

## $e^\pm, p, H$ atoms, $D$ atoms, and an $e^+$ and an $e^-$ in superfluid ${}^4\text{He}$

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### 〈Abstract〉

A model of  $e^\pm, p, H$  atoms,  $D$  atoms, and an  $e^+$  and an  $e^-$  in the liquid  ${}^4\text{He}$  is presented, here. This model is the extension of theories of Woo, Tan and Massey [and Massey, Woo and Tan], and Yim and Massey. The detailed numerical results are not presented here. However, it is reserved in the later time.

### 초유체 헬륨-4에서의 $e^\pm, p, H$ 원자들, $D$ 원자들 그리고 한개의 $e^+$ 와 한개의 $e^-$

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### 〈요 약〉

액체 헬륨에서의  $e^\pm, p, H$  원자들,  $D$  원자들, 그리고 한개의  $e^+$ 와 한개의  $e^-$ 의 모델이 여기에 발표되어진다. 이 모델은 우, 탄파 메시(그리고 메시, 우와 탄) 그리고 임파 메시의 이론들의 연장이다. 자세한 수치결과는 여기에 발표되지 않으나, 후에 발표될 예정이다.

### I. Introduction

There have been widely known experimental results and theories of  $e^\pm$  of outer surfaces of the liquid  ${}^4\text{He}$  [and also  $p$ -,  $H$ - and  $D$ - ${}^4\text{He}$  mixture].<sup>1</sup> However, there have been known few microscopic realistic theories of  $e^\pm$  [ $p, H$  atoms,  $D$  atoms, and an  $e^+$  and an  $e^-$ ] of the liquid  ${}^4\text{He}$  [i.e., the surface region and the bulk],<sup>2-4</sup> for example, the extensions of Woo, Tan and Massey<sup>2</sup> [WTM] [Massey, Woo and Tan<sup>3</sup> (MWT)] and Yim and Massey<sup>4</sup> [YM]. Here, we only present a realistic model of bulk  $e^\pm$ -,  $p$ -,  $H$  atoms-,  $D$  atoms-, and

an  $e^+$  and an  $e^-$  - ${}^4\text{He}$  mixture, the extension of theories of WTM[MWT]<sup>2,3</sup> and YM.<sup>4</sup> The detailed numerical results will be in the later paper.

This model is established by the following procedures:

(i) The properties of the liquid  ${}^4\text{He}$  are calculated. (ii) One  ${}^4\text{He}$  atom is replaced by one  $e^\pm$ , one  $p$ , one  $H$  atom, one  $D$  atom, and an  $e^+$  and an  $e^-$ . (iii) Now  $N_3$   ${}^4\text{He}$  atoms are replaced by  $N_3$   $e^\pm$ ,  $N_3$   $p$ ,  $N_3$   $H$  atoms,  $N_3$   $D$  atoms, and an  $e^+$  and an  $e^-$ .

And we construct the model-Hamiltonians. [Refer to Ref. (4) for the followings.]

## II. Theory: $e^\pm, p, H$ atoms, $D$ atoms, and an $e^+$ and an $e^-$ in the liquid ${}^4\text{He}$ -bulk

### 1. $e^\pm$ and $p$ in the liquid ${}^4\text{He}$

Pure liquid  ${}^4\text{He}$  is defined by the Hamiltonian

$$H = -\frac{\hbar^2}{2m_4} \sum_{i=1}^N \nabla_i^2 + \sum_{i < j=1}^N V(r_{ij}), \quad (1)$$

where  $N$  is the total number of  ${}^4\text{He}$  atoms, and

$$V(r) = 4\epsilon_0 \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right] \quad (2)$$

with  $\epsilon_0 = 10.22^\circ\text{K}$  and  $\sigma = 2.556\text{\AA}$ . When one  ${}^4\text{He}$  is replaced by one  $e^\pm$ , the two-body potential between ( ${}^4\text{He}, {}^4\text{He}$ ) particle pair is unchanged since the  $e^\pm$  is distributed homogeneously. Therefore, for the input information the properties of the liquid  ${}^4\text{He}$  either by Massey, Mcmillan and Schiff and Verlet [SV]<sup>5</sup> or in calculating in YM model<sup>4</sup> can be used.

