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Heat Storage in Packed Bed Subjected to Periodically Changing Input

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〈Abstract〉

This work investigates the transient characteristics and the dynamic responses of the specific-heat type storage unit which is subjected to the periodically changing inlet fluid temperature. Some analytic results and numerically calculated values are presented.

According to these, the transient behavior of the system is governed by the three parameters, time, τ, length, η, and angular frequency, ω. It is observed that higher values of ω yield faster response, while higher heat storage efficiency, Φ, is found for a system with higher values of η & ω and/or lower values of τ.

The optimum design and operating conditions for this type unit may be deduced from the results.

周期函數로 주어지는 入力에 對한 充塡層內의 蓄熱에 對하여

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〈要 約〉

顯熱利用方式 蓄熱系에서의 入口流體溫度가 周期的으로 變하는 境遇의 非定常狀態의 特性과 應答特性을 考察하였다. 몇가지의 解析結果과 數值的인 計算値들이 주어졌다.

이들에 依하면 系의 非定常特性은 세가지 無次元變數인 時間 τ, 位置 η, 그리고 周期ω에 依해 規定된다. 큰값의 ω에 對하여 系는 빠른應答을 가졌으며, 한편 ω와 η가 크고 τ가 작을 때 좋은 蓄熱効率을 나타내는 것을 보았다.

이들 結果로 부터 이러한 蓄熱系의 設計 및 運轉에 있어서의 最適條件을 決定할 수 있겠다.

I. Introduction

In order to simulate the storage of solar energe in solar heating system, an adequate model of the heat storage unit is required. One model of the heat store can be obtained by solving the simultaneous partial differential equation of the Schumann model.(1)

Most of previous work describe the system which is subjected to stepwise rising fluid temperature or single blow heating.(2,3,4) But in practical case, available quantity of direct solar radiation may be regarded as periodic function of time. And so, in this work, on the assumption that output of solar collector (i.e. inlet condition for storage unit) is given by the sinusoidal function of time, the transient temperature distribution and some characteristic parameters of the system are determined.

The physical system discussed here is shown in Fig. 1. The hot fluid with periodically changing temperature enters the inlet surface of the system, and flows through the fluid passage.

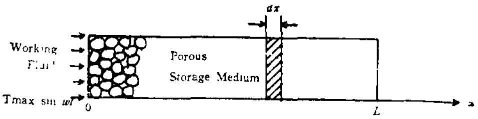

Fig. 1 Idealized Heat Storage System.

Ⅱ. Analysis

The main assumptions employed in the analysis are summarized as follows:

1. The system has constant thermophysical properties.
2. The temperature and velocity distributions are one dimensional in a direction normal to the bounding surface.
3. Axial heat conduction is negligible.
4. No heat losses to the environment occur.

Then, heat balances[5] can be described on the elemental slice of the system, dx, in Fig. 1, as

$$\frac{\partial T_f}{\partial t}+a(T_f-T_s)+u\frac{\partial T_f}{\partial x}=0. \qquad (1)$$

$$\frac{\partial T_s}{\partial t}+b(T_s-T_f)=0. \qquad (2)$$

The appropriate conditions for Eqs. (1) and (2) are

$$T_s(0,x)=T_0 \qquad (3)$$

$$T_f(t,0)=T_{\max}\cdot\sin wt. \qquad (4)$$

By introducing some nondimensionalized parameters[5], the governing equations and conditions are ultimately written as

$$\frac{\partial\nu}{\partial\eta}+(\nu-\theta)=0 \qquad (5)$$

$$\frac{\partial\theta}{\partial\tau}+(\theta-\nu)=0 \qquad (6)$$

$$\theta(0,\eta)=0 \qquad (7)$$

$$\nu(\tau,0)=\sin\omega\tau. \qquad (8)$$

A number of complicated problems of this model may be solved approximately by means of finite difference method, but in this work, Eqs. (5) to (8) are transformed into subsidiary equations by using the Laplace transformation technique. And the results are found to be

$$\frac{\partial\bar{\nu}}{\partial\eta}+\bar{\nu}-\bar{\theta}=0$$

$$s\bar{\theta}-\bar{\theta}(0)+\bar{\theta}-\bar{\nu}=0$$

$$\bar{\theta}(\eta)=0$$

$$\bar{\nu}(0)=\frac{\omega}{s^2+\omega^2}.$$

Solving for $\bar{\nu}$ and $\bar{\theta}$, we can obtain the solutions as

$$\bar{\nu}=e^{-\eta}\frac{\omega}{s^2+\omega^2}\exp\left(\frac{\eta}{s+1}\right)$$

$$\bar{\theta}=e^{-\eta}\frac{\omega}{s^2+\omega^2}\ \frac{1}{s+1}\exp\left(\frac{\eta}{s+1}\right).$$

