Algorithm for digital differential protection of power transformer

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〈Abstract〉

An inrush current detection algorithm for the protection of power transformer is described with a differential overcurrent algorithm. The algorithm is based on a least-square curve fitting of the sampled differential current by a decaying DC and two harmonic components.

The proposed method provides good discrimination between the differential current generated by the transformer energization-inrush current-and that produced by the internal faults. The digital simulation is performed on various inrush and internal conditions and the fault detection is done within 1-1.2 cycles based on a 60Hz waveform.

변압기의 디지탈 차동보호 알고리즘

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〈요 약〉

본 논문에서는 변압기의 차동과전류 보호를 위한 inrush 전류의 검출알고리즘이 제시되었다. 본 알고리즘은 최소자승법을 이용하여 샘플링된 차동전류로부터 제 1, 제 2 고조파 성분을 찾아 내었다. 여러 형태의 inrush 전류 및 내부 고장 전류에 대한 본 알고리즘의 시뮬레이션 결과, inrush에 의한 차동전류와 고장에 의한 차동전류에 대한 우수한 판별능력을 보여주었으며, 고장은 1-1.2 cycles (60Hz 기준)내에 검출되었다.

I. Introduction

The differential overcurrent protection of power transformer using digital techniques has been investigated by several authors [1-8] and most schemes have been focused on the development of algorithm which could discriminate the internal fault and magnetising inrush currents.

The harmonic restraint scheme is a standard means of transformer protection against internal faults and nonoperation for magnetising inrush currents. This is based on the assumption that the inrush current possesses a larger portion of second harmonic than the fault current.

Rockfeller [1] suggested that the inrush could be detected by monitoring the time durations between successive peaks of the differential current. Sykes and Morrison [2] proposed a digital filtering technique to calculate the fundamental and the second harmonic components of the differential current. They used a sampling frequency of 1KHz and the fault detection time was more than three cycles based on 60 Hz waveform. Malik, Dash and Hope [3] suggested a correlation technique to calculate the second harmonic component. They tested their relay off-line with a sampling frequency of 480 and 960 Hz. The finite duration impulse response (FIR) filters were developed by Schweitzer [4,9] to calculate the fundam ental and the second harmonic components, which are analogous to the square-wave correlation of the incoming samples. They improved the fault detection times varing from 1.25 to 1.5 cycles based on a 60 Hz waveform.

Rahman [8] suggested a rectangular transform technique in which the sine and cosine fourier cofficients are expressed in terms of the rectangular transform cofficients which are obtained from the data samples. The least square curre-fitting technique to find the second harmonic was proposed by Lucket, Munday and Murray [6]. They assumed that the differential current only contained a fundamental and a second harmonic and a decaying DC component of which the time constant is known. The sampling frequency of 1 KHz and a data windon of 20 samples were used. Degens [7] expanded the least square curre-fitting technique to the fifth harmonics to find the ratio between the fundamental and second harmonic components of the differential current. The fault detection time was more than two cycles of power frequency due to the one cycle delay introduced to prevent false trippings. He used the sampling frequency of 600 Hz and the data window of 13 samples.

The purpose of this paper is to present a improved algorithm based on a least square curre-fitting technique, which offers the faster computational speed and more accurate and faster discrimination between the internal fault and the magnetizing inrush currents compared with the earlier ones.

Ⅱ. Basic principle and development of algorithm

1. Differential overcurrent protection of power transformer

Differential overcurrent protection of transformer is based on a comparison of the primary and secondary currents. Under normal operating conditions the trasformer primary current is approximately equal to the secondary current appropriately adjusted by the turns ratio. Thus the difference between these currents is almost zero. If a fault internal to the transformer occurs, these currents are no longer equal and a large differential current results and the transformer is switched off. But when the transformer is energized, a large differential current may exist due to the magnetizing inrush current, while there is no current flowing on the secondary side of the transformer. This disproportion gives the trip command and results the maloperation of the differential relay. Therefore it is required to discriminate the inrush currents from the fault currents for the reliable operation.