The Hamiltonian of  $N_3$   $e^\pm$  in the liquid  ${}^4\text{He}$ , therefore, is defined by

$$\begin{aligned} H^m = & \sum_{i=1}^{N_3} \frac{P_i^2}{2m_e} + \sum_{i=N_3+1}^N \frac{P_i^2}{2m_4} + \sum_{i < j=1}^{N_3} V_{e^\pm - e^\pm}(r_{ij}) \\ & + \sum_{i=N_3+1}^N \sum_{i=1}^{N_3} V_{\text{He} - e^\pm}(r_{ii}) + \sum_{i < m=N_3+1}^N V(r_{im}), \end{aligned} \quad (3)$$

where  $m_e$  and  $m_4$  are the masses of an  $e^\pm$  and a  ${}^4\text{He}$  atom, respectively, coordinates 1 to  $N_3$  denote the  $e^\pm$  and  $N_3$  to  $N$  the  ${}^4\text{He}$  atoms,  $V(r)$  is defined by Eq. (2) and

$$V_{\text{He} - e^\pm}(r) = 4\epsilon_1 \left[ \left( \frac{\sigma_1}{r} \right)^{12} - \left( \frac{\sigma_1}{r} \right)^6 \right], \quad (4)$$

and

$$V_{e^\pm - e^\pm}(r) = 4\epsilon_2 \left[ \left( \frac{\sigma_2}{r} \right)^{12} + \left( \frac{\sigma_2}{r} \right)^6 \right], \quad (5)$$

where  $\epsilon_1 = 213.295^\circ\text{K}$ ,  $\sigma_1 = 1.8729\text{\AA}$ ,  $\epsilon_2 = 31957.607^\circ\text{K}$  and  $\sigma_2 = 1.30707\text{\AA}$ . We take into account the Pauli exclusion more explicitly in calculating  $V_{\text{He} - e^\pm}(r)$  and  $V_{e^\pm - e^\pm}(r)$ . Obviously,  $V_{\text{He} - e^\pm}(r)$  and  $V_{e^\pm - e^\pm}(r)$  are quite different from others.<sup>1</sup> However, we do believe ours are more realistic two-body potentials.<sup>4</sup> [Refer to the explanations outlined below Eq. (23).]

Now, the following variational procedure [the

extension of YM variational method<sup>4</sup>] is carried out as follows. We assume the properties of the liquid  ${}^4\text{He}$  are determined. We choose the trial ground-state wave function,  $\Psi_0^m$ , of  $e^\pm - {}^4\text{He}$  mixture [in this trial wave function fermions are considered bosons] as follows,

$$\begin{aligned} \Psi_0^m(\vec{r}_1, \dots, \vec{r}_{N_3}; \vec{r}_{N_3+1}, \dots, \vec{r}_N) = & \Psi_0^B(\vec{r}_1, \dots, \vec{r}_N) \\ & \times \exp \left[ \frac{1}{2} \sum_{i=1}^{N_3} \sum_{l=N_3+1}^N w(r_{il}) + \frac{1}{2} \sum_{i < j=1}^{N_3} s(r_{ij}) \right], \end{aligned} \quad (6)$$

where  $\Psi_0^B$  is the ground-state wave function for pure liquid  ${}^4\text{He}$  at the same number density as the mixture given below and  $w(r)$  and  $s(r)$  are a measure of the difference in correlations between ( ${}^4\text{He}, {}^4\text{He}$ ) and ( ${}^4\text{He}, e^\pm$ ) pairs, and between ( ${}^4\text{He}, {}^4\text{He}$ ) and ( $e^\pm, e^\pm$ ) pairs, respectively, and

$$\Psi_0^B(\vec{r}_1, \dots, \vec{r}_N) = \exp \left[ -\frac{1}{2} \sum_{i < m=1}^N u(r_{im}) \right], \quad (7)$$

