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On the continuity in bitopological spaces

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〈**Abstract**〉

In [1], the authors of [1] studied a continuity in bitopological spaces. We solve the open question in [1], that is, for two given quasi T_1 spaces X and Y, if $f: X \to Y$ is pairwisecontinuous, then the image set $f(X)$ taken as a subspace of Y is quasi T_1.

쌍위상공간 상의 연속성에 관하여

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〈요 약〉

논문 [1]에서 쌍위상공간상에서 연속성에 관하여 연구되었다. 우리는 논문 [1]에서 제안한 문제, 즉 두 quasi T_1공간에 대하여 만일 함수 $f: X \to Y$ 가 pairwise 연속면 부분공간 $f(X)$는 역시 quasi T_1임을 보였다.

I. Preliminary

In [1], authors observed some properties of quasiopen sets in a bitopological space and they defined the continuity of a map in a bitopological space. And also, they related distinct bitopological spaces.

We has come across meaninglessness and confusing in it. And so, we are under the necessary of modifying them in this paper.

In the introduction of [1], for given two topological spaces (X, T_X) and (Y, T_Y), a map $f: X \to Y$ which relates sets X and Y is meaningful but $f^{-1}: T_Y \to T_X$ has no particular meaningless. For,

Example 1. Consider, $X=\{a, b, c\}$, $T_X=\{X, \phi\}$ and $Y=\{x, y, z\}$ $T_Y=\{Y, \phi, \{x, y\}, \{y, z\}, \{y\}\}$. Define $f: X \to Y$ by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

In [1] for given two bitopological spaces (X, P_X, Q_X) and (Y, P_Y, Q_Y), the authors denote a P-open set and a Q-open set in X by P_X and Q_X respectively. Since P_X and Q_X are topologies on X and P_Y and Q_Y are topologies on Y, the denotations are confusing. So, in this paper we will denote a P-open set in X by a P_X-open set and a P-open set in Y by a P_Y-open set.

II. On the continuity in a bitopological space

Definition 1. Let (X, P_X, Q_X) and (Y, P_Y, Q_Y) be bitopological spaces. A map $f: X \to Y$ is said to be pairwise continuous if the inverse image of each P_Y-open set is P_X-open and that of each Q_Y-open set is Q_X-open.

Definition 2. Let (X, P_X, Q_X) and (Y, P_Y, Q_Y) be bitopological spaces. A map $f: X \to Y$ is said

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to be quasicontinuous if the inverse image of each quasiopen set in Y is quasiopen in X.

From the above, we get that the quasicontinuity implies the pairwisecontinuity, but the converse is not true. For,

Example 2. Let $X=\{a,b,c\}$, $P_X=\{X,\phi,\{a\}\}$, $Q_X=\{X,\phi,\{c\}\}$ and $Y=\{x,y,z\}$, $P_Y=\{Y,\phi,\{x\},\{x,y\}\}$, $Q_Y=\{Y,\phi,\{y\}\}$. Define $f:(X,P_X,Q_X)\longrightarrow(Y,P_Y,Q_Y)$ by $f(a)=y$, $f(b)=z$ and $f(c)=x$. Then f is quasicontinuous but not pairwisecontinuous.

Proposition 1. Let (X,P_X,Q_X), (Y,P_Y,Q_Y) and (Z,P_Z,Q_Z) be bitopological spaces. If $f:X\longrightarrow Y$ and $g:Y\longrightarrow Z$ are quasicontinuous, then $g\circ f:X\longrightarrow Z$ also is quasicontinuous.

Proof. Let V be a quasiopen set in Z. Then, since g is quasicontinuous, $g^{-1}(V)$ is quasiopen in Y. Moreover, since f also is quasicontinuous, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is quasiopen in X. The proof is completed.

Proposition 2. Let (X,P_X,Q_X) and (Y,P_Y,Q_Y) be bitopological spaces. If a map $g:X\longrightarrow Y$ is quasicontinuous and $(f(X),P_{f(X)},Q_{f(X)})$ is the subspace of Y, then the map $f:X\longrightarrow f(X)$ taken by restricting the range of g is quasicontinuous.

