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Efficient Bargaining Model: A Dynamic Approach

Jaeki Lee
Dept. of Economics

<Abstract>

This paper analyzes union's dynamic optimizing behavior when membership changes according to the change of employment level at the previous period, and compares the results with the static case. By using a dynamic model with endogenous membership rule, we can show that the amount of 'overemployment' in the static one period model is overestimated. In addition, the robustness of overemployment result is examined showing that overemployment is not robust and it depends on the union preferences. Finally, the paper analyzes the time consistency problem in the dynamic optimizing process.

효율적 협상모형에 대한 동태적 접근

이재기
경제학과

<요 약>

이 논문은 노동조합원수가 전기의 고용수준에 의해 결정될 경우의 노동조합의 동태적 최적화 행동을 분석한다. 이 동태적인 분석에서 나타난 결과를 지금까지 주로 행해진 정태적인 분석에서의 결과와 비교함으로써 정태적인 모형의 한계를 극복하고 보다 현실적인 효율적 협상모형을 모색한다. 특히 정태적인 모형의 문제점인 과잉고용의 문제가 동태적 접근에 의하면 정태적인 모형에서 나타나는 것처럼 크지는 않다는 결과가 도출된다. 그리고 동태적 모형의 결과에 대해서 Robustness와 Time Consistency 문제를 분석한다.

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I. Introduction

It is an empirical fact that unionized sector includes a large fraction of the total labor force in European countries and dominates many key industries in the United States and Korea. In this environment, the understanding of the economic behavior in trade unions is an important issue in economic research.

The current fashion of research is concentrated on the bargaining situation in which both union and firm have some monopoly power: therefore, it is very important to find an appropriate way to model the bargaining relationship in labor market analysis. Hall and Lilien[1979], and McDonald and Solow[1981] show some interesting results about the efficient bargaining model. Their results are, however, very restrictive because they assume the membership of union is fixed. In most literature, union membership is fixed and taken as exogenously determined. For some purposes, this may not matter, but for a general theory we need to be able to explain endogenous determination of union size. Grossman[1983] suggests a way to tackle this by considering a union with seniority rule. Another problem in the previous literature is their assumptions about union preferences. They assume that union maximizes current utility or welfare function under fixed membership. This assumption is also very restrictive; so we need to relax it and to consider a more general case.

In this paper, I analyze union's dynamic optimizing behavior when membership changes according to the change of employment level at the previous period, and compare the results with static case. In addition, robustness of overemployment result is examined. The result is that overemployment is not robust and it depends on the union preferences. Finally, I examine the time consistency problem in the dynamic optimizing process.

In section II, the basic results of one period model are reviewed. In section III, a dynamic model with endogenous membership rule is analyzed and the results about'overemployment' show that static one period model is misleading. Section IV examines the robustness of overemployment result in the efficient bargaining and shows that this result can be overturned in a different type of union preference. Section V analyses the time consistency problem of dynamic union behavior under endogenous membership rule. Section VI summarizes the results and suggests the directions for future research.

II. Static One-Period Model

The equilibrium in the monopoly union model is inefficient because the outcome of the bargaining does not lie on the bargaining contract curve. This idea was studied by McDonald and Solow[1981]. Efficient bargains lie on the locus of tangency points between union indifference curves and firm's isoprofit contours.[1)]

1) It is well known that efficient bargains are a first best contract. However, there is a question about feasibility because the terms of an optimal contract will not involve setting the wage equal to

Suppose that a firm's profit function is defined as follows:

(1) $\pi(sw,n) = sf(n) - wn,$

where s is productivity shock or output price, n is employment level, w is wage rate, and $f(\cdot)$ is production function which is twice differentiable with $f'>0$, $f''<0$.

Union's utility function is defined as a utilitarian type:

(2) $U(w,n) = u(w)n + (m-n)u(r),$

where m is the number of union members, r is alternative wage rate or unemployment benefit (or value of leisure) when a worker is unemployed, and $u(\cdot)$ is twice differentiable function with $u'>0$, $u''<0$.

