The EOQ Model with Permissible Delay in Payments

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(Recived September 30, 1985)

〈**Abstract**〉

This paper examined the impacts of permissible delay in payments, which was allowed by supplier, on EOQ model with price variable.

Using minimal cost criterion, it was traced that it was possible to fall down price by introduction of delay in payments. The cost function was formulated without delay and with delay, respectively. And the impacts was analyzed under the condition that the price of EOQ model without delay was prescribed.

It was assumed that demand was a function of price.

경제적 주문량 결정체계에서 지불연기의 영향에 관한 연구

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(1985. 9. 30. 접수)

〈요 약〉

본논문은 대금지불이 상품도착시점에서 상품공급자에 의해 지정된 일정한 시점까지 연기되어질 수 있을 때, 가격변수를 가진 경제적 주문량결정체계의 행태변화를 연구하였다.

비용최소화의 관점에서, 대금지불연기가 고려되는 경우와 그렇지 않은 경우에 대해 비용함수를 각각 구했으며 이로부터 체계의 최적치를 유도했다. 또한 지불연기의 영향으로 생길수 있는 판매가격의 인하가능성과 그 폭을 수리적으로 표현했다.

본논문의 연구진행은 수요가 가격의 함수라는 가정하에서 이루어졌음을 아울러 밝혀둔다.

I. Introduction

Pricing consideration of EOQ model was investigated by serveral authors(2, 4, 5, 6, 7). In retailing industries, inventory control must take into consideration pricing policies directed toward changing the demand level for a given elasticity of demand. And there are many industries where inventories are evaluated on their profit generating capability and where the interaction between inventory control and pricing policy must be considered.

Kotler(4) was one of the first to study the marketing inventory interface. However. this model does not attempt to optimize both sets of policies simultaneously. Rather, an optimal marketing strategy is derived first by finding the price that maximizes revenue for a given demand curve. The resulting demand and(or) price levels are then used as parameters for the EOQ model. Attempt to incorporate demand and(or) price as endogenous variables has been developed in terms of profit maximization framework(2, 5) and return on investment(ROI) (6, 7). However, these models still treat price and demand as parameters.

Arcelus and Srinivasan(1) attempted to present a model where marketing, inventory and accounting objectives were considered simultaneously within a ROI maximization framework. In this model, the order quantity and the price were considered as decision variables, and it was assumed that the demand was a function of price.

This paper presents other form of demand function which depends on price, and which are more general than the model of Arcelus and Srinivansan presented. With this demand function, the economic order quantity and the optimal price are derived by minimizing total annual variable cost.

And the effect of permissible delay in payments is investigated under the model proposed. Delay in payments represent postphonement of payment for replenished items to some date suplier permits.

Pecently, Goyal(3) studied EOQ model under the permissible delay in payments. He introduced new cost element named interest earned and modified the cost of interest charges for the items kept stock by the cost of interest charges after settlement of replenishment account.

But it is more reasonable to assume that the interest charges is also accumulated during accumulated during the period that the replenishment account is not settled, for the items in stock during unsettlement represent asset financed by debt and debt financed capital also requires the interest charges. Thus this paper does not modify the cost of interest charges for the items kept in stock and assume this is involved in holding cost.

We would now proceed by defining the problem.

Ⅱ. Definition of Problem

1. Problem

This paper investigates the optimal price and order quantity of EOQ model under price dependent demand function and studies the effect of delay in payments on price when delay is considered under above EOQ model.

We study the EOQ model with variable, price(PEOQ) first. And PEOQ model with delay in payments(DPEOQ) is studied. In all this procedure, price and replenishment interval(or order quantity, equivalently) are considered as decision variables and delay in payments is treated as parameter.

2. Assumptions

a. The demand for items is a function of price.

b. Shortaiges are not allowed.

c. During the tme the account is not settled, generated sales revenue is deposited in an interest bearing account.

d. Demand pattern for a replenishment period is uniform.

e. Time horizon is infinite.

3. Notations

D= annual demand

h=unit stock holding cost per item per year item per year inculding interest charges.

I=interest which can be earned per \$ in a year

P=unit sales price in \$

c=unit purchase price in \$

S=cost of placing one order

t=permissible delay in settleing accounts

T = time interval between successive orders (scheduling period)

Q = order quantity per replenishment

$Z(p, T)$ = total annual variable cost.

