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Almost Feebly continuous functions

S. N. Maheshwari* · Chae, Gyu-Ihn · P. C. Jain*
Dept. of Mathematics
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〈Abstract〉

A new function called almost feebly continuous function has been introduced and some properties of these functions have been obtained.

Almost feebly 연속함수에 관하여

S. N. Macheshwari* · 채규인 · P. C. Jain*
수 학 과
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〈요 약〉

새로운 Almost feebly 연속함수를 도입하여 이 함수들이 갖는 몇가지 성질을 알아 보았다.

I. Introduction

Levine [2] defined a subset A of a topological space X to be semiopen if there is an open set U such that $U \subset A \subset \mathrm{Cl}(U)$, where $\mathrm{Cl}(U)$ denotes the closure of U in X. Complement of a semiopen set is called semiclosed. The intersection of all the semiclosed sets containing a set A is called the semiclosure of A [1] and denoted by Scl A. Recently Maheshwari and Tapi [3] introduced the concept of feebly open sets as generalization of open sets. In a topological space X, a subset A is termed feebly open set if there is an open set U such that $U \subset A \subset \mathrm{Scl}\ U$. They also termed a function f from a topological space X into a topological space Y to be feebly continuous if the inverse image of every open subset of Y is feebly open in X. The purpose of the present paper is to introduce a new class of functions called almost feebly continuous function which contains the class of feebly continuous functions and to give some properties of these functions.

By $f: X \to Y$ we denote a function f from a topologica space X into a topological space Y and $\mathrm{Int}(A)$ denotes the interior of A.

II. Almost Feebly Continuous Functions

Definition 1. A function $f: X \to Y$ almost feebly continuous if the inverse image of every regular open subset of Y is feebly open in X.

Remark 1. Every feebly continuous function is almost feebly continuous because every regular open set is open. However the converse may be false.

Example 1. Let $X=\{a, b, c\}$, $P=\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $Y=\{x, y\}$, $Q=\{\phi, \{x\}, Y\}$. Define $f: X \to Y$ by $f(a)=f(c)=x$ and $f(b)=y$. Then f is almost feebly continuous but it is not feebly continuous, for $f^{-1}(\{x\})=\{a, c\}$ is not feebly open in X.

Dept. of Mathematics, University of Saugar, India.

Lemma 1: In a topological space X any union of feebly open sets is feebly open [3].

Proposition 1. Let $f: X \to Y$. Then the following conditions are equivalent.

(a) f is almost feebly continuous.

(b) For each $p \in X$ and each regular open set O in Y such that $f(p) \in O$, there exists a feebly open set A in X such that $p \in A$ and $f(A) \subset O$.

Proof: $(a) \Longrightarrow (b)$. Let O be regular open in Y and $f(p) \in O$. Then $P \in f^{-1}(O)$ and $f^{-1}(O)$ is feebly open. Set $A = f^{-1}(O)$. Thus, $p \in A$ and $f(A) \subset O$.

$(b) \Longrightarrow (a)$. Let O be regular open in Y and let $p \in f^{-1}(O)$. Then $f(p) \in O$. By (b), there is a feebly open set A_p in X such that $p \in A_p$ and $f(A_p) \subset O$. Andso, $p \in A_p \subset f^{-1}(O)$. Therefore $f^{-1}(O)$ is a union of feebly open sets in X. Hence, $f^{-1}(O)$ is feebly open in X by Lemma 1. Thus, (a) holds.

Lemma 2. A function $f: X \to Y$ is feebly continuous if and only if for any point $x \in X$ and any open set V of Y containing $f(x)$, there exists a feebly open set U in X such that $x \in U$ and $f(U) \subset V$.

The proof is similar to Proposition 1.

Proposition 2. An almost feebly continuous function $f: X \to Y$ is feebly continuous if Y is semiregular.

Proof. Let $x \in X$ and let A be an open set containing $f(x)$. Since Y is semiregular the family of regular open sets in Y forms a base for the topology on Y. So there is an open set M in Y such that $f(x) \in M \subset \mathrm{Int}\,(\mathrm{Cl}(M)) \subset A$. Since $\mathrm{Int}(\mathrm{Cl}(M))$ is regular open in Y and f is almost feebly continuous, by proposition 1, there is a feebly open set U in X containing x such that $f(x) \in f(U) \subset \mathrm{Int}\,(\mathrm{Cl}(M))$. Thus U is a feebly open set containing x such that $f(U) \subset A$. Hence f is feebly continuous by Lemma 2.

Definiton 2: A topological space X is said to be feebly T_2 if for $x, y \in X$, $x \neq y$, there exist disjoint feebly open sets U, V such that $x \in U$, $y \in V$ [5].

Proposition 3. Let $f: X \to Y$ be injective and almost feebly continuous. If Y is a T_2-space then X is feebly T_2.