This can be achieved by comparing the magnitudes of the fundamental and second harmonic components. If the second harmonic component of the differential current is estimated to be greater than a certain fraction of the estimate of the fundamental, then the inrush conditions are concluded to exist and blocks the trip command which may be initiated by a non-zero differential overcurrent-the fundamental component is used to give a trip command and a second harmonic restraint is used to block the trip command.

Fig. 1. shows a basic differential protection scheme of power transformer.

2. Development of algorithm

The differential transformer current, whether

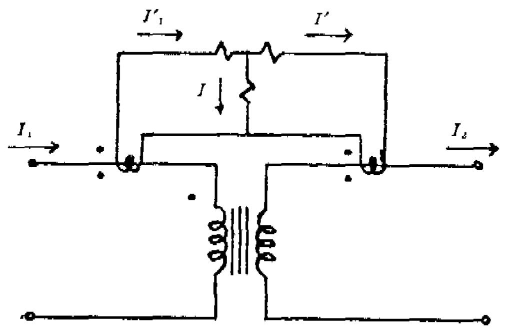

Fig. 1. Differential protection for Transformer

due to fault or inrush, can be represented as

$$I(t)=I_0\exp(-\alpha t)+\sum_{k=1}^{n} I_k \sin(k\omega t+\phi_k) \qquad (1)$$

where the time constant $1/\alpha$ is associated with decaying DC component. The harmonic components up to the second are allowed and the higher frequency components are filtered out of the waveform by using an analog lowpass filter. The cut-off frequency depends on the signal conditioning equipment, the sampling rate and the time of the cmputation available to the processor, however it doesn't have a big significance in this paper.

The filtered waveform can be written as

$$i(t)=I_0 \exp(-\alpha t)+\sum_{k=1}^{2} I_k\sin(k\omega t+\phi_k) \qquad (2)$$

Since this equation contains 6 unknowns, these unknowns can be found by sampling the current $i(t)$ at least 6 times and solving the independent equations given by each sampled values. Eq. (2) can be rewritten as

$$i(t)=I_0-I_0\alpha t+I_1\cos\phi_1\sin\omega t+I_1\sin\phi_1\cos\omega t$$
$$+I_2\cos\phi_2\sin2\omega t+I_2\sin\phi_2\sin2\omega t \qquad (3)$$

Note that decaying DC component is expressed by Taylor series of two terms in eq. (3).

By sampling the current m times for a certain time reference t_1 and a certain sampling frequency f_s, m linear independent equations are generated and formulated in matrix form as follows;

$$Ax=i \qquad (4)$$

where $x \triangleq [I_0-I_0\alpha \ \ I_1\cos\phi_1 \ \ I_1\sin\phi_1 \ \ I_2\cos\phi_2 \ \ I_2\sin\phi_2]^T$

$i \triangleq [i(t_1) \ \ i(t_2) \cdots i(t_m)]^T$

$$A \triangleq \begin{pmatrix} 1 & t_1 & \sin\omega t_1 & \cos\omega t_1 & \sin2\omega t_1 & \cos2\omega t_1 \\ 1 & t_2 & \sin\omega t_2 & \cos\omega t_2 & \sin\omega t_2 & \cos\omega t_2 \\ 1 & t_3 & \cdot & & & \\ \vdots & \vdots & & \cdot & & \\ 1 & t_m & & & \cdot & \cdot \end{pmatrix}$$

The solution of eq. (4) is given by using least square curve-fitting technique as

$$x=Bi \qquad (5)$$

where

$$B=A^{-1} \text{ if } m=6 \text{ (inverse)} \qquad (5\text{-a})$$
$$(A^TA)^{-1}A^T \text{ if } m>6 \text{ (pseudo-inverse)} \qquad (5\text{-b})$$