where  $u(r)$  measures the correlations between two  ${}^4\text{He}$  atoms. These differences are expected to be small over a large region of space because of the first terms of Eqs. (2), (4) and (5). This should enhance the convergence of YM variational method.<sup>4</sup> Now, the pair distribution functions,  $g^{(\alpha, \beta)}(r)$ 's, and their Fourier transformations,  $F^{(\alpha, \beta)}(k)$ 's,<sup>4</sup> are defined by

$$\begin{aligned} g^{(\alpha, \beta)}(r_{i\alpha j\beta}) = & \frac{N_\alpha(N_\beta - \delta_{\alpha\beta})}{n_\alpha n_\beta} \\ & \times \frac{\int |\Psi_0^m|^2 d(\vec{r}_{i\alpha}, \vec{r}_{j\beta})}{\int |\Psi_0^m|^2 d\vec{r}_1 \dots d\vec{r}_N}, \end{aligned} \quad (8)$$

and

$$F^{(\alpha, \beta)}(k) = n \int (g^{(\alpha, \beta)}(r) - 1) e^{i\vec{k} \cdot \vec{r}} d\vec{r}, \quad (9)$$

where  $\Psi_0^m$  is the ground-state wave function for bosonic  $e^\pm - {}^4\text{He}$  mixture;  $d(\vec{r}_{i\alpha}, \vec{r}_{j\beta})$  denotes  $d\vec{r}_1 \dots d\vec{r}_N$  with  $d\vec{r}_{i\alpha} d\vec{r}_{j\beta}$  omitted;  $n_3$  and  $n_4$  are partial number densities of  $e^\pm$  and  ${}^4\text{He}$ , respectively,  $\alpha$  or  $\beta$  can be either  $e^\pm$  or  ${}^4\text{He}$ , and

$$n = n_3 + n_4. \quad (10)$$

The ground-state expectation energy of the binary boson system can be expressed in terms

of  $g^{(\alpha,\beta)}(\mathbf{r})$ 's,  $u(\mathbf{r})$ ,  $w(\mathbf{r})$  and  $s(\mathbf{r})$  as [Refer to Ref. (4) for the followings.]

$$E_0^m = \frac{\langle \Psi_0^m | H^m | \Psi_0^m \rangle}{\langle \Psi_0^m | \Psi_0^m \rangle} = \langle T^m \rangle + \langle V^m \rangle, \quad (11)$$

where

$$\begin{aligned} \langle T^m \rangle = & -\frac{N_4 \hbar^2}{8m_4} \left[ n_4 \int g^{(4,4)}(\mathbf{r}) \nabla^2 u(\mathbf{r}) d\vec{r} + n_3 \right. \\ & \times \left. \int g^{(4,3)}(\mathbf{r}) \nabla^2 (u(\mathbf{r}) + w(\mathbf{r})) d\vec{r} \right] \\ & + \left( -\frac{N_3 \hbar^2}{8m_e} \right) \left[ n_4 \int g^{(4,3)}(\mathbf{r}) \nabla^2 (u(\mathbf{r}) \right. \\ & + w(\mathbf{r})) d\vec{r} + n_3 \int g^{(3,3)}(\mathbf{r}) \nabla^2 (u(\mathbf{r}) \\ & + s(\mathbf{r})) d\vec{r} \left. \right], \quad (12) \end{aligned}$$

and

$$\begin{aligned} \langle V^m \rangle = & N_4 \left( \frac{n_4}{2} \right) \int V(\mathbf{r}) g^{(4,4)}(\mathbf{r}) d\vec{r} \\ & + N_3 n_4 \int V_{\text{He}-e^+}(\mathbf{r}) g^{(4,3)}(\mathbf{r}) d\vec{r} \\ & + N_3 \left( \frac{n_3}{2} \right) \int V_{e^+-e^+}(\mathbf{r}) g^{(3,3)}(\mathbf{r}) d\vec{r}. \quad (13) \end{aligned}$$

