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A Study on the Two-Step Process Nuclear Reaction by Distorted Wave Born Approximation*

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〈Abstract〉

Using analysis based on the distorted wave Born approximation the mechanism of the two-step process nuclear reaction was studied explicitly. The transition amplitude of the two-step process reaction was derived and was used to calculate the reaction cross section. The ratio of the magnitude of the reaction cross section of the two-step process to single-step process was calculated to be approximately 10^{-4} at 10 Mev energy of incident gamma rays, which agreed with experiment.

Because of the intermediate state through which the two-step process reaction occured, this kind of reaction, together with the type of analysis reported here, promises to be a useful tool for the study of higher excited states of nucleus.

歪曲波 Born假定에 依한 二重過程 核反應 硏究*

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〈要 約〉

歪曲波 Born假定方法을 적용하여 二重過程 核反應의 反應機構를 살피보았다. 二重過程 核反應의 轉移振幅을 求하고 이를 單一過程核反應의 것과 比較하여 에너지 10Mev의 入射 Gamma線에 대한 反應斷面積比를 계산하여 약 10^{-4}의 값을 얻었으며 이것은 實驗値와 거의 일치하였다. 二重過程核反應은 中間核準位를 거쳐 일어나기 때문에 이 反應은 中間核準位의 특성 硏究에 크게 활용될수 있다고 생각된다.

I. Introduction

The measurement of nuclear reaction processes that occur when a nucleus is bombarded with nuclear projectiles produces much information about the nuclear states and leads to an understanding of nuclear many-body system. The two-step process nuclear reation, especially, is achieving increasing importance at this stage because it permits an understanding of the properties of higher nuclear excited states which is generally prohibited in the single-step (direct) process nuclear reactions.

In the past, heavy charged particles produced by accelerators have been most commonly used as nuclear projectiles to initiate the two-step process nuclear reactions. Broek et al,[1] using alpha particles, has shown experimentally that in the nickel region the two-phonon state 4^+ is excited by a two-step process through the well-defined one-phonon state 2^+; $0^+ \rightarrow 2^+ \rightarrow 4^+$. Since then, many experimental investigations[2] of the two-step process reaction on the many nuclei

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have been made by using the different charged particles as nuclear projectile. Recently, two-step process nuclear reactions induced by the gamma-rays (hereafter called the two-step process photonuclear reaction) have also been used[3] for the study of the higher excited states of nucleus. Since the electromagnetic field is relatively well understood, experiments involving electromagnetic quanta should be much easier to interpret. Thus, the photonulear reaction in two-step processes between gamma-rays and nucleus can provide better knowledge about the properties of nuclear states.

Much experimental data about the nuclear properties have been accumulated in recent years by the two-step process photonuclear reactions. The gamma-ray induced reaction of F^{19} (γ,α) N^{*15} has been studied by Shikazono and Kausarasaki[4], and Thomson[5]. In all these experiments, it was concluded that the reaction mechanism is believed to be a two-step process in which the incident photon is absorbed by a nucleon which then is reabsorbed and scatters from an α-particle cluster, the latter being emitted. Such a mechanism would leave N^{15} in a 3-particle 4-hole state (relative to O^{16}), and this concurs nicely with the theoretical expectations for the structure of the lowest $\frac{1^+}{2^-}$ and $\frac{5^+}{2^-}$ states of N^{15}. Patrick et al[6] measured the de-excitation γ-rays following the photodisintegration of N^{15} in N^{15} (γ,t) C^{12*} reaction. It was found that the mechanism of the reaction is a two-step one: photon absorption by a nucleon followed by nucleon-trition scattering. With gamma-ray energies less than the nuclear binding energy of (γ,n) reaction, the Cu^{63} nucleus has been studied[7] by the two-step process reactions. Here, the single-step (γ,n) reaction process was evidently forbidden because of the energy consideration, but the two-step process reaction occured through intermediate states of energies at 6.1 Mev and 6.7 Mev.

The purpose of this paper is to develop theoretically the reaction mechanism of the two-step process nuclear reaction, which can be used to account for the experimental data already accumulated. Lemmer et al[3] have used a perturbation theory for the study of this reaction and they simplified their considerations by using plane-wave Born approximation in which the perturbation theory lies in the use of plane waves for the incident and scattered particles. However, it was found that plane-wave Born approximation was certainly not an accurate theory. In particular, its predictions for the second order process are considered to be very misleading.