The fundamental and second harmonic components are of main concern and can be calculated from the vector of unknowns x in eq. (5). Let $C_i=I_i \cos \phi_i$ and $S_i=I_i \sin \phi_i$ $(i=1,2)$, then

$$C_1=\sum_k b(3,k)i(t_k) \qquad (6\text{-a})$$
$$S_1=\sum_k b(4,k)i(t_k) \qquad (6\text{-b})$$
$$C_2=\sum_k b(5,k)i(t_k) \qquad (6\text{-c})$$
$$S_2=\sum_k b(6,k)i(t_k) \qquad (6\text{-d})$$

where $b(i,k)$ is denoting the element of matrix B.

Since calculation of the elements $b(i,k)$ of matrix B is an off-line process, it has no influence on the calculation time of the algorithm.

The magnitudes of the fundamental and second harmonic components can be obtained by the eq. (7)

$$I_i=(I_i^2\cos^2\phi_i+I_i^2\sin^2\phi_i)^{1/2} \quad i=1,2 \qquad (7)$$

Since eq. (7) requires two squaring functions and one square root function, it is computationally inefficient for hardware implementation in a microprocessor or microcomputer level.

So a simple technique proposed by Schweitzer for reducing the computational overhead in eq. (7) is used to estimate the magnitude as the product of two quantities by a least-square estimation algorithm

If we define

$$u_i = \max(C_i, S_i) \text{ and} \qquad (8)$$
$$v_i = \min(C_i, S_i)$$

then the eq. (7) can be estimated by

$$I_i \fallingdotseq u_i + \frac{1}{4} v_i, \quad i=1,2 \qquad (9)$$

where max (a, b) denotes the larger of a and b and min (a, b) denotes the smaller.

Since the inrush current has a higher percentage of second harmonic than the internal fault current, it is expected that the ratio ξ of the second harmonic to the fundamental component(i.e. I_2/I_1) is larger during inrush than during the fault.

3. Computational requirements

If the algorithm is to be implemented in a digital processor, then four major factors-sampling frequency f_s, time reference, t_1, data window m and threshold ξ_0-should be determinded.

In order to prevent aliasing effect due to sampling, the sampling frequency should be greater than 240 Hz. As the sampling frequency increases, the time delay introduced by the analog filter can be reduced, but the time available for computation is also reduced. Three sampling frequencies of 360 Hz, 480 Hz, 600 Hz (6, 8 and 10 samples per cycle respectively) are used to study the effect of varing the sampling rates on the performance of the proposed algorithm.

Another important factor, the data window-the number of samples-which determines the number of equations given in eq. (4) should be greater than or equal to six, the number of unknowns in eq. (4). Various data windows are tested for each sampling frequency. The time reference is chosen as zero when nonzero differential current that exceeds the threshold current occurs.

The threshold value ξ_0 for the ratio of the second harmonic component to the fundamental component depends on the transformer that is to be protected.

The value of 0.125 is chosen for ξ_0 according to Schweitzer [4]. To secure the more reliable operation in case of the fault the time delay of a half cycle of power frequency is introduced.

Ⅲ. Simulation results

The proposed protection scheme has been programmed for off-line testing. Programs are also developed to compute the simulated inrush and the internal fault current waveforms where no-load is assumed [Appendix]. Fig.2 shows the flowchart of protection scheme. If ξ is smaller than a threshold ξ_0 during a half cycle of the power frequency then the trip command is gsnerated and if the differential current has the consecutive near-zero samples that do not exeed the threshold current, which is assumed to be zero, during one-third cycle of the power frequency, then the discriminating process is skipped and the trip is blocked. Note that the fundamental component is used as a operating signal and the second harmonic component is used as a restraining signal.

The digital simulations are preformed on various inrush and internal fault current models. [see Appendix] Various data windows-6 to 10 samples are considered in connection with three sampling frequencies.