Proof. Let $(f(X), P_{f(X)}, Q_{f(X)})$ be a subspace of Y where $P_{f(X)}=\{G\cap f(X)\mid G\in P_Y\}$ and $Q_{f(X)}=\{H\cap f(X)\mid H\in Q_Y\}$, U be a quasiopen set in $f(X)$ and $g:X\longrightarrow Y$ be quasicontinuous. Then $U=G\cap f(X)$, or $U=H\cap f(X)$, that is, $U=V\cap f(X)$ for some quasiopen set V of Y and since $f(X)$ is the entire image set by g, $f^{-1}(U)=g^{-1}(V)$. Since V is quasiopen in Y and $g:X\longrightarrow Y$ is quasicontinuous, $g^{-1}(V)$ is quasiopen in X. Hence $f^{-1}(U)$ is quasiopen in X. The proof is completed.

From the above and theorem 3 in [2], we get

Proposition 3. Let (X,P_X,Q_X) and (Y,P_Y,Q_Y) be bitopological spaces. If $g:X\longrightarrow Y$ is quasicontinuous and $(f(X), P_{f(X)}, Q_{f(X)})$ is a biopen subspace of (Y,P_Y,Q_Y), then $f:X\longrightarrow f(X)$ is quasicontinuous.

Proposition 4. Let (X,P_X,Q_X) and (Y,P_Y,Q_Y) be bitopological spaces and $g:X\longrightarrow Y$ be quasicontinuous. If Z is a subspace of Y containing the image set $f(X)$, then the map $f:X\longrightarrow Z$ is quasicontinuous.

Ⅲ. On the open questions

Theorem 1 of [1] appears to be false since it use the proposition 7 in [1]. For, if H is quasiopen in $f(X)$ it may not be quasiopen in Y and so we can not assert that $f^{-1}(H)$ is quasiopen in X by applying the hypothesis that f is quasicontinuous. So we will change the theorem 1 of [1] into the following proposition 5. On the base of proposition 5, we will show that those which authors [1] proposed are established when they are modified as the following proposition 6.

Proposition 5. If (X,P_X,Q_X) and (Y,P_Y,Q_Y) are two quasi T_1 spaces and $f:X\longrightarrow Y$ is quasicontinuous, then the image set $f(X)$ taken as a subspace of Y is quasi T_1.

Proof. It follows from theorem 6[2] and proposition 7[1].

Similarly the corollary 5 of [1] must be modified. And in the above proposition 5, it is also established if $f:X\longrightarrow Y$ is pairwisecontinuous. But if (Y,P_Y,Q_Y) is any bitopological space and not a quasi T_1 space, then the proposition 5 is not true in case that f is even quasicontinuous and also pairwisecontinuous. For,

Example 3. Let $X=\{a,b,c\}$, $P_X=\{X,\phi,\{a\},\{b\},\{a,b\}\}$, $Q_X=\{X,\phi,\{c\}$ and $Y=\{x,y,z\}$, $P_Y=\{Y,\phi,\{x\}\}$, $Q_Y=\{Y,\phi,\{y\}\}$. Define $f:X\longrightarrow Y$ by $f(a)=f(b)=y$, $f(c)=x$ and $g:X\longrightarrow Y$ by $g(a)=g(b)=x$, $g(c)=y$. Then f is quasicontinuous and g is pairwisecontinuous. And $f(X)=\{x,y\}$, $P_{f(X)}=\{f(X),\phi,\{x\}\}$, $Q_{f(X)}=\{f(X),\phi,\{y\}\}$. and $g(X)=\{x,y\}$, $P_{g(X)}=\{g(X),\phi,\{x\}\}$, $Q_{g(X)}=\{g(X),\phi,\{y\}\}$. Hence $(f(X), P_{f(X)}, Q_{f(X)})$ and $(g(X), P_{g(X)}, Q_{g(X)})$ are not quasi T_1 spaces respectively.

Proposition 6. Let (X, P_X, Q_X) and (Y, P_Y, Q_Y) be quasi T_1 spaces. If $f: Y \longrightarrow Y$ is pairwisecontinuous, then the image set $f(X)$ taken as a subspace of Y is quasi T_1.

Proof. Since a pairwisecontinuous map implies a quasicontinusus map, it follows from proposition 5.

References

1. Chae, Gyu-Ihn and Hong, Kul-Pyo: On the continuity in a bitopological space, U.I.T. Report 12(1981), 147—150.
2. S.N. Maheshwari, P.C. Jain and Chae, Gyu-Ihn: On quasiopen sets, U.I.T. Report 11 (1980), 291—292.
3. Murdeshwar, M.G. and S.A. Naimpally: Quasiuniform topological spaces. Monograph, P. Noordhoff (1966).
4. Dugundji, J.: Topology, Allyn and Bacon, Inc., Boston.