In this work, the two-step process excitations were analyzed based on the distorted-wave Born approximation. The distorted wave Born approximation method is a fairly straightforward perturbation expansion in powers of the interaction which causes transitions between nuclear states, taking the elastic process as the unperturbed process. The relative motion of the colliding pair before and after the inelastic event is described in this approximation by distorted waves.

Ⅱ. Mechanism of the Two-Step Process Reaction

1. Two-Step Process Excitation

For incident nuclear projectile energies less than nuclear binding energies, the only processes energetically possible are those inelastic ones which excite the target nucleus to an energy level within the incident projectile energies. The excited nucleus then returns to the ground state after a half-life characteristic of that energy level by emission of a gamma ray. However, if this nuclear state is well-defined and can live for a sufficiently long time to absorb a second projectile, the nucleus could again be raised into a still higher state where

enough energy is now available to break up the nucleus. As the reaction proceeds by the two steps through the intermediate state, it is named the two-step process excitation and thus aids in determining important characteristics of that intermediate excited states.

If we assume that the nuclear potential U seen by the nuclear projectile depends only on the distance from the surface we have expressions of U as follows:

$$U(h+\alpha, r)=U_0(h, r)+\Delta U(h, \alpha, r), \quad (1)$$

$$\Delta U(h, \alpha, r)=-\alpha \frac{\partial U}{\partial h}+\frac{1}{2!}\alpha^2\frac{\partial^2 U}{\partial h^2}, \quad (2)$$

the derivatives being evaluated at $\alpha=0$. Here, the parameter h is the nuclear radius defined in some suitable way and α is the angle-dependent displacement operator of collective deformations of the nuclear surface, The operator $\alpha(\theta, \phi)$ in this case is the distance by which the nuclear surface is displaced from equilibrium, if examined along the direction of observation. If expanded in multipoles

$$\alpha(\theta, \phi)=\sum_{L, M}\xi_{L, M}Y^M{}_L(\theta, \phi), \quad (3)$$

where $\xi_{L, M}$ are multiples of the canonical displacement operators for collective motions of nucleus and are considered for simplicity to be spin-independent.

Then $U_0(h, r)$ is interpreted as the spherical potential giving rise to elastic scattering and is taken to be the usual type of complex Saxon-Woods Potential. Eq. (2) defines two possible second-order excitation mechanisms for the final excited state, which may be designated two-step process excitation and single-step (or crossover) excitation. Two-step process excitation arises from the first term in Eq. (2), operating twice, and describes a process which goes through the intermediate state, whereas single-step excitation comes from the second term of Eq. (2), operating once.

2. Transition Amplitude

Since $U_0(h, r)$ does not contribute to the inelastic scattering, the transition amplitude for exciting a nucleus from an initial state i to a final state f inelastically can be written as

$$T_{fi}=\langle\Psi^{(-)}{}_f|\Delta U|\Psi^{(+)}{}_i\rangle. \quad (4)$$

It is assumed that the total wave functions for the initial and final states have the forms, respectively,

$$\Psi^{(+)}{}_i=[1+(E+i\epsilon-H_0)^{-1}\Delta U]a\chi^{(+)}, \quad (5a)$$

$$\Psi^{(-)}{}_f=b\chi^{(+)}{}_f, \quad (5b)$$

where a and b are eigenstates of the nuclear surface oscillation and the $\chi^{(\pm)}$ are the distorted waves for the problem. The $\chi^{(+)}$ is the exact solution for the scattering with potential $U_0(h, r)$ with the scattering boundary conditions and $\chi^{(-)}$ is the time-reversed solution which is related to $\chi^{(+)}$ through the equation

$$\chi^{(-)*}(\vec{k}, r)=\chi^{(+)}(\vec{k}, r). \quad (6)$$

Those disorted wave functions in the spherical harmonic expansions are.

$$\chi^{(+)}=\frac{(4\pi)^{\frac{1}{2}}}{kr}\sum_{l=0} i^l(2l+1)^{\frac{1}{2}}e^{i\delta_l}f_l(k, r)Y_l{}^0(\Omega) \quad (7a)$$

and

$$\chi^{(-)*}=\frac{(4\pi)^{\frac{1}{2}}}{kr}\sum_{l', m'} i^{-l'}e^{i\sigma_{l'}}f^{l'}(k, r)Y_{l'}^{m'}(\Theta, 0)Y_{l'}^{m'*}(\Omega). \quad (7b)$$