Fig.3-a shows the plots of the simulated inrush current waveform, where the value of a decay rate is chosen to give time of 200 ms (approx.) for the peak current to decay to 50% of the initial peak value. Fig.3-b, c show the output of the proposed algorithm in terms of the ratio ξ for the inrush current shown in Fig.3-a, where the sampling frequency of 360 Hz and 480 Hz and the data window of 6 and 8 are used. From Fig.3 it can be seen that the proposed protection scheme yields the adequate

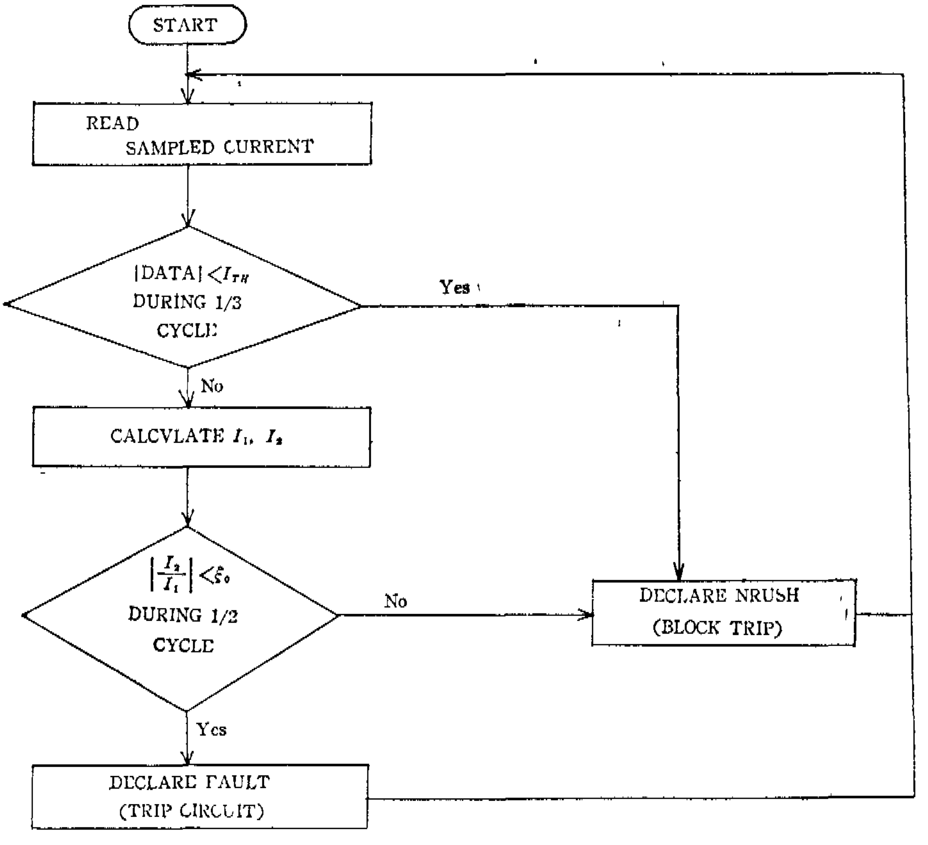

Fig. 2. Flowchart of the protection scheme

