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Almost continuous mapping and weakly continuous mapping

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〈**Abstract**〉

In this note we introduced a new class of mapping almost continuous and weakly continuous. In section 2, we consier the relation of almost continuous between Singal and Husain. In section 3, we investigated some propert of weakly continuous.

Almost continuous mapping과 weakly continuous mapping에 관한 소고

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〈요 약〉

우리는 이기서 almost continuous와 weakly continuous mapping을 소개하고, 제2절에서는 almost continuous에 관한 Stalling에 의한 정의와 Husain의 정의에 관하여 생각하고, 제3절에서는 weakly continuous에 관하여 몇 가지 성길을 조사하였다.

I. Introduction.

The object of the present note is to introduced a new class of mapping called almost continuous mapping in the sense of Singal and Singal [2] Husain [3], Stalling [4], and weakly continuous mapping [5], which turn out to be the natural tool for studying nearly compact space and locally nearly compact space and using weakly continuous mapping we prove almost continuous Singal and Singal if and only if almost continuous Husain. Throughout this note, let A be a subset of a topological space$(X, \mathscr{T})$. The closure of A and the interior of A are denoted by $\bar{A}$ and $(A)^\circ$ respectively, X and Y denote topological space, and by $f:X\rightarrow Y$ we denote a mapping f of a space X into a space Y.

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II. Almost continuous mappings.

Definition 2.1. A mapping $f:X\longrightarrow Y$ is almost continuous in the sense of Stalling if given any open set $W\subset X\times Y$ containg the graph f, there exists a continuous mapping $g:X\longrightarrow Y$ such that the graph of g is subset of W.

Definition 2.2. A mapping $f:X\longrightarrow Y$ is almost continuous at $x\in X$ in the sense of Husain if for each open set $V\subset Y$ containg $f(x)$, the closure of $f^{-1}(V)$ is a neighborhood of x. If f is almost continuous at each point of X, then f is called almost continuous.

Definition 2.3. A mapping $f:X\longrightarrow Y$ is called almost continuous at $x\in X$ in the sense of Singal and Singal if for each open set $V\subset Y$ containg $f(x)$, there exists an open set $U\subset X$ containg

x such that $f(U)$ is a subset of the interior of the closure of V. If f is almost continuous at each point of X, then f is called almost continuous.

Remark 2.1. Any continuous mapping satisfies above definitions, but mapping satisfying any one of the definitions need not be continuous. Example also show the independence of these definitions.

Remark 2.2. Example 2.1, show that an almost continuous function in the sense of Husain need not be almost continuous in the sense of Stalling. Example 2.2, show that an almost continuous mapping in the sense of Singal and Singal need not be almost continuous in the sense of Husain. Example 2.3, show that an almost continuous mapping in the sense of Stalling need not be almost continuous in the sense of Singal and Singal.

Example 2.1 Let R represent the reals with standard topology and $f:R\longrightarrow R$ be define by $f(x)=x$ if x is rational and $f(x)=-x$ if x irrational.

Example 2.2 Let R be the set of real numbers and let J consists of ϕ, R and the complements of all countable subsets of R. Let $X=\{a,b\}$ and let $\mathcal{T}^*=\{X,\phi,\{a\}\}$.

Let $f:(R,\mathcal{T})\longrightarrow(R,\mathcal{T}^*)$ be define by $f(x)=a$ if x is rational and $f(x)=b$ if x is irrational.

Example 2.3. Let R represent the reals with standard topology. Let $f:R\longrightarrow R$ define by $f(x)=\sin\left(\frac{1}{x}\right)$ if $x\neq 0$ and $f(x)=0$ if $x=0$.

Lemma 2.1. Let $f:X\longrightarrow Y$ be an open mapping given any subset $S\subset Y$, and any colsed A containg $f^{-1}(S)$, there exists a colsed $B\supset S$ such that $f^{-1}(B)\subset A$.