Using a cluster expression the pair distribution functions,  $g^{(\alpha,\beta)}(\mathbf{r})$ 's, can be expressed in terms of  $g_0(\mathbf{r})$ ,  $S_0(\mathbf{k})$ ,  $w(\mathbf{r})$  and  $s(\mathbf{r})$  as in the bosonic  ${}^3\text{He}$ - ${}^4\text{He}$  mixture<sup>4</sup>

$$g_1^{(4,4)}(\mathbf{r}) = g_0(\mathbf{r}) + x g_1^{(4,4)}(\mathbf{r}) + x^2 g_2^{(4,4)}(\mathbf{r}), \quad (14)$$

$$g_1^{(4,3)}(\mathbf{r}) = g_0^{(4,3)}(\mathbf{r}) + x g_1^{(4,3)}(\mathbf{r}) + \dots, \quad (15)$$

and

$$g_1^{(3,3)}(\mathbf{r}) = g_0^{(3,3)}(\mathbf{r}) + x g_1^{(3,3)}(\mathbf{r}) + \dots, \quad (16)$$

where  $g_0(\mathbf{r})$  and  $S_0(\mathbf{k})$  are, respectively, the pair distribution and the liquid structure functions of pure liquid  ${}^4\text{He}$ ,  $x = (N_3/N)$ , and

$$\begin{aligned} g_1^{(4,4)}(\mathbf{r}) = & \frac{g_0(\mathbf{r})}{(2\pi)^3 n} \left[ \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} \left\{ 2(S_0(\mathbf{k}) - 1) K_{\vec{k}} \right. \right. \\ & \left. \left. + K_{\vec{k}}^2 \right\} + \Delta g_1^{(4,4)}(\mathbf{r}) \right], \quad (17) \end{aligned}$$

$$\begin{aligned} g_0^{(4,3)}(\mathbf{r}) = & g_0(\mathbf{r}) e^{w(\mathbf{r})} \left[ 1 + \frac{1}{(2\pi)^3 n} \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} \left( \right. \right. \\ & \left. \left. S_0(\mathbf{k}) - 1 \right) K_{\vec{k}} + \Delta g_0^{(4,3)}(\mathbf{r}) \right], \quad (18) \end{aligned}$$

$$\begin{aligned} g_1^{(4,3)}(\mathbf{r}) = & \frac{g_0(\mathbf{r}) e^{w(\mathbf{r})}}{(2\pi)^3 n} \left[ \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} \left\{ L_{\vec{k}} (S_0(\mathbf{k}) \right. \right. \\ & \left. \left. - 1) + L_{\vec{k}} K_{\vec{k}} \right\} + \Delta g_1^{(4,3)}(\mathbf{r}) \right], \quad (19) \end{aligned}$$

$$g_0^{(3,3)}(\mathbf{r}) = g_0(\mathbf{r}) e^{s(\mathbf{r})} \left[ 1 + \frac{1}{(2\pi)^3 n} \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} \right.$$

$$\left. \times \left\{ 2(S_0(\mathbf{k}) - 1) K_{\vec{k}} + K_{\vec{k}}^2 \right\} + \Delta g_0^{(3,3)}(\mathbf{r}) \right], \quad (20)$$

$$\begin{aligned} g_1^{(3,3)}(\mathbf{r}) = & \frac{g_0(\mathbf{r}) e^{s(\mathbf{r})}}{(2\pi)^3 n} \left[ 2 \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} (S_0(\mathbf{k}) \right. \\ & \left. - 1) (L_{\vec{k}} - K_{\vec{k}}) + \int d\vec{k} e^{-i\vec{k}\cdot\vec{r}} \right. \\ & \left. \times (L_{\vec{k}}^2 - K_{\vec{k}}^2) + \Delta g_1^{(3,3)}(\mathbf{r}) \right], \quad (21) \end{aligned}$$

where the  $\Delta g_i^{(\alpha,\beta)}(\mathbf{r})$ 's are complicated expressions involving the  $n$ -particles distribution function ( $n \geq 4$ ).  $K_{\vec{k}}$  and  $L_{\vec{k}}$  are given by

$$K_{\vec{k}} = n \int d\vec{r} e^{i\vec{k}\cdot\vec{r}} g_0(\mathbf{r}) (e^{w(\mathbf{r})} - 1),$$

and

$$L_{\vec{k}} = n \int d\vec{r} e^{i\vec{k}\cdot\vec{r}} g_0(\mathbf{r}) (e^{s(\mathbf{r})} - 1). \quad (22)$$

