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On Quasiopen Sets

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〈Abstract〉

In a bitopological space, we define a quasiopen set and a quasi T_1-space as well. And we show that the theorem satisfied in a general topological space (A topological space is T_1-space iff every singleton subset is closed) is also constitued in the bitopological space.

準개집합에 관하여

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〈요　약〉

双위상공간상에서 準개집합은 정의하고 또한 準 T_1공간을 정의하여, 일반위상공간상에서 만족되는 정리 (T_1공간이 되기위한 필요충분조건은 singleton subset 모두가 closed이다)가 双위상공간상에서도 마찬가지로 만족됨을 보인다.

Ⅰ. Introduction

A subset S of a bitopological space (X, P, Q) is quasiopen [1] if for every $x \in S$ there exists a P-open set U such that $x \in U \subset S$ or a Q-open set V such that $x \in V \subset S$. Every P-open (resp. Q-open) set is quasiopen [1]. Every quasiopen set in a space (X, P, Q) is a union of a P-open set and a Q-open set [1]. Hence it is clear that any union of quasiopen sets is quasiopen. Complement of a quasiopen set is termed quasiclosed [1].

The purpose of the present note is to study some more properties of quasiopen sets and obtain the nature of singleton sets in pairwise T_1 spaces. Throughout the note $X \sim A$ denotes the complement of A in X.

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Ⅱ. Quasiopen sets

A quasiopen set need not be P-open or Q-open. For,

Example 1. Let $X=\{a,b,c\}$, $P=\{\phi, \{a\}, X\}$ and $Q=\{\phi, \{b\}, X\}$. Then the set $\{a, b\}$ is quasiopen but it is neither P-open nor Q-open.

Datta [1] remarked that the intersection of two quasiopen sets may not be quasiopen. The following example shows that the intersection of even a P-open (resp. Q-open) set and a quasiopen set may not be quasiopen.

Example 2. Let $X=\{a, b, c\}$, $P=\{\phi, \{a, b\}, X\}$ and $Q=\{\phi, \{b, c\}, X\}$. Then the intersection of the sets $\{a, b\}$ and $\{b, c\}$ is not quasiopen.

However we have,

Theorem 1. If O is biopen and A is quasiopen in a space (X, P, Q) then $O \cap A$ is quasiopen.

Proof: Let W be a P-open set and V be a Q-open set such that $A = W \cup V$. Then $O \cap A = (O \cap W) \cup (O \cap V)$. Since O is biopen, $O \cap W$ is P-open and $O \cap V$ is Q-open. Therefore, $O \cap A$ is quasiopen.

Theorem 2. Let (Y, P^*, Q^*) be a subspace of a space (X, P, Q). If A is quasiopen in X then $A \cap Y$ is quasiopen in Y.

Proof. There is a P-open set W and a Q-open set V such that $A = W \cup V$. Then, $A \cap Y = (W \cap Y) \cup (V \cap Y)$. It is clear that $W \cap Y$ is P^*-open and $V \cap Y$ is Q^*-open. Consequently, $A \cap Y$ is quasiopen in Y.

Remark: Let (Y, P^*, Q^*) be a subspace of a space (X, P, Q). If A is quasiopen in Y and Y is even if P-open (resp. Q-open) A may not be quasiopen in X. For,

Example 3: Let $X = \{a, b, c, d\}$, $P = \{\phi, \{a, b\}, X\}$ and $Q = \{\phi, \{b, c\}, X\}$. Let $Y = \{a, b\}$. Then, $\{b\}$ is quasiopen in Y, but it is not quasiopen in X.

However, we have,

Theorem 3: Let (Y, P^*, Q^*) be a biopen subspace of a space (X, P, Q). If A is quasiopen in Y, then A is quasiopen in X.

Proof: Let W^* be P^*-open and V^* be Q^*-open such that $A = W^* \cup V^*$. Since $W^* \subset Y$, $V^* \subset Y$ and Y is biopen, W^* is P-open and V^* is Q-open. Consequently, A is quasiopen in X.

Ⅲ. Singletons in pairwise T_1

The term pairwise T_1 is due to Murdeshwar and Naimpally [2].

Definition 1. A space (X, P, Q) is pairwise T_1 [2] if for $x, y \in X, x \neq y$, there exist a P-open set U and a Q-open set V such that $x \in U, y \notin U$, and $x \notin V$, $y \in V$.

It is well known that the singletons in T_1 spaces are closed. About the nature of singleton sets in pairwise T_1 spaces nothing is known in the literature so far. In this section we deal with this problem.

Definition 2. A space (X, P, Q) is termed quasi T_1 if for $x, y \in X$, $x \neq y$, there exist quasiopen sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

We get,

Theorem 4: Every pairwise T_1 space is quasi T_1.

The converse of Theorem 4 may be false. for,

Example 4. Let $X = \{a, b, c\}$,
$P = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and
$Q = \{\phi, \{c\}, X\}$.

Then the space (X, P, Q) is quasi T_1 but it is not pairwise T_1.

Theorem 5. A space (X, P, Q) is quasi T_1 iff singletons are quasiclosed.

Proof: Suppose that (X, P, Q) is quasi T_1 and $x \in X$. For, $y \in X$, $y \neq x$, there exists a quasiopen set V containing y but not x. Then $X \sim V$ is quasiclosed which contains x but not y. Therefore the intersection of all the quasiclosed sets containing x does not contain any point other than x. Since any intersection of quasiclosed sets is quasiclosed it follows that $\{x\}$ is quasiclosed.

The sufficiency is evident.

In view of Theorem 2 we have,

Theorem 6. Every subspace of a quasi T_1 space is quasi T_1.

Theorem 7. In a pairwise T_1 space singletons are quasiclosed.

This follows from Theorems 4 and 5.

※ 이 論文은 印度 Saugar大學校 敎授들(論文提出者 參照)과의 共同 論文임.

References

1. Datta, M.C.
Contributions to the theory of bitopological spaces, Ph. D. Dissertation, B.I.T. Pilani(India), 1971.

2. Murdeshwar, M.G. and S.A. Naimpally
Quasiuniform topological spaces, Monograph, P. Noordoff (1966).