Proof. Let $B=Y-f(X-A)$, since $f^{-1}(S)\subset A$, it follows that $S\subset B$, and because f is an open mapping, B is closed in Y. Observing $f^{-1}(B)=X-f^{-1}[f(X-A)]\subset X-[X-A]=A$ completes the

Lemma 2.2. Let $f:X\longrightarrow Y$ be an open mapping. Then for every subset of $B\subset Y$, $f^{-1}(\overline{B})\subset\overline{f^{-1}(B)}$.

Proof. This is a direct consequence of Lemma 2.1.

Definition 2.4. A set A is called regularly-open, if it is the interior of its own closure or equivalently, if it is the interior of some closed set. A is called regular-closed, if it is the closure of its own interior or equivalently, if it is the closure of some open set [2].

Theorem 2.1. Let $f:X\longrightarrow Y$ be an open almost continuous in the sense of Singal and Singal. Then f is almost continuous Husain.

Proof. Let $x\in X$ and $V\subset Y$ be an open set containg $f(x)$ By the Lemma 2.2, $f^{-1}(\overline{V})\subset\overline{f^{-1}(V)}$ now we observe that $(\overline{V})^\circ$ is a regularly-open set and that $V\subset(\overline{V})^\circ\subset\overline{V}$. Since f is almost continuous Singal and Singal, $f^{-1}((\overline{V})^\circ)$ is open in X by Theorem 2.2, of [2]. Thus, $f^{-1}((\overline{V})^\circ)\subset f^{-1}(\overline{V})\subset\overline{f^{-1}(V)}$ and consequently $\overline{f^{-1}(V)}$ is a neighborhood of x. Hence, f is almost continuous Husain.

Theorem 2.2. Let $f:X\longrightarrow Y$ be almost continuous Singal and Singal and let $V\subset Y$ be open If $x\notin f^{-1}(V)$ but $x\in\overline{f^{-1}(V)}$, then $f(x)\subset\overline{V}$.

Proof. Let $x\in X$ be such that $x\not\subset f^{-1}(V)$ but $x\in\overline{f^{-1}(V)}$ and suppose $f(x)\notin\overline{V}$. Then there exists an open set W such that $f(x)\in W$ and $W\cap V=\phi$. Thus $\overline{W}\cap V=\phi$ and $(\overline{W})^\circ\cap V=\phi$. Since f is almost continuous Singal and Singal, there exists an open set $U\subset X$ such that $x\in U$ and $f(U)\subset(\overline{W})^\circ$. As a consequence. $f(U)\cap V=\phi$ However, since $x\in\overline{f^{-1}(V)}$, $U\cap f^{-1}(V)\neq\phi$, so that $f(U)\cap V\neq\phi$. We have a contraction. It follows that $f(x)\in\overline{V}$.

Theorem 2.3. Let $f:X\longrightarrow Y$ be an open almost continuous in the sense of Singal and Singal. Then for each open $V\subset Y$, $\overline{f^{-1}(V)}\subset f^{-1}(\overline{V})$.

Proof. Let $V\subset Y$ be open, by Theorem 2.2, $f(\overline{f^{-1}(V)})\subset\overline{V}$. Since $\overline{f^{-1}(V)}\subset f^{-1}(f(\overline{f^{-1}(V)}))$ for any function, we have $\overline{f^{-1}(V)}\subset f^{-1}(f(\overline{f^{-1}(V)}))\subset f^{-1}(\overline{V})$.

Corollary 2.1. Let $f:X\longrightarrow Y$ be an open almost continuous in the sense of Singal and Singal. Then for each open $V\subset Y$, $\overline{f^{-1}(V)}=f^{-1}(\overline{V})$.

Ⅲ. Weakly continuous mapping.

Definition 3.1. A mapping $f:X\longrightarrow Y$ is said to be weakly continuous [5] if for each point $x\in X$ and each open set $V\subset Y$ containg $f(x)$, there exists an open set $U\subset X$ containg x such that $f(U)\subset \overline{V}$.