In order to obtain the temperature distribution, the inverse Laplace transform has to be operated on the above solutions. Appropriate handling of some formulae[6] gives the exact solutions of Eqs. (5) to (8) as

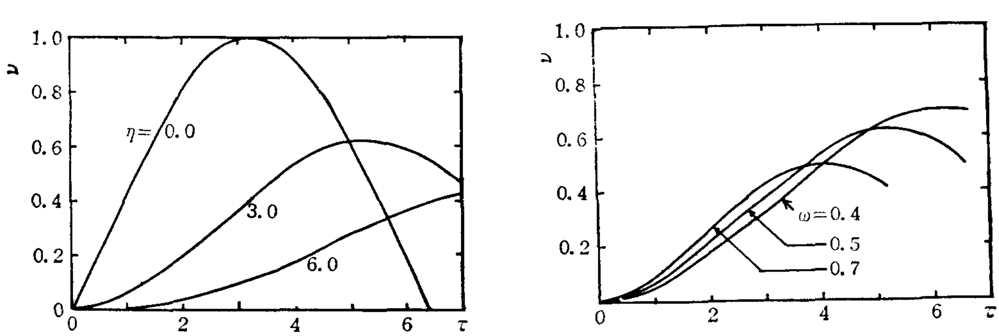

Fig. 2 Dimensionless Fluid Temperature Distribution.

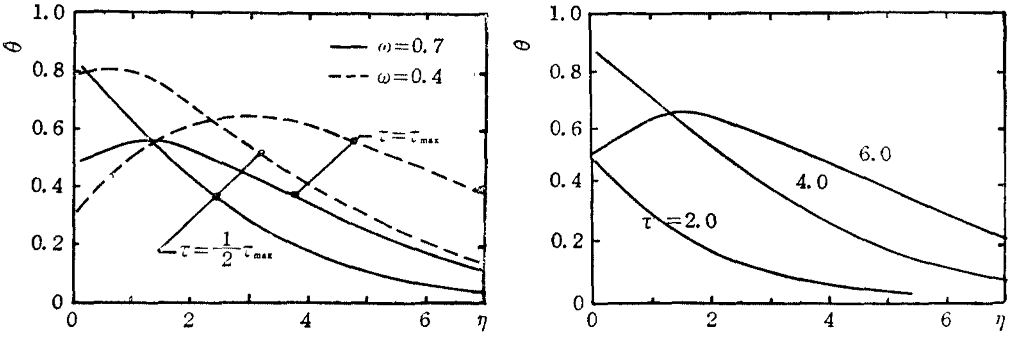

Fig. 3 Dimensionless Solid Temperature Distribution.

$$\nu=\sqrt{1+\omega^2}e^{-\eta}\int_0^{\tau}\sin\,\omega\left\{\tau-\left(\zeta+\frac{1}{\omega}\cos^{-1}\frac{1}{\sqrt{1+\omega^2}}\right)\right\}e^{-\zeta}I_0(2\sqrt{\eta\zeta})d\zeta \quad (9)$$

$$\theta=e^{-\eta}\int_0^{\tau}\sin\,\omega(\tau-\zeta)e^{-\zeta}I_0(2\sqrt{\eta\zeta})d\zeta. \quad (10)$$

The process to evaluate the numerical values of the solutions is programed for the digital computer. Some results of this operation are shown in Figs. 2 and 3.