Whenever the 3-particles distribution function enters in these cluster expansions, the Kirkwood Superposition approximation [KSA] was used. In the calculations  $w(\mathbf{r})$  and  $s(\mathbf{r})$  are chosen to be short-range functions similar to  $u(\mathbf{r})$  in pure  ${}^4\text{He}$ , i. e.,

$$w(\mathbf{r}) = -(a\sigma/r)^5,$$

and

$$s(\mathbf{r}) = -(c\sigma/r)^5, \quad (23)$$

where  $a$  and  $c$  are variational parameters. The contribution of  $g_1^{(3,3)}(\mathbf{r})$  to the total energy should be small to enhance the convergence of YM variational method.<sup>4</sup> Also, it can be justified in the previous calculations.<sup>4</sup> [For example,  $L_{\vec{k}} \sim K_{\vec{k}}$ ] This gives the terms by Pauli exclusion principle of Eqs. (4) and (5). [It is the direct consequence of YM theory 4] If we substitute the  $g^{(\alpha,\beta)}(\mathbf{r})$ 's from the cluster expressions of Eq. (14) to Eq. (16) into from Eq. (11) to Eq. (13), the ground-state energy is expressed as follows:

$$\begin{aligned} E_0^m(n, x) = & N_4 e_0(n) + N_3 e_1(n) + (N_3^2/N) e_2(n) \\ & + \mathcal{O}(N_3^3/N^2), \quad (24) \end{aligned}$$

where

$$\begin{aligned} e_0(n) = & -\frac{\hbar^2 n}{8m_4} \int g_0(\mathbf{r}) \nabla^2 u(\mathbf{r}) d\vec{r} + \\ & \frac{1}{2} n \int g_0(\mathbf{r}) V(\mathbf{r}) d\vec{r}, \quad (25) \end{aligned}$$

$$\begin{aligned}
e_1(n) = & -\hbar^2 n [1/(8m_4) + 1/(8m_e)] \\
& \times \int g_0^{(4,3)}(r) \nabla^2(u(r) + w(r)) d\vec{r} \\
& + n \int g_0^{(4,3)}(r) V_{\text{He}-e^\pm}(r) d\vec{r} - e_0(n),
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
e_2(n) = & -\frac{\hbar^2 n}{8m} \int g_0^{(3,3)}(r) \nabla^2(u(r) + s(r)) d\vec{r} \\
& + \frac{1}{2} n \int g_0^{(3,3)}(r) V_{e^\pm-e^\pm}(r) d\vec{r} \\
& + \frac{\hbar^2 n}{8m_4} \int g_1^{(4,4)}(r) \nabla^2 u(r) d\vec{r} \\
& - \frac{n}{2} \int g_1^{(4,4)}(r) V(r) d\vec{r} \\
& - \hbar^2 n \left( \frac{1}{8m_4} + \frac{1}{8m_e} \right) \int g_1^{(4,3)}(r) \nabla^2(u(r) \\
& + w(r)) d\vec{r} + n \int g_1^{(4,3)}(r) V_{\text{He}-e^\pm}(r) d\vec{r} \\
& - \frac{\hbar^2 n}{8m_4} \int g_2^{(4,4)}(r) \nabla^2 u(r) d\vec{r} \\
& + \frac{1}{2} n \int g_2^{(4,4)}(r) V(r) d\vec{r} - e_1(n); \tag{27}
\end{aligned}$$