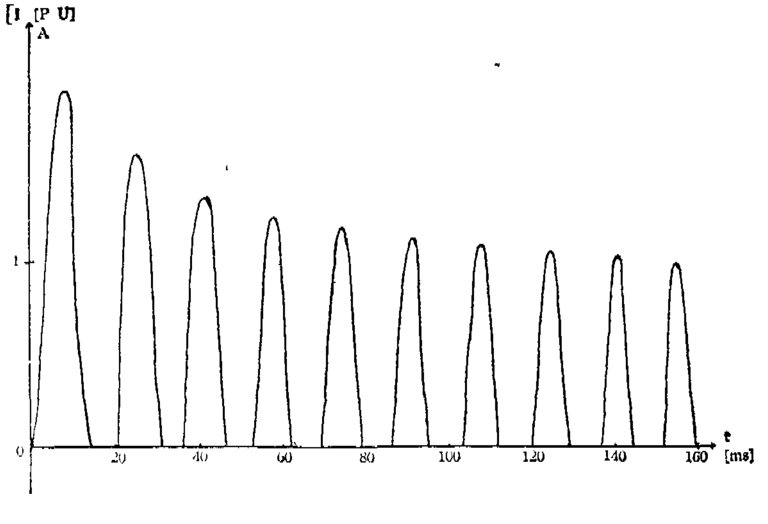

Fig. 3-a Magnetizing inrush current waveform

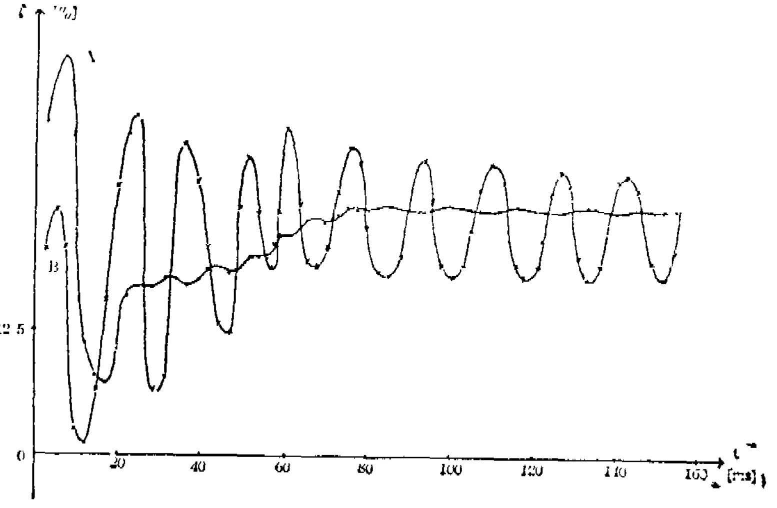

Fig. 3-b. Performance for magnetizing inrush: $f_s=480$ Hz
A: 8 samples B: 6 samples

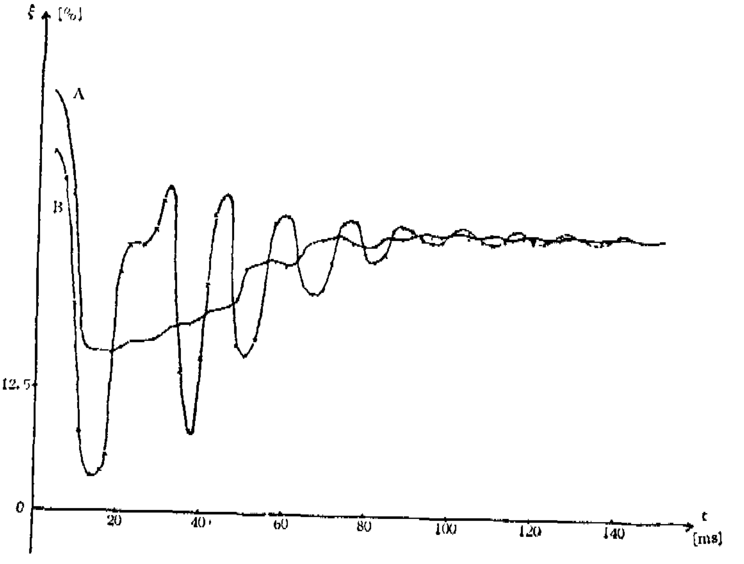

Fig. 3-c. Performance for magnetizing inrush: $f_s=360$ Hz
A: 8 samples B: 6 samples

restraint signal on the inrush current.

The simulated internal fault current waveform is illustrated in Fig. 4-a and Fig. 4-b, c show the performance of the proposed algorithm, where Q and λ are chosen as 20 and 0 respectively.