Here, f_l is the radial wave function corresponding to angular momentum lh, Θ is the scattering angle, and k is the momentum of the beam. Substitution of Eqs. (5a) and (5b) in Eq. (4) gives the transition amplitude

$$T_{fi}=\langle b|\langle\chi_f{}^{(-)}|\tau(\hat{1}, \hat{2})|\chi_i{}^{(+)}\rangle|a\rangle \quad (8)$$

with the scattering operator for distorted waves

$$\tau(\hat{1}, \hat{2})=\Delta U+\Delta U G_0\Delta U \quad (9)$$

and the zero-order Green's function

$$G_0(\hat{1}, \hat{2})=(E+i\epsilon-H_0)^{-1}. \quad (10)$$

The spherical harmonic expansion of the Green's function can be written as

$$G_0(\hat{1}, \hat{2})=\frac{1}{r_1 r_2}\sum_{\lambda, \nu} g_\lambda(r_1, r_2)Y_\lambda{}^\nu(\hat{1})Y_\lambda{}^{\nu*}(\hat{2}), \quad (11)$$

where $g_\lambda(r_1, r_2)$ is the radial Green's function for partial wave λ.

It is desirable to expand τ in powers of α.

Moreever now α is an operator which creates or annihilates nuclear excitations, so that arranging the terms of τ according to powers of α corresponds to arranging them in terms of elementary excitations. If our α is defined as exciting collective motions, then α acting once on the ground state just gives the simple collective excited states, then α^2 acting on the ground state gives double collective excitation. The first few terms of the expansion of τ in powers of α are

$$\tau_1=\alpha\frac{\partial U}{\partial h} \tag{12}$$

$$\tau_2=\frac{1}{2}\alpha^2\frac{\partial^2 U}{\partial h^2}+\alpha\frac{\partial U}{\partial h}G_0\alpha\frac{\partial U}{\partial h}. \tag{13}$$

Eq. (12) is the expression for simple excitation, while Eq. (13) is for the double excitation. The α^2 acting on the ground state gives both direct (or crossover) and two-step process excitation. The first term of Eq. (13) is for the direct excitation, by operating $\frac{\partial^2 U}{\partial h^2}$, whereas the second one is the two-step process term by repeated operation of $\frac{\partial U}{\partial h}$, the latter involves an excitation through an intermediate state. Thus, further discussions contained herein will be limited to the α^2 terms.

Upon substituting Eq. (13) into Eq. (8) we have for the transition amplitude,

$$T_{fi}=\langle b|\langle\chi_f^{(-)}|\frac{1}{2}\alpha^2\frac{\partial^2 U}{\partial h^2}+\alpha\frac{\partial U}{\partial h}G_0\alpha\frac{\partial U}{\partial h}|\chi_f^{(+)}|a\rangle. \tag{14}$$

The direct excitation, which is related to the first term of Eq. (14), is not taken into account in this work because of the energy consideration.

If α commutes with the Green's function, some simplication is obtained. This approximation is at least reasonable for strongly absorbed projectiles which have short wavelength. Upon using this approximation we have identity

$$\alpha\frac{\partial U}{\partial h}G_0\alpha\frac{\partial U}{\partial h} \tag{15}$$
$$=\frac{1}{2}\left[\alpha^2(\hat{1})\frac{\partial U}{\partial h}G_0\frac{\partial U}{\partial h}+\frac{\partial U}{\partial h}G_0\frac{\partial U}{\partial h}\alpha^2(\hat{1})\right].$$

With the use of Eq. (15) the transition amplitude for the two-step process excitation, T_{fi} (2), becomes

$$T_{fi}(2)=\frac{1}{2}\langle\chi_f^{(-)}\Big\{\langle b|\alpha^2(\hat{1})|a\rangle\frac{\partial U}{\partial h}G_0\frac{\partial U}{\partial h}+\frac{\partial U}{\partial h}G_0\frac{\partial U}{\partial h}\langle b|\alpha^2(\hat{2})|a\rangle\Big\}|\chi_i^{(+)}\rangle. \tag{16}$$

The collective excitation of odd-A nuclei can be expressed in term of spectroscopic factors which multiply the amplitude for even target nuclei, Therefore, consideration will be limited for the nuclei for which the initial state a has zero angular momentum and the final state b has angular momentum I with projection M.