$e_0(n)$  is the ground-state energy per particle of pure liquid  ${}^4\text{He}$  at density  $n$ . At zero concentration and zero pressure  $e_1(n)$  becomes the chemical potential of a single  $e^\pm$  and the fourth term,  $O(N_3^3/N^2)$ , is shown to be unimportant in Ref. 4. [Refer to the explanations outlined below Eq. (23).] The variational calculation is carried out in three steps: (a) The correlation function,  $u(r)$ , for pure liquid  ${}^4\text{He}$  is assumed to be known; (b) We consider the problem of an  $e^\pm$  in the liquid  ${}^4\text{He}$ . The trial wave function then reduces to

$$\begin{aligned}
\Psi_0^m(\vec{r}_1; \vec{r}_2, \dots, \vec{r}_N) = & \Psi_0^B(\vec{r}_1, \dots, \vec{r}_N) \\
& \times \exp\left[\frac{1}{2} \sum_{l=2}^N w(r_{1l})\right]. \tag{28}
\end{aligned}$$

$w(r)$ ,  $g_0^{(4,3)}(r)$ , and  $g_1^{(4,4)}(r)$  are then determined by minimizing  $e_1(n)$ ; (c) Assuming that  $w(r)$  has been fixed in step (b) we finally determine  $s(r)$ ,  $g_0^{(3,3)}(r)$ ,  $g_1^{(4,4)}(r)$  and  $g_2^{(4,4)}(r)$  by minimizing  $e_2(n)$ . Variational procedures for other systems to be given below are all same.

For  $p$ - ${}^4\text{He}$  mixture the mass of  $e^\pm$  in the Hamiltonian, Eq. (3), is replaced by  $m_p$ .

## 2. $H$ atoms and $D$ atoms in liquid ${}^4\text{He}$

The Hamiltonian of  $H$  atoms- ${}^4\text{He}$  mixture is defined by

$$\begin{aligned}
H^m = & \sum_{i=1}^{N_3} \frac{P_i^2}{2m_H} + \sum_{l=N_3+1}^N \frac{P_l^2}{2m_4} + \sum_{i < j=1}^{N_3} V_{H-H}(r_{ij}) \\
& + \sum_{i=1}^{N_3} \sum_{l=N_3+1}^N V_{H-\text{He}}(r_{il}) + \sum_{l < m=N_3+1}^N V(r_{lm}), \tag{29}
\end{aligned}$$

where  $m_H$  and  $m_4$  are the masses of a  $H$  atom, and a  ${}^4\text{He}$  atom, respectively,  $V(r)$  is given in Eq. (2) and  $V_{H-\text{He}}(r)$  and  $V_{H-H}(r)$  are given by

$$V_{H-\text{He}}(r) = 4\epsilon_1 \left[ \left( \frac{\sigma_1}{r} \right)^{12} - \left( \frac{\sigma_1}{r} \right)^6 \right], \tag{30}$$

and

$$V_{H-H}(r) = 4\epsilon_2 \left[ \left( \frac{\sigma_2}{r} \right)^{12} - \left( \frac{\sigma_2}{r} \right)^6 \right] \tag{31}$$

with  $\epsilon_1 = 35.7713^\circ \text{K}$ ,  $\sigma_1 = 2.17338\text{\AA}$ ,  $\epsilon_2 = 39.1263^\circ \text{K}$  and  $\sigma_2 = 2.24337\text{\AA}$ .

The difference between the Hamiltonian of  $H$  atoms- ${}^4\text{He}$  mixture and  $D$  atoms- ${}^4\text{He}$  mixture is only  $m_D = 2m_H$ , where  $m_D$  is the mass of a  $D$  atom. However, the former is boson mixture and the latter the fermion mixture. [The former obeys bose statistics and the latter fermi statistics.]