The fault detection time for the fault current shown in Fig. 4-a, varing the sampling frequency and data window is illustrated in Table 1.

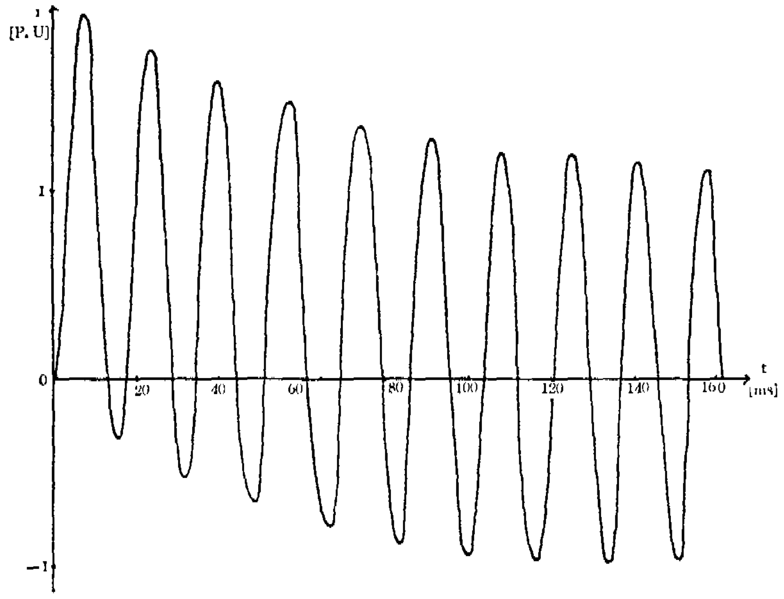

Fig. 4-a. Internal fault waveform

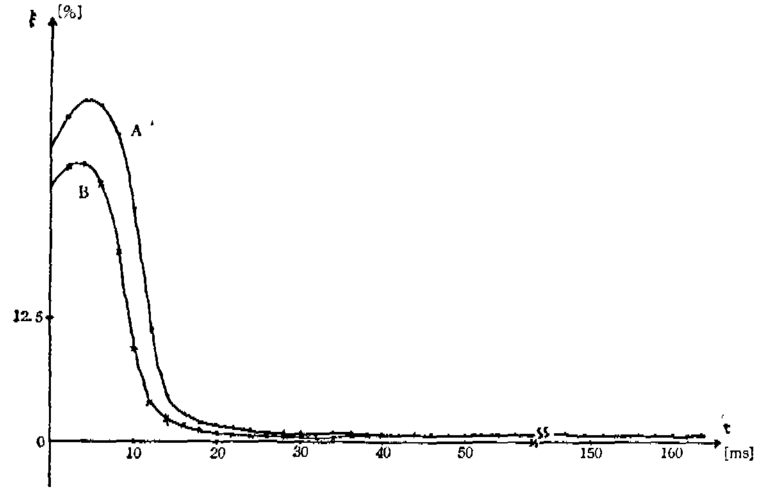

Fig. 4-b. Performance for internal fault: f_s=480 Hz
A: 8 samples B: 6 samples

Table 1. Fault detection time.

data window \ sampling freq	360	480	600Hz
6	1.00	1.00	0.90
7	1.17	1.00	1.00
8	1.17	1.13	1.10
9		1.13	1.10
10			1.20

* unit: cycle based on a 60 Hz waveform

From Table 1, it can be seen that the fault detection time is getting reduced as the sampling frequency increases on the same data window and as the data window decreases on the same sampling frequency.