Theorem 3.1. Let $f:X\longrightarrow Y$ is a weakly-continuous open mapping, then f is almost continuous in the sense of Singal and Singal.

Proof. Let $x\in X$ and V be any neighborhood of $f(x)$. Since f is weakly-continuous, there is an open neighborhood U of x such that $f(U)\subset \overline{V}$. Since f is open, therefore $f(U)$ is open. Then $f(U)\subset(\overline{V})^{\circ}$ and consequently f is almost continuous.

Theorem 3.2. An open almost continuous Husain

$f:X\longrightarrow Y$ is almost continuous Singal and Singal if and only if $\overline{f^{-1}(V)}=f^{-1}(\overline{V})$ for every open $V\subset Y$.

Proof. If f is almost continuous Singal and Singal, then Theorem 2.1, corollary 2.1, gives the condition. Conversely if f is almost continuous Husain, open and has the condition, we show f is almost continuous Singal and Singal. Let $x\in X$ and let $V\subset Y$ be open containg $f(x)$. Since f is almost continuous Husain, there exist an open set $U\subset X$ such that $x\subset U\subset\overline{f^{-1}(V)}$ $\subset f^{-1}(\overline{V})$. It follows that $f(U)\subset f(f^{-1}(\overline{V}))\subset \overline{V}$. So that f is weakly continuous, and since f is open.

By Theorem 3.1, f is almost continuous Singal and Singal.

Theorem 3.3. Let $f:X\longrightarrow Y$ is weakly continuous, then $\overline{f^{-1}(V)}\subset f^{-1}(\overline{V})$ for each open set $V\subset Y$.

Proof. Suppose there exists an open set W containg $f(x)$ such that $W\cap V=\phi$. Since V is open, we have $V\cap\overline{W}=\phi$.

Since f is weakly continuous, there exists an open set $U\subset X$ containg x such that $f(U)\subset\overline{W}$. Thus we obtain $f(U)\cap V=\phi$. On the other hand, since $x\in\overline{f^{-1}(V)}$, we have $U\cap f^{-1}(V)\neq\phi$ and hence $f(U)\cap V\neq\phi$. We have contraction. Thus we have $\overline{f^{-1}(V)}\subset f^{-1}(\overline{V})$.

Theorem 3.4. Let $f:X\longrightarrow Y$ is almost continuous and $\overline{f^{-1}(\overline{V})}\subset f^{-1}(\overline{V})$ for each open set $V\subset Y$, then f is weakly continuous

Proof. For any point $x\in X$ and open set $V\subset Y$ containg $f(x)$, by the hypothesis we have $\overline{f^{-1}(V)}$ $\subset f^{-1}(\overline{V})$. Since f is almost continuous Husain, there exists an open set $V\subset X$ such that $x\subset U$ $\subset\overline{f^{-1}(V)}$.

Thus we have $f(U)\subset\overline{V}$. This implies that f is weakly continuous.

Corollary 3.1. An almost continuous Husain mapping $f:X\longrightarrow Y$ is weakly continuous if and only if $\overline{f^{-1}(V)}\subset f^{-1}(\overline{V})$ for every open set $V\subset Y$.

Theorem 3.5. Let $f:X\longrightarrow Y$ is weakly continuous if and only if for each open set $V\subset Y$, $f^{-1}(V)\subset(f^{-1}(\overline{V}))^{\circ}$.

Proof. Sufficency. Let $x\in X$ and $f(x)\in V$. Then $x\in f^{-1}(V)\subset(f^{-1}(V))^{\circ}$. Let $U=(f^{-1}(\overline{V}))^{\circ}$, $f(U)=f((f^{-1}(V))^{\circ})\subset ff^{-1}(V)\subset\overline{V}$. Necessity. Let $x\in f^{-1}(V)$. Then there exists an open set U such that $x\in U$ and $f(U)\subset\overline{V}$. Hence $x\in U\subset$ $f^{-1}(\overline{V})$ and $x\in f^{-1}(\overline{V})$.

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