## 3. $e^-$ and $e^+$ in liquid ${}^4\text{He}$

Here we are interested in an  $e^+$  and an  $e^-$  in the liquid  ${}^4\text{He}$  since this system combined with Sec. (II-1) can generate the general system what we want

$$\begin{aligned}
H^m = & \frac{P_1^2}{2m_e} + \frac{P_2^2}{2m_e} + \sum_{l=3}^N \frac{P_l^2}{2m_4} + V_{e^\pm-e^\pm}(r_{12}) \\
& + \sum_{i=1}^2 \sum_{l=3}^N V_{e^\pm-\text{He}}(r_{il}) + \sum_{l < m=3}^N V(r_{lm}).
\end{aligned}$$

where  $V(r)$  is shown in Eq. (2), and

$$V_{e^\pm-\text{He}}(r) = 4\epsilon_1 \left( \frac{\sigma_1}{r} \right)^{12},$$

and

$$V_{e^\pm-e^\pm}(r) = 4\epsilon_2 \left[ \left( \frac{\sigma_2}{r} \right)^{12} - \left( \frac{\sigma_2}{r} \right)^6 \right] \tag{32}$$

with  $\epsilon_1 = 213.295^\circ \text{K}$ ,  $\sigma_1 = 1.8729\text{\AA}$ ,  $\epsilon_2 = 31957.607^\circ \text{K}$  and  $\sigma_2 = 1.30707\text{\AA}$ .

## 4. Applications of these calculations [Sec.

(II-1)] to Sec. (II-3)] to  $e^\pm, p, H$  atoms,  $D$  atoms, and  $e^+$  and  $e^-$  in liquid  ${}^4\text{He}$

We refer to Refs. (2) to (4) for the application of these models to these systems. However,  $H$  atoms- ${}^4\text{He}$  mixture should be considered to be two-components bose liquid. [Refer to Refs. (2) to (4).]

### III. Numerical results-bulk

We present only the approximate results of the chemical potential of a particle using the properties of the  ${}^3\text{He}$ - ${}^4\text{He}$  mixture.<sup>4</sup> The Hamiltonian of  $e^\pm$ ,  $p$ ,  $H$ ,  $D$ , and an  $e^+$  and an  $e^-$  can be written as

$$H^m = H_0^m + H_I^m, \quad (33)$$

where for  $e^\pm$ - ${}^4\text{He}$  mixture

$$H_0^m = \sum_{i=1}^{N_3} \frac{P_i^2}{2m_3} + \sum_{i=N_3+1}^N \frac{P_i^2}{2m_4} + \sum_{i < j=1}^N V(r_{ij}),$$

[ $m_3$  is the mass of a  ${}^3\text{He}$ .] (34)

and

$$H_I^m = \sum_{i=1}^{N_3} \left( \frac{P_i^2}{2m_e} - \frac{P_i^2}{2m_3} \right) + \sum_{i < j=1}^{N_3} (V_{e^\pm, i^\pm}(r_{ij}) - V(r_{ij})) + \sum_{i=N_3+1}^N \sum_{i=1}^{N_3} (V_{H, e^\pm}(r_{ii}) - V(r_{ii})), \quad (35)$$

for  $p$ - ${}^4\text{He}$  mixture in Eq. (35)  $m_e$  is replaced by  $m_p$ , for  $H$  atoms- ${}^4\text{He}$  mixture  $H_0^m$  is given by Eq. (34) and

$$H_I^m = \sum_{i=1}^{N_3} \left( \frac{P_i^2}{2m_H} - \frac{P_i^2}{2m_3} \right) + \sum_{i < j=1}^{N_3} (V_{H-H}(r_{ij}) - V(r_{ij})) + \sum_{i=N_3+1}^N \sum_{i=1}^{N_3} (V_{H, H}(r_{ii}) - V(r_{ii})), \quad (36)$$

and for  $D$  atoms- ${}^4\text{He}$  mixture  $H_0^m$  is also given by Eq. (34) and

$$H_I^m = \sum_{i=1}^{N_3} \left( \frac{P_i^2}{2m_D} - \frac{P_i^2}{2m_3} \right) + \sum_{i < j=1}^{N_3} (V_{H-H}(r_{ij}) - V(r_{ij})) + \sum_{i=N_3+1}^N \sum_{i=1}^{N_3} (V_{H, H}(r_{ii}) - V(r_{ii})), \quad (37)$$

and for an  $e^+$  and an  $e^-$ - ${}^4\text{He}$  mixture  $H_0^m$  is also given by Eq. (34) and

$$H_I^m = \left( \frac{P_1^2}{2m_e} - \frac{P_1^2}{2m_3} \right) + \left( \frac{P_2^2}{2m_e} - \frac{P_2^2}{2m_3} \right)$$

$$+ (V_{e^+ - e^-}(r_{12}) - V(r_{12})) + \sum_{i=1}^2 \sum_{i=3}^N (V_{e^\pm - H}(r_{ii}) - V(r_{ii})). \quad (38)$$