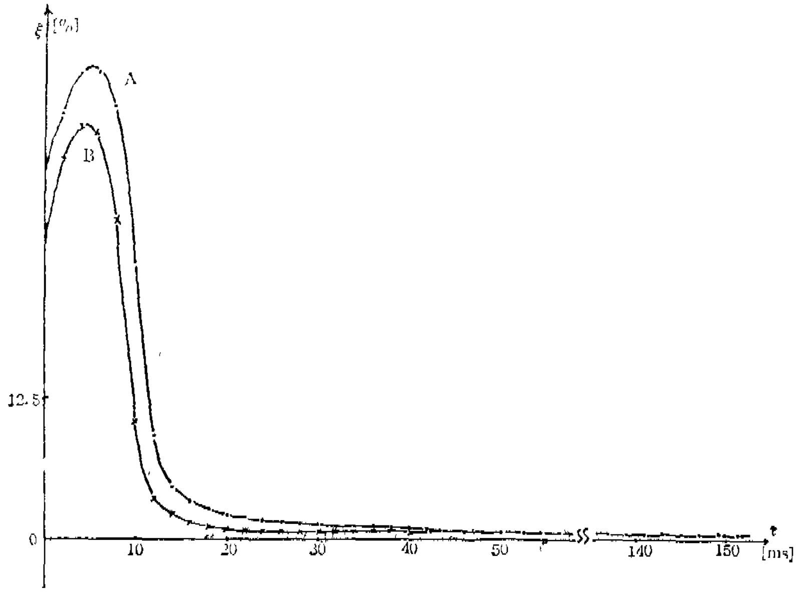

Fig. 4-c Performance for internal fault: f_s=360Hz
A: 8 samples B: 6 samples

Ⅳ. Conclusions

An inrush current detection algorithm based on a least square curve-fitting of the sampled differential current is described with a differential overcurrent algorithm.

The differential current is estimated by a decaying DC component of which the time constant is unknown and the fundamental and second harmonic components.

The proposed algorithm demonstrates the eccelent function of distinguishing the inrush from the internal fault at a low sampling frequency (6 samples per cycle) and the fast fault detection time varing from 1 to 1.2 cycles based on a 60 Hz waveform. The fault detection time is getting short as the sampling frequency increases and the time windom decreases.

One half cycle of time delay successfully secures the reliable trip operation for the fault without sacrificing the fault detection time and futhermore, automatic blocking initiated by the successive differential currents that are within threshold level during more than one-third cycle improves the decision scheme in speed.

Appendix

1. Simulated internal fault current

A *differential current* due to an internal fault in an unloaded single phase transformer is given by

$$i(t)=\sin(\omega t+\lambda-\gamma)-\exp\{(-\omega t+\lambda)/Q\}\sin(\lambda-\gamma) \qquad (1)$$

where $Q=X/R$ and $\gamma=\tan Q$

λ=switching angle

The simulated fault current data are generated by eq. (1) with $\lambda=0$, $\pi/8$, $\pi/4$ and $Q=10, 20$.

2. Simulated *inrush current*

A differential current due to an inrush in an unloaded single-phase transformer is given by

$$i(t)=\frac{1}{Z}\exp(-\alpha t)\sin(\omega t+\lambda-\gamma)$$
$$-\exp\{(-\omega t+\theta_s)/Q\}$$
$$\sin(\theta_s+\lambda-\gamma)$$

: above saturation (2-a)

$i(t)=0$: below saturation (2-b)

where $X=X_s+X_{SAT}$

$Z=|R+jX|$

$Q=X/R$

$1/\alpha$: time constant for decay of remnant flux

θ_s: saturation angle

It is necessary to determune when $|\phi|>\phi_s$ and this is accomplished using the following relationships for core flux:

$\phi=-\phi_{Max}\cos(\omega t+\lambda)+\exp(-\alpha t)\ (\phi_R+\phi_{Max}\cos\lambda)$

where ϕ_R: remnant flux

ϕ_{Max}: peak steady-state amplitude of flux

The simulated inrush current data are obtained by eq (2) with $Q=10, 20$, $\theta_s=0.3$ and $\lambda=0$, $\pi/6$, $\pi/3$.

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