Efficiency requires that firm's profit function or union's utility function be maximized subject to an arbitrary level of the other. The condition for efficient contract is

(3) $[u(w)-u(r)]/u'(w) = w - sf'(n).$

Under a similar setting, McDonald and Solow[1981] shows various interesting results as follow. Among other things, the last two results are related with this paper.(The first three results still hold in a dynamic approach of this paper.)

1. The contract curve slopes upwards in wage-employment space.
   proof: $dw/dn = sf''(n)u'(w)/[w-sf'(n)]u''(w) > 0.$

2. A rise in unemployment benefit shifts the contract curve upward.
   proof: $dw/db = -u(r)/[w-sf'(n)]u''(w) > 0.$

3. An increase in the productivity shock shifts the contract curve downward.
   proof: $dw/ds = u'(w)f'(n)/[w-sf'(n)]u''(w) < 0.$

4. Equilibrium employment level is higher than that in the equivalent competitive labor market.
   proof: Add $u'(w)r$ to both sides of equation (3). Then,
   $u(w)-u(r)+u'(w)(r-w) = u'(w)[r-sf'(n)].$

The left hand side is positive by strict concavity of $U(\cdot)$. According to Figure 1, $u(w)-u(r)+u'(w)(r-w) = bc > 0$. Hence, $r > sf'(n)$, so the value of marginal product of labor is lower than the competitive reservation wage r.

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marginal revenue product for the firm. This ex post enforceability problem is outlined in Farber[1986].

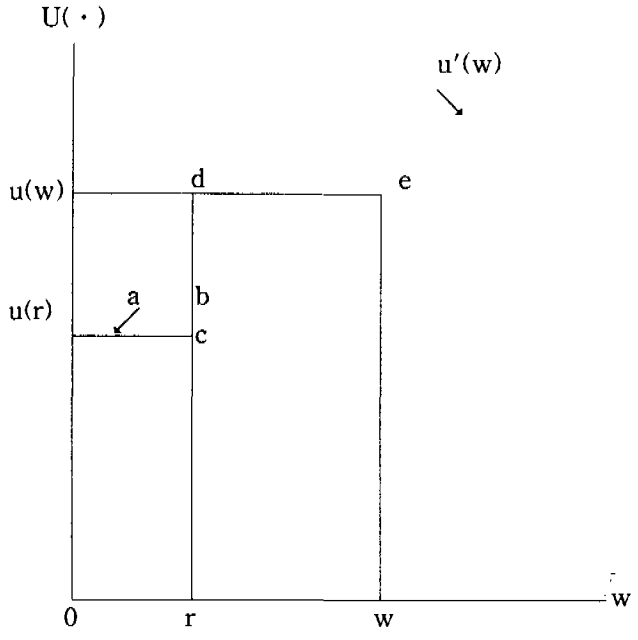

Figure 1. Concavity of u(w)

5. If the elasticity of labor demand curve is constant, and the bargaining outcome is fixed by the "fair shares" rule wn = ksf'(n), where k is the share parameter, the wage will be unaffected by changes in the price of output.
proof: Differentiate the two equations, the contract curve (3) and the sharing rule, to give

$$\begin{vmatrix} z-sf''(n) & \\ n(w-ksf'(n)) & \end{vmatrix} \begin{vmatrix} dw/ds \\ dn/ds \end{vmatrix} = \begin{vmatrix} f'(n) \\ kf(n) \end{vmatrix}$$

where z stands for $(d/dw)[w-(u(w)-u(r))/u'(w)] = ((u(w)-u(r))u''(w)/u'(w)^2$. The determinant is negative. Calculation and sustitution of the value of k from the fair share rule shows that

$$\text{sgn } dw/ds = -\text{ sgn } \{f'(n)(1 - nf'(n)/f(n)) + nf''(n)\}$$

The first term is positive and the second negative, confirming the indeterminacy of the sign of dw/ds.[2] However, if $\varepsilon = -(dn/dw)(w/n)$ is constant, namely if $f'(n)/f''(n)n$ is a constant, $dw/ds = 0$.[3]

The last result is especially simple form of the "wage stickiness" which McDonald and Solow wish to highlight.