The nuclear matrix element of α^2 is then given by

$$\langle b(IM)|\alpha^2|a(100)\rangle=C_2(I)Y_I^{M^*}(\hat{1}) \tag{17}$$

and the transition amplitude may be explicitly expressed as

$$T_{fi}(2)=\frac{1}{2}C_2(I)\langle\chi_f^{(-)}|Y_I^{M^*}(\hat{1})\frac{\partial U(1)}{\partial h}\cdot G_0(k_b,\cdot\hat{1},\hat{2})\frac{\partial U(2)}{\partial h}+\frac{\partial U(1)}{\partial h}G_0(k_a:1,2)\cdot\frac{\partial U(2)}{\partial h}Y_I^{M^*}(\hat{2})|\chi_i^{(+)}\rangle. \tag{18}$$

Introducing the distrorted wave expansions Eqs. (7a) and (7b) and the r_1, r_2 dependence of Green's function (Eq. 11), respectively, Eq. (18) becomes

$$T_{fi}(2)\frac{1}{2}=C_2(I)\frac{(4\pi)^{\frac{3}{2}}}{k_ak_b}\sum_{l,l'}(2l+1)^{\frac{1}{2}}e^{i(\sigma_l+\sigma_{l'})}\cdot Y_{l'}^{m'}(\Theta,0)\sum_{\lambda,\nu}\Big\{\int_0^\infty dr_1\int_0^\infty dr_2\int d\Omega_1\int d\Omega_2\Big[f_{l'}(k_b,r)\cdot\frac{\partial U(r_1)}{\partial h}g_\lambda(k_b:r_1r_2)\frac{\partial U(r_2)}{\partial h}f_l(k_a,r_2)\cdot Y_{l'}^{m'*}(1)Y_I^{M^*}(1)Y^\nu{}_{\lambda'}(1)Y_\lambda^{\nu^*}(2)Y^0{}_l(2)+f_{l'}(k_b,r_1)\cdot\frac{\partial U(r_1)}{\partial h}g_\lambda(k_a:r_1,r_2)\frac{\partial U(r_2)}{\partial h}f_l(k_a,r_2)\cdot Y_{l'}^{m'*}(1)Y_\lambda{}^\nu(1)Y_I^{M^*}(2)Y_\lambda^{\nu^*}(2)Y^0{}_l(2)\Big]\Big\} \tag{19}$$

If we perform one of the angular integrations and use the expansion of the Clebsh-Gordan coefficient for a product of spherical harmonics

$$Y_{l_1}^{m_1}(\hat{1})Y_{l_2}^{m_2}(\hat{2})=\sum_l\Big[\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}\Big]^{\frac{1}{2}}$$

$$C(l_1l_2l\,;\,m_1m_2)\cdot C(l_1l_2l\,:\,OO)Y_l^{m_1+m_2}, \quad (20)$$

the transition amplitude of the two-stepp rocess reaction can be written as

$$T_{fi}(2)=-\frac{1}{2}\,C_2(I)\frac{4\pi}{k_ak_b}(2I+1)^{\frac{1}{2}}\sum_{l,l'}\frac{(2l+1)}{(2l'+1)^{\frac{1}{2}}}$$
$$\cdot e^{i(\sigma_l+\sigma_{l'})}Y_{l'}^{m'}(\Theta,0)C(lIl'\,:\,Om')C(lIl'\,:\,OO)$$
$$\cdot\int_0^\infty dr_1\int_0^\infty dr_2\Big[f_{l'}(k_b,r_1)\frac{\partial U(r_1)}{\partial h}g_l(k_a\,;\,r_1,r_2)$$
$$\cdot\frac{\partial U(r_2)}{\partial h}f_l(k_a,r_2)+f_{l'}(k_b,r_1)\frac{\partial U(r_1)}{\partial h}$$
$$\cdot g_{l'}(k_a\,;\,r_1,r_2)\frac{\partial U(r_2)}{\partial h}f_l(k_a,r_2)\Big]. \quad (21)$$

This is the final expression of the transition amplitude of two-step process reaction and thus could help us understand better some of the qualitatives of the two-step process for the study of the higher excited state of nucleus.