To provide a check on the response of the system, the solutions for infinite time which remain finite periodic form are studied. The obtained relations are given as

$$\nu_\infty=\exp\left(\frac{-\omega^2}{1+\omega^2}\eta\right)\sin\,\omega\left(\tau_\infty-\frac{\eta}{1+\omega^2}\right) \quad (11)$$

$$\theta_\infty=\frac{1}{\sqrt{1+\omega^2}}\exp\left(\frac{-\omega^2}{1+\omega^2}\eta\right)\sin\,\omega\left\{\tau_\infty-\left(\frac{\eta}{1+\omega^2}+\frac{1}{\omega}\cos^{-1}\frac{1}{\sqrt{1+\omega^2}}\right)\right\} \quad (12)$$

From Eqs. (11) and (12), some characteristic parameters, time lag, φ, and amplitude ratio, Ω, are defined as follows

$$\varphi_f=\frac{\eta}{1+\omega^2} \quad (13)$$

$$\varphi_s=\varphi_f+\frac{1}{\omega}\cos^{-1}\frac{1}{\sqrt{1+\omega^2}} \quad (14)$$

$$\Omega_f=\exp\left(\frac{-\omega^2}{1+\omega^2}\eta\right) \quad (15)$$

$$\Omega_s=\Omega_1\cdot\frac{1}{\sqrt{1+\omega^2}} \quad (16)$$

Evaluated values of these parameters are also presented in Figs. 4 to 6.

To find some further results, two kinds of heat storage efficiencies, Φ_A and Φ_B, are determined. That is,

$$\Phi_A=\frac{Q_s}{Q_c}=\frac{\int_0^{\eta_L}\theta d\eta}{\eta_L} \quad (17)$$

$$\Phi_B=\frac{Q_s}{Q_i}=\frac{\int_0^{\eta_L}\theta d\eta}{\int_0^{\tau}\nu_e d\tau} \quad (18)$$

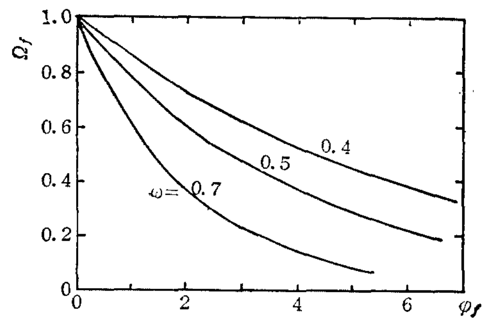

Fig. 4 Amplitude Ratio vs. Time Lag.

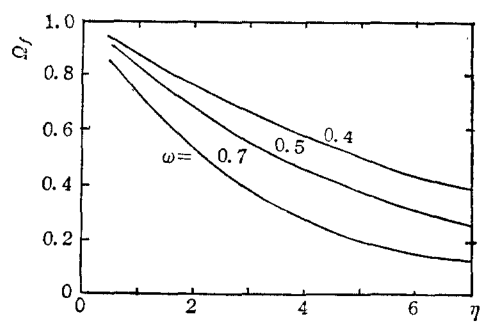

Fig. 5 Amplitude Ratio vs. Dimensionless Distance.

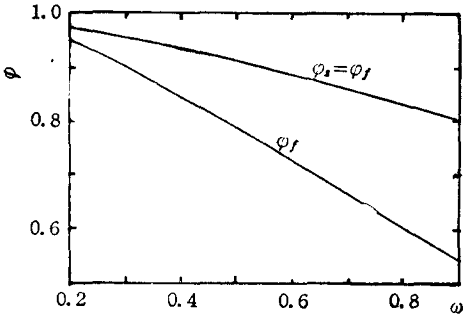

Fig.6 Time Lag vs. Dimensionless Angular Frequency.

where Q_c; the maximum heat capacity of storage medium, $(S_s\cdot L\cdot(\rho c_p)_s\cdot(T_{max}-T_0))$

Q_t; the heat supplied to the system during time t, $\left(\int_0^t u\cdot S_f\cdot dt\cdot(\rho c_p)_f(T_{fe}-T_0)\right)$

Q_s; the heat stored in storage medium during time t $\int_0^L S_s\cdot dx\cdot(\rho c_p)_s(T_s-T_0)$).

The efficiency, Φ_A, in the model of this work seems to have less significance than in the system with constant inlet temperature. However, of particular interest is the time at which Φ_A has the maximum value. The time, which can be also obtained by the relation $\frac{\partial Q_s}{\partial\tau}=0$, is regarded as one of the effective working time.