Ⅲ. Mathematical Formulation

In this section, we formulate total annual variable cost function in terms of price and time interval between successive orders.

It is evident that four cost elements, inventory holding cost, replenishment cost, sales profit and interest earned during delay, are influenced by the two decision variables. Thus the cost function is formulated as sum of these four cost elements(or three cost elements when the delay is not considered).

We formulate three kind of cost function, one for the model which exclude the possibility of delay in payments(PEOQ) and two for the model including delay in payments(DPEOQ).

Before we proceed formulation, let's remark on demand function. Demand decreases as price increases. And this phenomena can be traced by exponential function with minus exponent generally. Thus, we represent demand function as follows

$$D = z \cdot \exp(-m \cdot p)$$

where z and m are constants known.

1. Cost function of PEOQ model

Total annual variable cost consists of the following elements.

a. Replenishment cost $= S/T$.

b. Cost of stock holding.

The average stock equals $D \cdot T/2$(see figure 1) hence stock holding cost per year is given by

$$D \cdot T \cdot h/2 = z \cdot \exp(-m \cdot p) \cdot T \cdot h/2.$$

c. Sales profit per year is given by

$$D \cdot (p-c) = z \cdot \exp(-m \cdot p) \cdot (p-c).$$

Thus, total annual variable cost is given by

$$Z(p, T) = S/T + z \cdot \exp(-m \cdot p) \cdot T \cdot h/2 - z \cdot \exp(-m \cdot p) \cdot (p-c) \quad (1)$$

2. Cost function of DPEOQ model

When delay in payments is considered, cost function has one additional cost element which are the interest earned during the permissible settlement period. And the interest earned has two different form, one for $T \geq t$ and one for $T < t$.

When $T \geq t$,

the interest earned in one cycle $= D \cdot p \cdot t^2 \cdot I/2$. (shown in figure 1)

the interest earned in one year

$$= D \cdot p \cdot t^2 \cdot I/(2T)$$
$$= z \cdot \exp(-m \cdot p) \cdot p \cdot t^2 \cdot I/(2T)$$

Thus, total annual variable cost is given by

$$Z(p, T) = S/T + z \cdot \exp(-m \cdot p) \cdot T \cdot h/2 - z \cdot \exp(-m \cdot p) \cdot (p-c) - z \cdot \exp(-m \cdot p) \cdot p \cdot t^2 \cdot I/(2T) \quad (2)$$

When $T < t$,

the interest earned in one cycle

$$= (D \cdot T^2 \cdot p/2 + D \cdot T \cdot p \cdot (t-T)) \cdot I$$
$$= D \cdot T \cdot p \cdot I \cdot (t - T/2)$$

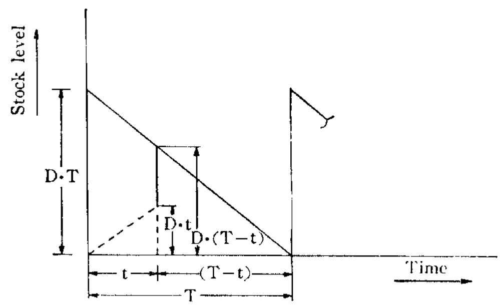

Fig. 1. Time-weighted inventory when $T \geq t$.

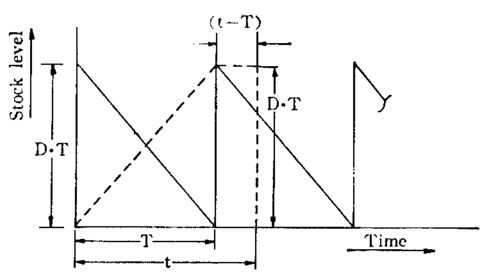

Fig. 2. Time-weighted inventory when $T \leq t$

(shown in figure 2).

the interest earned in one year

$$=D\cdot p\cdot I\cdot (t-T/2)$$
$$=z\cdot \exp(-m\cdot p)\cdot p\cdot I\cdot (t-T/2)$$

Thus, the total annual variable cost is given by

$$Z(p, T)=S/T+z\cdot \exp(-m\cdot p)\cdot T\cdot h/2-z\cdot \exp(-m\cdot p)\cdot (p-c)-z\cdot \exp(-m\cdot p)\cdot p\cdot I\cdot (t-T/2) \quad (3)$$

Ⅳ. Derivation of Optimal Solution

1. PEOQ model

From (1).

