UIT Report Vol.15, No.1, pp.19~22, 1984 연구논문집 제15권 제1호 pp.19~22, 1984.

An Optimal Inspection and Preventive Replacement Policy

Gong Moung-Bock
Dept. of Industrial Engineering
(Received September 30, 1983)

〈Abstract〉

This paper presents a new maintenance policy for a preparedness system in which a failure is detected by inspection only. The policy treated is that replace upon detection of failure or at time x_n as a preventive maintenance.

최적 검사 및 예방교체 정책

공 명 복
산업공학과
(1983. 9.30 접수)

〈요 약〉

본 논문은 고장을 검사에 의해서만 탐지할 수 있는 준비태세 시스템의 새로운 정비정책을 제시하고 있다. 정책은 고장을 발견즉시 교체하거나 예방정비로서 시간 x_n에 교체하는 것이다.

I. Introduction

The problem of inspection and preventive replacement often arises in connection with equipments which are deteriorating. We assume that deteriorating is stochastic and that the failure of the equipment is known only if it is inspected; known in reliability literature as the preparedness model(4, 6). We assume further that the equipment is replaced upon detection of the failure or at time x_n as a preventive maintenance. The typical examples of this type are defensive weapons, drugs stored for use during epidemics and production machines. This paper concerns with the problem of determining the optimal inspection times and preventive replacement time. There are three costs involved: (1) each inspection entails a cost c_1; (2) the time elapsed between equipment failure and its discovery at the next inspection or its preventive replacement results a cost c_2 per unit of time; (3) each replacement incurs a cost c_3.

Barlow et al. (1,2,8) discussed an optimal inspection policy for the minimum expected loss in one life cycle of an equipment while taking into account the cost of an inspection and the cost per unit of time elapsing between equipment failure and its discovery at the next inspection. Brender(2,3) added a replacement cost and calculated an optimal inspection policy for the expected minimum loss per unit of time. In this paper, Brender assumed that replacement occurs upon detection of failure only. Watanapanom and Shaw(7) discussed optimal inspection policies forgoing the assumption that inspections do not degrade an equipment. Menipaz(5) considered various inspection policies with variable maintenance costs while taking into consideration a positive discounnt factor, and found various optimal inspection policies for the minimum expected loss in one life cycle of an equipment. The author suggested a truncated inspection policy where a replacement occurs at a pre-determined time x_n only.

Ⅱ. Assumptions

We make the following assumptions:

(a) failure time density function is known;

(b) equipment failure is discovered only through inspection;

(c) inspection takes negligible time and does not influence the equipment;

(d) replacement occurs upon detection of failure or at time x_n as a preventive maintenance.

Ⅲ. Model

Consider an equipment with a known failure distribution $F(t)$ having finite mean. Inspections are made at times $x_1<x_2<\cdots<x_{n-1}$ and a preventive replacement occurs at time x_n. According to standard results from the renewal theory, the expected loss per unit of time over an infinite time span, $L(X)$ is given to be

$$L(X)=\frac{E[C(X)]}{E[T(X)]},$$

where $X=(x_1,x_2,\cdots,x_n)$,

$E[C(X)]$ =expected total loss per cycle following policy X,

and

$E[T(X)]$ =expected lenght of a cycle following policy X.

Now

$$E[C(X)]=\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}\{ic_1+c_2(x_i-t)\}dF(t)-\int_{x_{n-1}}^{x_n}c_1dF(t)+c_3, \tag{1}$$

and

$$E[T(X)]=\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}(x_i-t)dF(t)+\int_{x_0}^{x_n}\{1-F(t)\}dt, \tag{2}$$

where $x_0=0$.

From (1) and (2), $L(X)$ is given by

$$L(X)=\frac{\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}\{ic_1+c_2(x_i-t)\}dF(t)-\int_{x_{n-1}}^{x_n}c_1dF(t)+c_3}{\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}(x_i-t)dF(t)+\int_{x_0}^{x_n}\{1-F(t)\}dt}. \tag{3}$$

Both the numerator and denominator of (3) are affected by the policy X. However, it is possible to find the X that minimizes (3) by considering an optimization problem having the loss function $\mathscr{L}(\alpha,X)=E[C(X)]-\alpha E[T(X)]$. These two problems are related as follows.

THEOREM 1. (2,3,7) If there exists an α such that $0<\alpha=\alpha^*<c_2$ for which $\min_X \mathscr{L}(\alpha^*,X)=\mathscr{L}(\alpha^*,X^*)=0$, then the policy X^* also minimizes (3).

The existence of such an α^* is quite evident for the present problem.

THEOREM 2. (1,2) If $f(t)$ has an increasing failure rate, then $f(t)$ is unimodal.

Now we shall prove the following important theorem.

THEOREM 3. If $f(t)$ has an increaring failure rate, then there exists an optimal policy X^* having a finite number of inspections. If $f(t)$ has a constant or a decreasing failure rate, then there exists an optimal policy X^* having an infinite number of inspections.