Numerically calculated results for a wide range of parameters, τ, η, and ω, are plotted in Figs.7 and 8.

III. Results

The foregoing analytical studies show that;

1. The transient behavior of the system is governed by the three parameters τ, η, and ω.

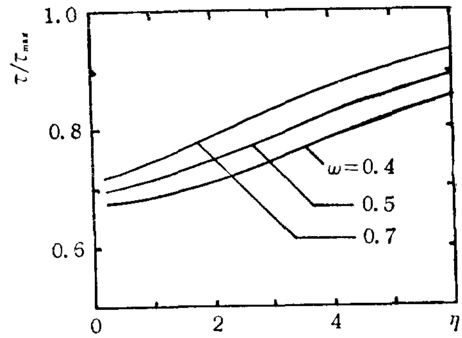

Fig.7 Conditions for Maximum Φ_A.

2. ω, or time constant $1/b$, is an important system parameter describing the response characteristics of the system. In addition to it, the responses are also controlled by η. It is observed in Figs.4 throguh 6 that large values of ω yield faster response in ν and θ.

3. Effects of parameters τ, η, and ω on heat storage efficiencies, Φ_A and Φ_B, can be examined in Figs.7 and 8. Higher efficiency, Φ_B, was

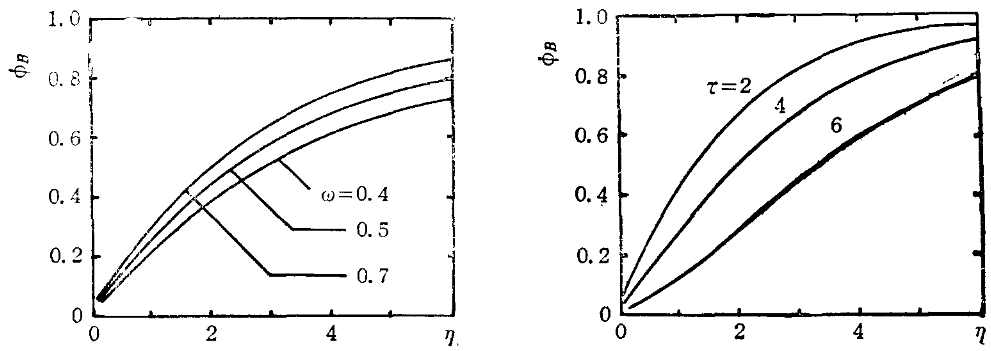

Fig.8 Heat Storage Efficiency.

found for a system with higher values of η and ω and/or lower value of τ. Physically, this means that higher value of a/b (i.e., the ratio of the heat capacity of the storage medium to that of the working fluid) contributes to the good efficiency.

4. These results may be applicable to the heat storage system which is subjected to the periodically changing inlet fluid temperature. The optimum design and operating conditions for the system can be also detected in the results 2 and 3.

Nomenclature

a: $hA/(\rho_c V)_f$ [1/sec]
b: $hA/(\rho_c V)_s$ [1/sec]
c: specific heat [J/kg·K]
S: sectional area [m²]
T: temperature [°K]
t: time [sec]
u: flow velocity of the fluid [m/sec]
w: angular frequency [rad/sec]
x: distance measured from inlet [m]
η: dimensionless distance $\left(\frac{x \cdot a}{u}\right)$
θ: dimensionless solid temperature $\left(\frac{T_s - T_0}{T_{max} - T_0}\right)$
ν: dimensionless fluid temperature $\left(\frac{T_f - T_0}{T_{max} - T_0}\right)$
ρ: density [kg/m³]
τ: dimensionless time $\left(t - \frac{x}{u}\right) \cdot b$
φ: time lag given by Eqs. (13) & (14)
Φ: heat storage efficiency given by Eqs. (17) & (18)
ω: dimensionless angular frequency (w/b)
Ω: amplitude ratio given by Eqs. (15) & (16)

Superscript
-: Laplace transformed variable

Subscript
e: fluid exit surface
f: for working fluid
i: fluid inlet surface
o: for initial state
s: for storage medium
∞: for infinite time

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