Maglin(1963).

requirements (nk). At this capital intensity, denoted on the graph as $\overset{*}{k}$ which reprents an equilibrium growth path, consumption per head is a maximum for all generations. This society has chosen that saving rate $\overset{*}{s}$ (the one interecting the capital requirement curve at $\overset{*}{k}$ in Figure 1), that yields the highest path, *recognizing that consumption per head will* grow at the same rate n along all paths. If this society saved less (s_1), both output and consumption per head would be less. If this society save more (s_2), output per head would be greather but consumption per head would be less.

The relevance of this optimal saving and investment lesson for the evaluation of a public investment project can be drawn directly in terms of amount saved and invested, or *indirectly in terms of discount rates, and one* could also argue that public investment should be discounted not by the actual high rate of interest r, but by the lower socially optimal rate, n, and vice versa. The optimal saving policy implicit in this analysis is sometimes called the Golden Rule of capital accumulation-each generation is doing as others(saving that proportion of national output) as it would have others do unto it.

What is the social rate of discount for public expenditure? The slope of the production function equals the marginal product of capital (dy/dk), which is the private market interest rate r.[14]

Furthermore, the requirements slope is nk/k or simply, the economy's growth rate. Thus the optimal saving condition is simply that society should accumulate capital until the markets real interest rate, r, equals the economy's real growth rate, n. If an economy is undersaving, say at s_1 an k_1, capital is scarer than it should be, $r>n$, and there would be consmption benefits to all future generations in devoting more present output to investment. Fiscal and monetary policy should be designed to encourage more saving and investment in this economy. In Korea, we have seen that n has average about 7.6 percent since 1960, just below the before-tax real investment rate(r) of about 13 percent.[15] Albeit that Korea has kept the vast amount of foreign savings, Korea seems to have been undersaving to a high degree in recent years.

This growth model has made many simplifying assumptions and one must proceed with great care in applying its results to the discounting of public investment. Alternative technology, population, and labor-force growth rates could alter its results. But despite these drawbacks, it does help us make decisions on *many difficult matters. Unlike all other rules.* it gives us a standard for discounting that is independent of the possibly inappropriate current fiscal and regulatory policies affecting r. It also shows how a society could save too much as well as too little, and it works this right into the discounting rule.

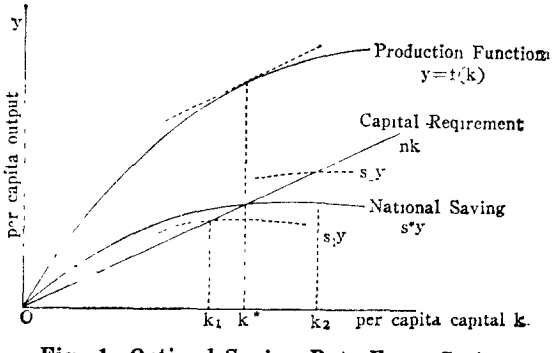

Fig. 1. *Optimal Saving Rate For a Society*

(14) If private investors are rational and capital markets are reasonably competitive, they will invest to the point where this marginal product of capital equals the oppotunity cost of capital.

(15) Since the before-tax real interest rate (r) cannot be determmined in the Korean market, we cannot but use a proxy. This rate of return here refers to the ratio of all non-labor value added to total capital stock. So. strictly speaking, it is not a rate of return to purely physical capital, but an indicator of how much a unit of capital could earn in Korea. See Koo Bohn Young(1978)

Ⅳ. Conclusion

There is a close relationship between the problem of optimal social rate of discount and the market saving. In section Ⅱ some problems concerning individual and social saving behaviors were studied. Here we concluded that the private rate of discount is not any longer so easy to identify the correct rate of discount to use in the public sector.

In Section Ⅲ, introducing the Phelps' golden capital accumulation model, we found the optimal discount rate to be used for public investment. Even though the model is simple, the implied discounting rule turns out to give a reasonably sensible estimate of what a social discount rate ought to be. In this model we can obtain the prime discount rate, e.g. social rate of time preference(call it n) in our country where savings and the capital-labor ratio is relatively low. To some extent, we can differentiate this rate among the public projects.[16]

In order to concentrate solely on the saving issue, I have ignored several other problems: the open economy assumption, imbalance or other distortions (tax, risk etc) on the capital market. But, whether one adopts these constraints or not, any choice of the social rate of discount will to some extent be arbitrary.[17]

There was an attempt to estimate the shadow price of capital in Korea.[18] Koo suggested 12—14%. Considering the undersaving situation in Korea, I think it is overestimated.