(Proof) Since the proof of the second statement is similar as that of the first, we shall prove the first statement only. We can write

$$\mathscr{L}(\alpha,X)=\sum_{i=1}^{n}\int_{x_{i-1}}^{x_i}\{ic_1+(c_2-\alpha)\}dF(t)-\int_{x_{n-1}}^{x_n}c_1dF(t)-\alpha\int_{x_0}^{x_n}c_1dF(t)\}dt.$$

A necessary condition for an optrmum x_n for

$\mathscr{L}(\alpha, X)$ in obtained by setting $\partial\mathscr{L}(\alpha, X)/\partial x_n = 0$, yielding

$$\frac{\partial\mathscr{L}(\alpha, X)}{\partial x_n} = (n-1)c_1 f(x_n) + (c_2-\alpha)(F(x_n) - F(x_{n-1})) - \alpha(1-F(x_n)) = 0. \quad (4)$$

Since $\lim_{n\to 0} x_n = 0$ and $\lim_{n\to\infty} x_n = \infty$, we have the followings.

$$\lim_{n\to 0}\left[\frac{\partial\mathscr{L}(\alpha, X)}{\partial x_n}\right] = -\alpha,$$

and

$$\lim_{n\to\infty}\left[\frac{\partial\mathscr{L}(\alpha, X)}{\partial x_n}\right] = 0. \quad (5)$$

We shall show that (4) is uniquely satisfied for a finite n so that (4) is necessary and sufficient for the minimum. From (4)

$$\frac{\partial^2\mathscr{L}(\alpha, X)}{\partial x_n^2} = (n-1)c_1 f'(x_n) + c_2 f(x_n), \quad (6)$$

$$\lim_{n\to 0}\left[\frac{\partial^2\mathscr{L}(\alpha, X)}{\partial x_n^2}\right] > 0,$$

and

$$\lim_{n\to\infty}\left[\frac{\partial^2\mathscr{L}(\alpha, X)}{\partial x_n^2}\right] = 0. \quad (7)$$

By THEOREM 2 and (7), (6) is unimodal, and the left of the mode is strictly decreasing from above zero and the right of the mode is strictly increasing to zero. By (5), (6) and (7), (4) is unimodal, and the left of the mode is strictly increasing from $-\alpha$ and the right of the mode is strictly decreasing to zero. The x_n satisfying(4) is the unique optimal preventive replacement time. Proof is complete.

Now we shall consider the procedure for finding X^*. A necessary condition for optima $x_1, x_2, \cdots, x_n$ obtained by setting each $\partial\mathscr{L}(\alpha, X)/\partial x_i = 0$ $(i=1, 2, \cdots, n-1)$ is given to be

$$x_{i+1} - x_i = \frac{F(x_i) - F(x_{i-1})}{f(x_i)} - \frac{c_1}{c_2-\alpha}, \quad i = 1, 2, \cdots, n-2,$$

and

$$x_n - x_{n-1} = \frac{F(x_{n-1}) - F(x_{n-2})}{f(x_{n-1})}. \quad (8)$$

By THEOREM 3, if these x_i's satisfy (4), then these are optima. From these facts, where $f(t)$ has an increasing failure rate, we suggest the following computational procedure for finding the optimal policy.

(a) For given α, choose x_1 and obtain recursively $x_2, x_3, \cdots$, from (8).

(b) Check them satisfying (4).

(c) Otherwise, change α and repeat the procedure (a) through (c) until (4) is satisfied.

IV. Concluding remarks

This paper presents a new maintenance policy for a preparedness equipment in which a failure is detected only through inspection. Brender(2, 3) investigated a failure replacement policy in a preparedness model. The policy treated in this paper is an age replacement policy in the same model. It is proved that if $f(t)$ has an increasing failure rate, then the age replacement policy is optimal, and that if $f(t)$ has a constant or a decreasing failure, then the failure replacement policy is optimal. The results in a preparedness model are same as those of a non-preparedness model.

References

1. R. Barlow, L. Hunter and F. Proschan, "Optimum Checking Procedure," J. Soc. Indust. Appl. Math., Vol. 11, No. 4, 1078—1095(1963)
2. R. Barlow and F. Proschan, Mathematical Theory of Reliability, 84—118, 227—231, John Wiely & Sons, New York(1965)
3. D. Brender, "A Surveillance Model for Recurrent Events," IBM Watson Research Center Report(1963)
4. I. Gertsbakh, Models of Preventive Maintenance, North-Holland, Amsterdam(1978)
5. E. Menipaz, "Optimization of Stochastic Maintenance Policies," European J. Oper. Res., 2, 97—106(1978)
6. Y. Sherif and Smith, "Optimal Maintenance Models for Systems Subject to Failure-A

Review," Nav. Res. Log. Q., 28, 4.—74 (1981)

7. N. Wattanapanom and Shaw, "Optimal Inspection Schedules for Failure Detection in a Model Where Tests Fasten Failures," Oper. Res., Vol.27, No.2, 303—317(1979)

8. M. Zelen, Statistical Theory of Reliability, 3—54, 75—113, Univ. Wiscosin Press, Madison(1964)