UIT Report Vol. 13, No. 1, pp. 171~173, 1982 연구논문집 제13권 제 1 호 pp. 171~173, 1982

Quasi semiopen sets and quasi semicontinuity in bitopological spaces

Lee, Il-Yong · Lee, Je-Yoon
Dept. of Mathematics
(Received December 30, 1981)

〈Abstract〉

In a bitopological space, the authors present a study of quasi semiopen sets and that of a property of a quasi semicontinuity by means of quasi semiclosure.

쌍위상공간에서 Quasi semiopen 집합과 Quasi semi 연속성에 관하여

이 일 용 · 이 제 윤
수 학 과
(1981. 12. 30 접수)

〈요 약〉

쌍위상공간상에서 Quasi semiopen 집합의 성질을 알아보고, Quasi semiclosure을 이용하여 quasi semi 연속이 될 필요충분 조건을 구해 보았다.

I. Introduction

Levine (1963) [1] has stated that, in a topological space, a set A is semiopen if and only if there exists an open set O such that $O \subset A \subset \overline{O}$ where $\overline{(\ )}$ denotes the closure in the topological space. A subset A is semiopen iff $A \subset \overline{A^\circ}$, $(\ )^\circ$ denoting the interior in the topolcgical space. Any union of semiopen sets is semiopen. All open sets are semiopen.

A subset A of a topological space X is said to be semiclosed iff $X-A$ is semiopen. Since all open sets are semiopen, all closed sets are semiclosed. And since any union of semiopen sets is semiopen, any intersection of semiclosed sets is semiclosed.

For any set A in a topological space X, the semiclosure of A is defined by $\bigcap_{D \subset S} D$, where $S = \{D : D \text{ is semiclosed in } X,\ A \subset D\}$. That is, the semiclosure of A is the intersection of all semiclosed sets containing A.

M.C. Datta [2] has stated that, a subset S of a bitopological space (X, P, Q) is quasiopen if for every $x \in S$ there exists a P-open set U such that $x \in U \subset S$ or a Q-open set V such that $x \in V \subset S$. Any union of quasiopen sets is quasiopen. Every P-open (resp. Q-open) set is quasiopen. Every quasiopen set in a bitopological space X is a union of a P-open set and a Q-open set. Complement of a quasiopen set is termed quasiclosed.

In a bitopological space (X, P, Q), P-cl$(A) \cap$ Q-cl(A) is the quasiclosure of A, where P-cl(A) (resp. Q-cl(A)) donotes the closure of A

relative to P(resp. Q). Let us denote the quasiclosure of A by qcl(A).

The purpose of this note is to present a study of some properties of quasi semiopen sets and quasi semicontinuous mappings.

II. Quasi semiopen sets

Definition 1. A set A in a bitopological space (X, P, Q) is termed quasi semiopen iff for every $x \in A$ there exists either a P-semiopen set U such that $x \in U \subset A$ or a Q-semiopen set V such that $x \in V \subset A$.

From the above definition, it is clear that every P-semiopen (resp. Q-semiopen) set is quasi semiopen. But the converse may be false.

Proposition 1. A set A in a bitopological space (X, P, Q) is quasi semiopen iff it is a union of a P-semiopen set and a Q-semiopen set.

Proof. The sufficiency is obvious.

Necessity: Let A be quasi semiopen and $p \in A$. Then there exists either a P-semiopen set U such that $p \in U \subset A$ or a Q-semiopen set V such that $p \in V \subset A$. Let G be the subset of A such that, for every $x \in G$, there is a P-semiopen set U_x such that $x \in U_x \subset G \subset A$. Then $A - G = H$ is a subset of A such that, for every $y \in H$, there is a Q-semiopen set V_y such that $y \in V_y \subset H \subset A$. And since any union of semiopen sets is semiopen, $G = \cup\{U_x : x \in G\}$ is P-semiopen and $H = \cup\{V_y : y \in H\}$ is Q-semiopen. And also $A = G \cup H$, the proof is completed.

From the above proposition, we get

Proposition 2. The union of any collection of quasi semiopen sets is quasi semiopen.

Intersection of two quasi semiopen sets may not be quasi semiopen. For,

Example 1. Let $X = \{x, y, z\}$, $P = \{X, \phi, \{x\}, \{y, z\}\}$ and $Q = \{X, \phi, \{x, y\}\}$, Then $\{y, z\}$ and $\{x, y\}$ are quasi semiopen but their intersection $\{y\}$ is not quasi semiopen.