I believe that the cost-benefit analysis arrive at theoretically more well-founded and also empirically realistic studies in regard to the future in Korea.

References

K. J. Arrow, "Discounting and Public Investment Criteria" in A. V. Kneese and S. C. Smith, eds. ***Water Research***, Baltimore, 1966.

William J. Baumol, "On the Social Rate of Discount," ***American Economic Review***, September, 1968.

D. D. Ramsey, "On the Social Rate of Discount," ***American Economic Review***, December, 1969.

David F. Bradford, "Constraints on Government Investment Opportunities and the Choice of Discount Rate," ***American Economic Review***, December, 1975.

Dasgupta and Pearce, ***Cost Benefit Analysis***, London, 1972.

Diamond, "The Oppotunity Costs of Public Investment: comment." ***Quarterly Journal of Economics***, Vol. 82, 1968.

Martin S. Feldstin, "The Inadequacy of Weighted Discount Rates," in Richard Layard, ed., ***Cost-Benefit Analysis***, New York: Penguin, 1972.

——— "Financing in the Evaluation of Public Expenditure," in ***Essays in Public Finance and Stabilisation Policy***, ed. W. Smith, Amsterdam, 1974.

J. S. Flemmings, "What discount rate for public expenditure", in Michael Posner, ed., ***Public Expenditure***, Cambridge Univ. Press, 1977.

Edward M. Gramlich, ***Benefit Cost Analysis of Government Programs***, Prentice-Hall, 1981.

Kare P. Hagen, "Optimal Shadow Prices and Discount Rates for Budget-Constrained Public

(16) The differentials in the rate of return among sectors would persist for the following reasons: 1) risks and uncertainty are different; 2) market structure(degree of monopoilstic control) differs; 3) growth rate of demand would differ among sectors.
(17) See Baumol (1968)
(18) See Koo Bohn Young (1978)

Firms," *Journal of public Economics*, 22, 1983.

Per-Olov Johansson, "Cost-Benefit Rules in General Disequilibrium," *Journal of Public Economic*, 18, 1982.

Harry G. Johnson, "Money in a Neo-classical One Sector Growth Model," in R. W. clower, ed. *Monetary Theory*. New York: penguin, 1969.

Koo Bohn Young, *"An Estimate of Shadow Price Parameters in Korea*, Korea Development Institute, January, 1978.

John Krutilla and Otto Eckstein, *Mutiple Purpose River Development*, Baltimore, Md.: Johns Hopkins University Press, 1958.

Layard, *Cost-Benefit Analysis*, ed, New York: Penguin, 1972.

Luce, R.D, and Raitta, H., *Games and Decision*, Wiley, 1958.

Marglin, "The Social Rate of Discount and the Optimal Rate of Investment," *Quarterly Journal of Economics*, February, 1963.

Raymond F. Mikesell, *"The Rate of Discount for Evaluating Public Projects*, American Enterprise Institute for Public Policy Research, Washington, D.C., 1977.

Mishan, *Cost Berefit Analysis*, 3rd ed. London., 1982.

Richard, A. Musgrave, "Cost-Benefit Analysis and the Theory of public Finance," *Journal of Economic Literature*, September, 1969.

Phelps, "The Golden Rule of Accumulation: A Fable for Growthmen," *American Economic Review*, September, 1961.

A.C. Pigou, *The Economic of Welfare*, New York: MaCmillan, 1932.

A.R. Prest and R. Turvey, "Cost-Benefit Analysis: A. Survey," *Economic Journal*, 1965.

Agnar Sando & Jacques H, Dreze, "Discount Rates for Public Investment in Closed and Open Economies. *Economica*, November, 1971.

Larry A. Sjaastad & Daniel L. Wisecarver, "The Social Cost of Public Finance." *Journal of political Economy*, June, 1977.

A.K. Sen, "On Optimising the Rate of Saving," *Economica*, Vol. 58, 1971.

A.K. Sen, "Isolation, Assurance and the Social Rate of Discount," *Quarterly Journal of Economics*, Vol. 81, 1967.

James Tobin, "Economic Growth as an Objective of Government Policy," *American Economic Review*, May, 1964.

Tullock, G. "The Social Rate of Discount and the Optimal Rate of Investment; comment, *Quartely Journal of Economics*, Vol. 1978.