$$Z(p, T)=S/T+z\cdot \exp(-m\cdot p)\cdot T\cdot h/2 -z\cdot \exp(-m\cdot p)\cdot (p-c)$$

And

$$\partial Z(p, T)/\partial T=-S/T^2+z\cdot \exp(-m\cdot p)\cdot h/2 \quad (4)$$

$$\partial Z(p, T)/\partial p=z\cdot \exp(-m\cdot p)\cdot(-m\cdot T\cdot h/2 +m\cdot (p-c)-1) \quad (5)$$

It is not easy-to obtain the general optimum which minimizes total anual variable cost in terms of two decision variables. But if the value of one decision variable is prescribed, the optimal value of other decision variable can be easily obtained from(4) and (5) as follows

$$T=\sqrt{2S/(z\cdot \exp(-m\cdot p)\cdot h)} \quad (6)$$

$$p=(m\cdot T\cdot h+2)/(2m)+c \quad (7)$$

From (6) and (7), we can observe the fact that price increases linearly as replenishment interval increases.

2. DPEOQ model

(a) Case 1: $T\geq t$

From (2).

$$Z(p, T)=S/T+z\cdot \exp(-m\cdot p)\cdot T\cdot h/2 -z\cdot \exp(-m\cdot p)\cdot (p-c) -z\cdot \exp(-m\cdot p)\cdot p\cdot t^2\cdot I/(2T)$$

And

$$\partial Z(p, T)/\partial T=-S/T^2+z\cdot \exp(-m\cdot p)\cdot h/2 +z\cdot \exp(-m\cdot p)\cdot p\cdot t^2\cdot I/(2T^2) \quad (9)$$

$$\partial Z(p, T)/\partial p=z\cdot \exp(-m\cdot p)\cdot (-m\cdot T\cdot h/2 +m\cdot(p-c)-1+t^2\cdot I(m\cdot p-1)/(2T)) \quad (10)$$

For given value of one decision variabie, the optimum of the other variable can be obtained from (9) and (10) as follows

$$T=\sqrt{(2S-z\cdot\exp(-m\cdot p)\cdot p\cdot t^2\cdot I)/ z\cdot\exp(-m\cdot p)\cdot h)} \quad (11)$$

$$p=((m\cdot T\cdot h+2)/2+t^2\cdot I/(2T)+m\cdot c)/ (m\cdot(1+t^2\cdot I/(2T)))) \quad (12)$$

It may be pointed out that the effect of use of money is to decrease the replenishment interval by the term,

$$z\cdot \exp(-m\cdot p)\cdot p\cdot t^2\cdot I$$

(b) Case 2: $T<t$

From (3).

$$Z(p, T)=S/T+z\cdot \exp(-m\cdot p)\cdot T\cdot h/2 -z\cdot\exp(-m\cdot p)\cdot (p-c) -z\cdot\exp(-m\cdot p)\cdot p\cdot I\cdot (t-T/2)$$

And

$$\partial Z(p, T/\partial T=-S/T+z\cdot \exp(-m\cdot p)h/2 +z\cdot \exp(-m\cdot p)\cdot p\cdot I/2 \quad (13)$$

$$\partial Z(p, T)/\partial p=z\cdot \exp(-w\cdot p)\cdot (-m\cdot T\cdot h/2 +m\cdot (p-c)-1+m\cdot p\cdot I\cdot (t-T/2)-I\cdot(t-T/2)) \quad (14)$$

For a prescribed value of price or replenishment interval, the optimal value of the other one can be obtained from (13) and (14) as llows

$$T=\sqrt{(2S-z\cdot\exp(-m\cdot p)\cdot p\cdot I)/ (z\cdot\exp(-m\cdot p)\cdot h)}$$

$$p=((m\cdot T\cdot h+2)/2+I\cdot(t-T/2)+m\cdot c)/ (m\cdot c)/(m\cdot(1+I(t-T/2)))$$

As pointed out in Case 1, the scheduling period decreases by the use of money.

(c) For given price, the optimal replenishment interval is obtained by following the procedure below.

Step 1: Determine T_1 from (11).
If $T_1\geq t$. obtain $Z(p,T_1)$ from(2).

Step 2: Determine T_2 from (15).
If $T_2<t$. obtain $Z(p,T_2)$ from (3)

Step 3: If $T_1<t$ and $T_2\geq t$. then evaluate $Z(p,t)$ from (3) for $T=t$.