Proof. Let x and y be any two distinct points of X. Since f is injective $f(x) \neq f(y)$. Now, Y being a T_2-space there exist two disjont open sets U and V such that $f(x) \in U$, $f(y) \in V$. Since U and V are disjoint open sets, we have $U \cap \mathrm{Cl}(V) = \phi$ and hence $U \cap \mathrm{Int}(\mathrm{Cl}(V)) = \phi$. Similarly, we obtain, $\mathrm{Int}(\mathrm{Cl}(U)) \cap \mathrm{Int}\,(\mathrm{Cl}(V)) = \phi$. Evidently, $f(x) \in \mathrm{Int}(\mathrm{Cl}(U))$ and $f(y) \in \mathrm{Int}(\mathrm{Cl}(V))$. Since $\mathrm{Int}\,(\mathrm{Cl}(U))$ and $\mathrm{Int}\,(\mathrm{Cl}(V))$ are regular open sets in Y and f is almost feebly continuous it follows that $f^{-1}(\mathrm{Int}(\mathrm{Cl}(U))$ and $f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))$ are disjoint feebly open sets containing x and y respectively. Hence X is feebly T_2.

Remark 2. Restriction of an almost feebly continuous function may not be almost feebly continuous. For

Example 2. Let $X = \{a, b, c\}$ be equipped with the topology $\mathscr{T} = \{\phi, \{a\}, X\}$ and $Y = \{x, y, z\}$ be equipped with the topology $\mathscr{T}^* = \{\phi, \{x\}, \{y\}, \{x, y\}, Y\}$. Define $f: X \to Y$ by $f(a) = f(b) = x$ and $f(c) = z$. Then f almost feebly continuous (in fact feebly continuous). But in the case $A = \{b, c\}$, then $f|A: A \to Y$ is not almost feebly continuous.

Lemma 3. If U is open in X and A is feebly open in X, then $U \cap A$ is feebly open in U [5].

Proposition 4. If $f: X \to Y$ is almost feebly continuous and A is open in X then the restriction $f|A: A \to Y$ is almost feebly continuous.

Proof. Since f is almost feebly continuous, for any regular open set V in Y, $f^{-1}(V)$ is feebly open in X. Hence by Lemma 3, $f^{-1}(V) \cap A$ is feebly open in A, because A is open Since $(f|A)^{-1}(V) = f^{-1}(V) \cap A$, therefore it follows that $f|A$ is almost feebly continuous.

Remark 3. Composition of two almost feebly continuous functions may not be almost feebly coutinuous. For

Example 3. Let $X=\{a,b,c\}$, $T_1=\{\phi, \{a\}, \{b,c\}, X\}$, $Y=\{a,b,c,d\}$, $T_2=\{\phi, \{a\}, \{b\}, \{a,b\}, Y\}$; $Z=\{u,v,w\}$, $T_3=\{\phi, \{u\}, \{v\}, \{u,v\}, Z\}$ Define $f: X\to Y$ by $f(a)=a$, $f(b)=c$, $f(c)=d$ and $g: Y\to Z$ by $g(a)=g(b)=g(c)=u$. $g(d)=w$. Then f and g are almost feebly continuous functions but gof is not almost feebly continuous because $\{u\}$ is regular open in Z but $(gof)^{-1}\{u\}=\{a,b\}$ is not feebly open in X.

Definition 3. A function $f: X\to Y$ is almost continuous if the inverse image of every regular open subset of Y is open in X [4].

Definition 4. A function $f: X\to Y$ is termed ß-continuous if the inverse image of every regular open set in Y is regular open in X.

Remark 4. It is clear that ß-continuity $\Longrightarrow$ almost continuity. That the implication is not reversible in general is shown by the following example.

Example 4. Let $Y=X=\{a,b,c\}$, $T_X=T_Y=\{\phi, \{a\}, \{b\}, \{a,b\}, X\}$. Define $f: X\to Y$ by $f(a)=f(b)=a$, $f(c)=c$. Then f is almost continuous. but not ß-continuous, for $f^{-1}(\{a\})=\{a,b\}$ is open in X but not regular open in X

Proposition 5. Let. $f: X\to Y$, $g: Y\to Z$. If g is almost continuous and f is feebly continuous then gof is almost feebly continuous.

Proof. Let U be regular open in Z. Then $g^{-1}(U)$ is open in Y because g is almost continuous. Therefore, $f^{-1}(g^{-1}(U))$ is feebly open in X because f is feebly continuous. Since $(gof)^{-1}(U)=f^{-1}(g^{-1}(U))$, gof is almost feebly continuous.

Proposition 6. Let $f: X\to Y$ and $g: Y\to Z$. If g is ß-continuous and f is almost feebly continuous then gof is almost feebly continuous.

The proof is similar to proposition 5.

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