2) Note that the sign of dw/dp does not depend explicitly on the utility function, except that it helps determine the point at which the revenue function is evaluated.

3) For details, see McDonald and Solow[1981].

An important problem in this static model is that the bargains along the contract curve are efficient only from the viewpoint of a firm and the fixed membership of a union. As Pencavel[1985] says, this fixed membership assumption is not appropriate. In many industrial unions, union membership accompanies employment and the termination of employment status also terminates union membership. We, therefore, need to extend the static model to a dynamic model with endogenously determined membership.

III. Dynamic Multi-period Model

According to Pencavel[1985], it is assumed that union membership depends on employment status. A possible way to formulate this assumption is that membership in period t depends on the employment level in period t-1.[4)]

(4) $m_t = n_{t-1}$

Union maximizes the expected value of $\sum_{t=0}^{\infty} b^t[u(w_t)n_t + (m_t-n_t)u(r)]$ where b is the discount factor ($0<b<1$) and unemployment benefits r are assumed constant over time. Firm maximizes profits in each period. (This assumption will be relaxed later.) This imposes the constraint that profits in each period be some minimum amount: then we can assume the existence of isoprofit contour which is the constraint of union's maximization problem. Without loss of generality, we can assume that firm's profit level is 0.

(1)' $s_t f(n_t) - w_t n_t = 0$

The other assumptions in static model are still effective.[5)]

We now have a complete specification of the preferences of a union and a firm, technology structure, and membership dynamics. The union solves the following problem:

(5) Max E $\{\sum_{t=0}^{\infty} b^t[u(w_t)n_t + (m_t-n_t)u(r)]\}$

subject to

(1)' $s_t f(n_t) - w_t n_t = 0$,

4) This type of membership rule is also used in Blanchard and Summer[1986] in which they show how temporary shocks can have a permanent effect on the level of employment in contexts where wages are set by currently employed members (insiders) of union.

5) If we assume that productivity shock s_t is serially correlated, e. g., s_t is assumed to follow a first order Markov process, we can explain the persistence of employment level fluctuations.

(4) $m_t = n_{t-1}$ or $m_t - m_{t-1} = n_{t-1} - m_{t-1}$.

The u and f functions have been specified so that corner solutions will be avoided, so the following first order conditions are necessary for a maximum:

(6a) $s_t f(n_t) - w_t n_t = 0$,
(6b) $m_t - n_t = n_{t-1} - m_{t-1}$,
(6c) $w_t - s_t f'(n_t) = [u(w_t)-(1-b)u(r)]/u'(w_t)$.

By comparing (2) and (6c), we can get the following result.

Proposition 1: Equilibrium employment level in dynamic model is higher than that in static model.

proof: Since $u(r) > (1-b)u(r)$, the right hand side of (6c) must be greater than the left hand side of (2). Strict concavity of production function proves the result.

This proposition shows that static efficient bargaining model is misleading: it understates the employment distortion compared to the results in the competitive market. The intuitive interpretation of this result is that, since union is maximizing the sum of utilities of present and future members, it wants to increase current employment level. Since there are imperfect insurance markets, economic agents try to make their decisions such a way to offset the effects of that imperfection. In an ideal world, each union member would like to purchase full insurance against the risk of unemployment. The possibility is assumed away here. Hence union sets employment above the level which it would desire if insurance markets are perfect. Overemployment is rational and it is the optimal way to reduce risk at the expense of technical efficiency.

Steady state equilibrium of this model can be derived from the first order conditions.

(7) $n = m$
(8) $w - sf'(n) = [u(w)-(1-b)u(r)]/u'(w)$

The following Figure 2 shows the steady state equilibrium of this model.