Step 4: Compare $Z(p, T_1)$, $Z(p, T_2)$ and $Z(p, t)$. Select the replenishment interval and the order quantity associated with the least annual cost value evaluated in step 1 and 2 or 3.

Ⅴ. Analysis of Model

In previous section, we studied the EOQ model which had two decision variables, price and replenishment interval. The emphasis was given to the mathematical formulation of cost function and the derivation of optimal values of decision variables.

Now, it is the time to analyze the impact of the model in terms of delay in payments. The focus of analysis is given to investigation of possible decrease in price which does not decrease total annual variable cost by considering delay in payments.

1. When $T \geq t$

First of the procedure is determination of optimal replenishment interval and calculation of minimum cost under the given price of PEOQ model.

Secondly, we introduce delay in payment and calculate the price of DPEOQ model which equalizes the minimum total cost of PEOQ to the total cost of DPEOQ.

Let the given price of PEOQ be p_1, then optimal replenishment interval is given from (6) by

$$T^* = \sqrt{2S/(z\cdot\exp(-m\cdot p_1)\cdot h)} \tag{15}$$

And the minimum total annual variable cost of PEOQ model is given as follows

$$Z(p_1, T^*) = 2S\cdot z\cdot\exp(-m\cdot d)\cdot h - z\cdot\exp(-m\cdot p)\cdot(p-c) \tag{16}$$

By using the replenish interval of PEOQ, we can represent total annual variable cost of DPEOQ model as function of price.

$$\begin{aligned} Z(p, T^*) = &\sqrt{S\cdot z\cdot\exp(-m\cdot p_1)\cdot h/2} \\ &+\sqrt{S\cdot z\cdot\exp(-m\cdot(2p-p_1))\cdot h/2} \\ &-z\cdot\exp(-m\cdot p)\cdot(p-c) \\ &-z\cdot\exp(-m\cdot p)\cdot p\cdot t^2\cdot I/2 \\ &\cdot\sqrt{z\cdot\exp(-m\cdot p_1)\cdot h/(2S)} \end{aligned} \tag{17}$$

From (16) and (17), it can be easily shown that the price of (17) must fall down to equalize the two. The amount of decrease can be calculated numerically.

And if we deviate the replenishment interval of DPEOQ model, the more decrease in price is realized.

2. When $T < t$

By following the same procedure above, we can obtain optimal replenishment interval and minimal total annual variable cost of PEOQ model and represent total annual variable cost of DPEOQ model as a function of price as follow

$$T^* = \sqrt{2S/(z\cdot\exp(-m\cdot p)\cdot h)}$$

$$\begin{aligned} Z(p_1, T^*) = &\sqrt{2S\cdot z\cdot\exp(-m\cdot p)\cdot h} \\ &-z\cdot\exp(-m\cdot p)\cdot(p-c) \end{aligned}$$

$$\begin{aligned} Z(p, T^*) = &\sqrt{S\cdot z\cdot\exp(-m\cdot p_1)\cdot h/2} \\ &+\sqrt{S\cdot z\cdot\exp(-m\cdot(2p-p^1))\cdot h/2} \\ &-z\cdot\exp)-m\cdot p)\cdot(p-c) \\ &-z\cdot\exp(-m\cdot p)\cdot p\cdot I\cdot(t \\ &-\sqrt{S/(2z\cdot\exp(-m\cdot p_1)\cdot h)} \end{aligned}$$

It is not difficult to show the realization of decrease of price without increase in total cost. And the amount of decrease can be calculated numerically.

Ⅵ. Concluding Remarks

In this paper, we studied two models of EOQ with two decision variables, price and replenishment interval.

The mathematical formulation of total annual variable cost function was derived to obtain the optimal values of decision variables and to investigate the impact of delay in payments on price. Because of the difficulties to optimize the two variables simultaneously, the optimal solutions were described in terms of only one variable under the condition that the value of the other variable was prescribed. As delay in payments allowed the inventory holder to earn interest for the items sold during unsettlement period, it was expected that we could decrease price. Through our procedure, we could observe this decrease and described this mathematically.

The two topics which is not solved satisfactorily in this paper, is recommended for further study:

(1) the problem to optimize the two decision variables, price and replenishment interval, simultaneously

(2) the representation of the amount of decrease in price as a closed form.

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