For the approximate calculation, a bilinear form for the Green's function can be used. It is

$$g_l(k\,;\,r_1,r_2)\simeq\frac{2}{\pi}\int_0^\infty dk\frac{f_l(k,r)f_l(k,r')}{y_l(k)(E_0+i\epsilon-E)}, \quad (22)$$

where $y_l(k)$ is the scattering amplitude for the partial wave l at the momentum k. The g_l-function is based on the orthogonality relation for the optical model radial wave functions,

$$\int_0^\infty f_\lambda(k,r)f_\lambda(k',r)dr=\frac{1}{2}\pi y_\lambda(k)\delta(k-k'). \quad (23)$$

With the use of Eq. (22), $T_{fi}(2)$ takes the form

$$T_{fi}(2)\simeq\frac{2}{\pi}\int_0^\infty\frac{F(k,\Theta)}{E_0+i\epsilon-E}dk, \quad (24)$$

where the explicit expression of $F(k,\Theta)$ gives,

$$F(k,\Theta)=-\frac{1}{2}C_2(I)\frac{4\pi}{k_ak_b}(2I+1)^{\frac{1}{2}}\sum_{l,l'}e^{i(l-l')}$$
$$\cdot\frac{(2l+1)}{(2l'+1)^{\frac{1}{2}}}e^{i(\sigma_l+\sigma_{l'})}Y_{l'}^{m'}(\Theta,0)(lIl'\,:\,Om')$$
$$\cdot C(lIl'\,:\,OO)\frac{iE\xi}{2}\Big[\int_0^\infty dr_1f_l(k,r_1)\frac{\partial U(r_1)}{\partial h}$$
$$\cdot f_l(k,r_1)+\int_0^\infty dr_2f_l(k,\,r_2)\frac{\partial U(r_2)}{\partial h}f_l(k,r_2)\Big], \quad (25)$$

with $$y\equiv\frac{1}{y_l}\,\frac{\partial y_l}{\partial l_0} \quad (26)$$

for l-values near but below the sharp cut off l_0.

The amplitude for the direct (or crossover) excitation of the target nucleus can also be derived: At the large r values, the U is essentially exponential over the important region of integration and thus the $\frac{\partial^2U}{\partial h^2}$ can be expressed as

$$\frac{\partial^2U}{\partial h^2}\simeq-\frac{1}{a}\,\frac{\partial U}{\partial h} \quad (27)$$

over this region. Here a gives the rate of fall off of the exponential optical potential. The transition amplitude of the direct excitation becomes

$$T_{fi}(1)=\frac{1}{2}\,C_2(I)\,\frac{4\pi}{k_ak_b}(2I+1)^{\frac{1}{2}}\sum_{l,l'}\frac{(2l+1)}{(2l'+1)^{\frac{1}{2}}}$$
$$\cdot e^{i(\sigma_l+\sigma_{l'})}Y_{l'}^{m'}(\Theta,O)C(lIl'\,:\,Om')C(lIl'\,:\,OO)$$
$$\cdot\frac{1}{a}\int_0^\infty f_{l'}(k_b,r_1)\frac{\partial U(r_1)}{\partial h}f_l(k_a,r_1)dr_1. \quad (28)$$

The dominance of $T_{fi}(1)$ in the outer region of r is due to the distortion effects, particularly the strong absorption and thus plane wave calculations are completely unreliable.

A further simplication can be achieved if $k_a\simeq k_b$. In this case, the two radial integral terms in $T_{fi}(2)$ of Eq. (25) become equal, and $T_{fi}(1)$ may be given in terms of $F(k,\Theta)$ by employing Eqs. (25) and (28). It is

$$T_{fi}(1)\simeq\frac{F(k,\Theta)}{iE\xi a}. \quad (29)$$

It is also interesting at this stage to see the ratio of the magnitude of the cross sections of the two-step to direct (or crossover) process reaction, $\frac{\sigma(2)}{\sigma(1)}$, which is the square of the transition amplitude ratio, $\frac{|T_{fi}(2)|^2}{|T_{fi}(1)|^2}$. For 10 Mev gamma rays, the ratio of the magnitude of the cross sections was approximately catculated to be 10^{-4}. A more correct calculation, however, would be expected to give a slightly smaller value. This magnitude is almost consistent with some of experiments[7].

Ⅲ. Conclusion

The distorted-wave Born approximation was used to develop the reaction mechanism of the two-step porcess nuclear reaction which leads

to a final state of the nucleus through an intermediate state. The transition amplitude for exciting a nucleus from an initial state to a final state was studied explicitly using distorted waves and was compared with that of single-step process reaction. By using the transition amplitudes of the two different reactions, the ratio of the reaction cross section of the two-step process to one-step process was calculated to be 10^{-4}, which has approximately the right magnitude to agree with experiment.

Therefore, we can say that the distorted-wave Born approximation method is quite successful for explanation of the two-step process reaction. The two-step process reaction, together with the analysis reported here, promises to be very useful for the study of the higher excited states of nucleus because it can allow excitations which are prohibited in the single-step process reaction.

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