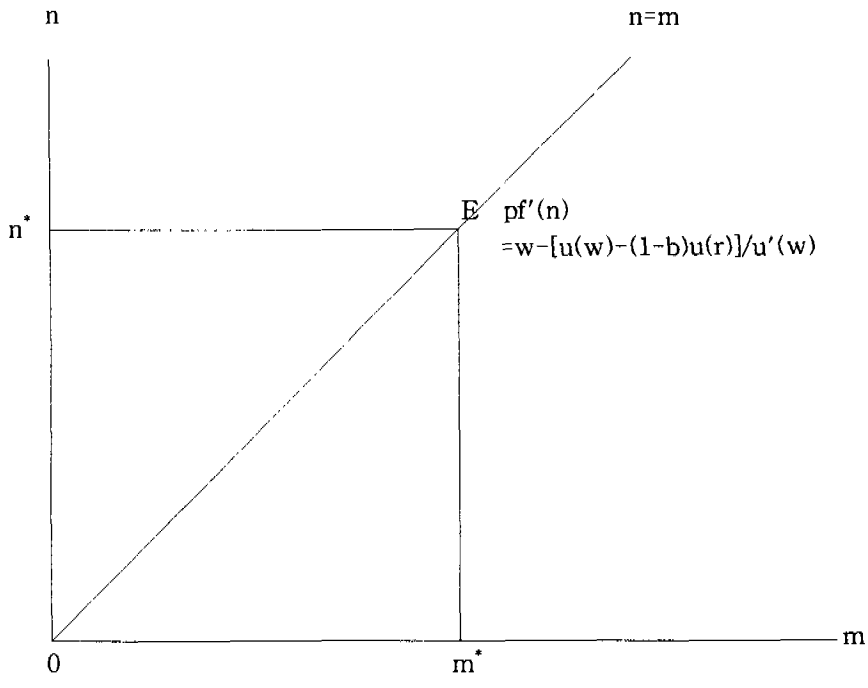

Figure 2. The Steady State Equilibrium

A different version of efficient bargaining model can be constructed by the following way. We can assume that the firm also maximizes the discounted value of future profit stream, then (1)' can be changed to the following:

(9) $E\{\sum_{t=0}^{\infty} b^t[s_t f(n_t) - w_t n_t]\} > 0.$

The first order conditions for this optimization problem are

(10) $u(w_t) - (1-b)u(r) + \lambda[s_t f'(n_t) - w_t] = 0,$
(11) $u'(w_t) - q = 0,$

where λ and q are Lagrange multipliers which do not depend on time.

Proposition 2: Wage level is fixed over time.
proof: It is obvious from equation (11).

From the proposition 2, we can get "stickiness of wage" without assuming constant elasticity of labor demand and fair share rule in McDonald and Solow[1981].

The above first order condition (10) and (11) are formally similar to the famous results in implicit contract theory. By combining (10) and (11), we can confirm that proposition 1 still holds in this case.

IV. Robustness

The 'utilitarian' type objective function is widely used and the empirical studies of Farber[1978] and Brown and Ashenfelter [1986] are based on empirical specifications that are more or less consistent with utilitarian union. Nevertheless, the utilitarian type objective function is quite restrictive because it imposes a special kind of separability. The previous overemployment result may be derived from this special assumption in union's objective function. To clarify this point, let's consider one other objective function which is popular and worth considering such that:

(12) $\phi = nu(w)/m + (m-n)u(r)/m.$

This class of union preference is plausible when the jobs are allocated randomly so that each member has the same probability of having a job.[6] In static one period model, efficient combination of wage rate and employment level under the expected utility type union objective function is equal to the previous utilitarian type case because (12) is just a monotonic transformation of (1). From equation (3), it is obvious that the most preferred wage of each worker is not affected by the size of union m, even if the expected utility of individual members is inversely related to the size of membership.

If the same dynamic procedure is applied to (12), union's optimization problem is

$$\text{Max } E\ \{\textstyle\sum_{t=0}^{\infty} b^t[n_t u(w_t)/m_t + (m_t-n_t)u(r)/m_t]\}$$

subject to (1)' and (4).