Definition 2. In a bitopological space(X, P, Q) a set A is termed quasi semiclosed if $X - A$ is quasi semiopen.

By the proposition 1, it is clear that the intersection of an arbitrary collection of quasi semiclosed sets is quasi semiclosed.

Definition 3. The intersection of all quasi semiclosed sets containing a set A is termed quasi semiclosure of A. We will denote it by qscl(A).

From the above, it is clear that qscl(A) is the smallest quasi semiclosed set containing A. And we get

Proposition 3. A set A is quasi semiclosed iff A=qscl(A).

III. Quasi semicontinuous mapping

The authors of [3] has introduced some new mappings in bitopological spaces.

Definition 4. Let (X, P, Q) and (X^*, P^*, Q^*) be bitopological spaces. A mapping $f : X \to X^*$ is said to be quasi continuous if the inverse image of every quasi open subset of X^* is quasiopen in X.

Definition 5. Let (X, P, Q) and (X^*, P^*, Q^*) be bitopological spaces. A mapping $f : X \to X^*$ is said to be quasi semicontinuous if the inverse image of each quasi open subset of X^* is quasi semiopen in X.

Remark. Quasi continuity implies quasi semicontinuity, of course, but not conversely. For

Example 2. Let $X = \{a, b, c\}$, $P = \{X, \phi, \{a\}\}$, $Q = \{X, \phi, \{b\}\}$; $X^* = \{x, y, z\}$, $P^* = \{X^*, \phi, \{x\}, \{x, y\}\}$. $Q^* = \{X^*, \phi, \{y\}\}$. Define $f : X \to X^*$ by $f(a) = f(c) = x$, $f(b) = y$. Then f is quasi semicontinuous, but it is not quasicontinuous.

Proposition 4. Let X and Y be bitopological spaces and $f : X \to Y$ be a mapping. Then the following statements are equivalent.

(a) f is quasi semicontinuous

(b) The inverse image under mapping f of every quasiopen subset of Y is quasi semiopen subset of X.

(c) For each $x \in X$ and each quasiopen set O in Y such that $f(x) \in O$, there is a quasi semiopen set A in X such that $x \in A$ and $f(A) \subset O$.

(d) The inverse image under mapping f of each quasi semiclosed subset of Y is quasi semiclosed in X.

Proof. It follows from the proposition 4 in [3].

Proposition 5. Let X and Y be bitopological spaces and $f: X \to Y$ be a quasi semicontinuous mapping. Then the followings are equivalent.

(a) The inverse image under mapping f of each quasi semiclosed subset of Y is quasi semiclosed in X

(b) For each $A \subset X$, $f(\text{qscl}(A)) \subset \text{qscl}(f(A))$

(c) For each $B \subset Y$, $\text{qscl}(f^{-1}(B)) \subset f^{-1}(\text{qscl}(B))$

Proof. The road map for the proof is (a)⇔(b)⇔(c).

(a)⇒(b): Let $A \subset X$. Then $A \subset f^{-1}(f(A)) \subset f^{-1}(\text{qscl}(f(A)))$. Since $\text{qscl}(f(A))$ is quasi semiclosed in Y, by (a), $f^{-1}(\text{qscl}(f(A)))$ is quasi semiclosed in X and contains A.

Therefore, $\text{qscl}(A) \subset f^{-1}(\text{qscl}(f(A)))$. Hence $f(\text{qscl}(A)) \subset \text{qscl}(f(A))$.

(b)⇒(a): Let F be quasi semiclosed in Y. Then $f(\text{qscl}(f^{-1}(F))) \subset \text{qscl}(f(f^{-1}(F)) \subset \text{qscl}(F) = F$. Therefore, $\text{qscl}(f^{-1})) \subset f^{-1}(F)$. Hence by the proposition 3, $f^{-1}(F)$ is quasi semiclosed in X.

(b)⇔(c): Obvious.

References

1. N. Levine: Semiopen sets and semicontinuity in topological spaces, Amer. Math. Monthly, Vol. 70(1963), 36–41.
2. M. C. Datta: Contribution to the theory of bitopological spaces, Ph. D. Dissertation, B. I. T. Pilani(India), 1971
3. S. N. Maheshwari, Chae, Gyu-Ihn and S. S. Thakur: Some new mappings in bitopological spaces, U. I. T. Report, Vol. 12, No. 2 (1981), 301–304.