For these mixtures

$$E_0^m = \langle \Psi_0^m | H_0^m + H_I^m | \Psi_0^m \rangle / \langle \Psi_0^m | \Psi_0^m \rangle = \frac{\langle \Psi_0^m | H_0^m | \Psi_0^m \rangle}{\langle \Psi_0^m | \Psi_0^m \rangle} + \frac{\langle \Psi_0^m | H_I^m | \Psi_0^m \rangle}{\langle \Psi_0^m | \Psi_0^m \rangle} = N_4 e_0(n) + N_3 e_1(n) + O(N_3^2/N), \quad (39)$$

where  $\Psi_0^m$  is the ground-state wave function of mass-3 bosons-the liquid  ${}^4\text{He}$  mixture.<sup>4</sup> Using the properties of  ${}^3\text{He}$ - ${}^4\text{He}$  mixture<sup>4</sup> the chemical potentials of an  $e^\pm$  [Sec. (II-1)], a  $p$ , a  $H$  atom, a  $D$  atom and an  $e^\pm$  [Sec. [II-3]] in the liquid  ${}^4\text{He}$ ,  $\mu$ , are, respectively, (7.45eV, 8.112eV, 8.854eV), ( $-113.067^\circ\text{K}$ ,  $-108.932^\circ\text{K}$ ,  $-104.868^\circ\text{K}$ ), ( $-2.16^\circ\text{K}$ ,  $3.196^\circ\text{K}$ ,  $8.303^\circ\text{K}$ ) ( $-25.74^\circ\text{K}$ ,  $-22.4765^\circ\text{K}$ ,  $-19.714^\circ\text{K}$ ) and (7.47eV, 8.129 eV, 8.871eV), where in the brackets the firsts, the seconds and the thirds represent, respectively, at 0 atm., 4.6 atm. and 7.4 atm., and zero concentration, and

$$\mu = e_1(n) + n \frac{\partial e_0(n)}{\partial n}. \quad (40)$$

### IV. Conclusion

The present approximate result of the chemical potentials particularly for a  $H$  atom in the liquid  ${}^4\text{He}$  at zero pressure and zero concentration is in agreement with the recent experimental result by Jochemsen et. al..<sup>6</sup> [This binary boson system is stable.<sup>4</sup>] The experimental result of injecting energy of  $e^\pm$  into the bulk  ${}^4\text{He}$  at zero pressure from the vacuum is 1.2eV.<sup>1</sup> In order to obtain this result from the present results  $m_e$  in Eq. (35) is replaced by  $6m_e$  [in surface regions  $3m_e$ ].<sup>4</sup> It may be explained these many  $e^\pm$  are paired like Cooper's pairings because of the largeness of attractive phonon couplings in the bulk and also in the surface regions. Also,  $e^\pm$  may exist in the form of the quasi-classical solid-like lattice.<sup>4,7</sup> These models can be simply applied to  $e^\pm$ ,  $p$ ,

$H$  atoms,  $D$  atoms, and  $e^+$  and  $e^-$  in  $^3\text{He}$ - $^4\text{He}$  mixture, and also surface regions. All detailed numerical results are reserved in the later time.

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The author insists on ceasing the behaviours of insulting directly and mostly indirectly the papers and publications published by him or him and the other author without any knowledges, any bases and any laws, using mostly the unlawful agents employed and spoiled by persons acted in those ways outlined above and by themselves not to be able to be controlled by laws. These behaviours much effect th elife and personalities, particularly, of me toward the worst direction. These should be restricted or guilted by some kinds of true laws.

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