Since u and f functions are strictly concave (The maximand is also strictly concave because convex combination of strictly concave functions is also strictly concave.), the following first order conditions are necessary for a interior solution:

(13a) $[u(w_t)-u(r)]/n_{t-1} + b[u(r)n_t-[n_{t-1}u(w_{t-1}) + (n_t-n_{t-1})u(r)]]/n_t - \lambda_t[s_t f'(n_t)-w_t] = 0,$

(13b) $(n_t/n_{t-1})u'(w_t) + \lambda_t n_t = 0,$

where λ_t is a Lagrange multiplier at time t. By combining (13a) and (13b). the following equilibrium condition can be derived.

(14) $[u(w_t)-u(r)]/n_{t-1} + b[u(r)n_t-[n_{t-1}u(w_{t-1}) + (n_t-n_{t-1})u(r)]]/n_t + u'(w_t)[s_t f'(n_t)-w_t]/n_{t-1} = 0.$

6) Farber[1978] used this type of objective function, a median voter arguement, to derive the optimal mix in the compensation package as that preferred by the median aged member of the union.

The above condition is quite complicated, but the steady state equilibrium condition can be characterized as follows:

(15) $w - (1-b)[u(w)-u(r)]/u'(w) = sf'(n)$.

From equation (15), we can have the following proposition.

Proposition 3: For $0<b<1$, at steady state the employment level in dynamic model is less than that in static model if union maximizes the discounted value of expected utility.

In spite of the different assumptions in union preferences, equilibrium conditions for efficient bargains are equal in static approaches. As we see in Proposition 1 and 3, however, the results in dynamic approaches are different. The overemployment result in utilitarian union assumption is overturned if we assume a different type of union maximizing expected utility.[7]

V. Time Consistency

An Important problem attached to dynamic model is time consistency. In particular, the endogeneity of membership rule is closely related with this problem. In this paper, it is obvious that at any given time the currently employed would find it optimal to commit union or the employed group to maximizing their interests indefinitely, while ignoring the welfare of those currently laid off. That is, they would like to apply the rule $m_t = n_{t-1}$ this period and $m = m_t$ hereafter. If the currently employed decide about membership, the only time consistent rule is $m_t = n_{t-1}$, which is always the best current period rule for the currently employed. The issue is, therefore, whether the currently employed can commit the union to take care of their interest in the future whether they are still employed by the firm or not.[8] If time period is finite, time consistency problem is trivial because decision making at the last period is independent of membership rule and by recursive process we can prove decision makings of all prior periods are also independent of membership rule.

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7) The other type of objective function which brings somewhat interesting results is $U^*=wn+\int_n u(i)di$, where i is an index across workers with the lowest value representing the worker with the lowest value of leisure. This is similar to the union preference used by Hall and Lillien[1979] and Anderson and Devereux[1988]. In this case, u(i) represents the standard labor supply curve and the employment level at the cooperative efficient bargaining solution is equivalent to the competitive market solution.

8) The arguement in this section is basically equal to the argument in Blanchard and Summers[1986]. Time consistency problem in their dynamic monopoly union model is, in my opinion, completely equal to that in dynamic efficient bargaining model.

VI. Concluding Remarks

A simple static efficient bargaining model of trade union was extended to incorporate a membership rule in which the employment level in the previous period determines the size of union. I showed that the results about employment level in static model are misleading regardless of the different assumptions about union preferences.

The overemployment result in utilitarian union is not robust because the result is overturned under the assumption that union maximizes representative agent's expected utility. This result suggests that the criticism about the outcomes of efficient bargaining model - Underemployment phenomenon is what we need to explain - can be inappropriate. I also showed that in a reasonable setting, wage stickiness can be derived without assuming several impractical conditions. Finally, I briefly examined time consistency problem and showed that my model is time consistent under the membership rule $m_t = n_{t-1}$.

Future research should be directed to empirical investigation about union preferences. As we see in this paper, correct specification of union preference is very important to study wage and employment dynamics in the labor market. Another important problems are how to construct correct way to incorporate strike threats and how to specify union membership rule. Simple rules used in this paper is in its infancy.

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