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Comparison of Sales Response Models

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<Abstract>

The objective of this paper is to compare sales response models based on the Koyck model Advertising is assumed to have distributed carryover effects on sales Brand Loyal model without serial correlation of error terms was compared with the reduced Koyck model in terms of recovery of parameters by using the simulated data Also consumer-specific variables were added to the model in the estimation with the real data. In the simulated data generated by using the Koyck model, the Koyck model was better in recovering the parameters than the Brand Loyal model. Therefore, if the wrong model is used for estimation, the wrong result may come out. In the real data of IRI (Information Resources Incorporated), the Koyck model is better than the Brand Loyal model but not much different in R^2 and root mean square error terms. In addition, some signs of variables are different from the expected ones.

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<요 약>

이 논문의 목표는 코익(Koyck) 모형에 기초하여 매출반응모형을 비교하는 것이다. 광고는 매출에 이월효과를 갖는 것으로 가정된다. 모의(Simulated) 자료를 이용하여 자기상관 오차항을 갖지않는 Brand Loyal 모형이 축소된 코익 모형과 모수의 추정값에 의해 비교되었다. 또한 소비자 특성변수들이 실제 자료를 이용한 모형추정에서 추가적으로 이용되었다. 코익모형에 기초하여 모의추출된 자료를 이용하여 각 모형들을 추정한 결과 코익모형

이 모수추정에 있어서 우수하게 나타났다. 따라서 자료와 일치하지 않는 모형이 이용되어질 경우 잘못된 결과가 산출될 수 있다 IRI사(Information Resources Incorporated)의 실제 자료를 이용하여 모형추정을 한 결과 코익모형이 Brand Loyal model과 R^2 값이나 RMSE (root mean square error)값에 있어서 보다 우수하게 나타났으나 크게 차이가 없었으며 여러 변수의 부호가 기대와 다르게 나타났다

Introduction

The relationships between sales and marketing mix variables have been of considerable interest to both marketing managers and marketing model builders Marketing model builders are interested in marketing response functions, that is, in explaining how marketing mix variables affect sales According to Vidale and Wolfe (1957), the optimum budget size and allocation of resources can be determined once the relation between sales response and marketing mix variables has been established Therefore, it is important to specify the model that reflects the true relation in the world

The objective of this paper is to compare performances of sales-response models and to identify which model performs better in parameter recovery or fit The simulated data were generated by using the Koyck model Therefore, the true model for the simulated data was known By using the simulated data, alternative models were estimated and compared. The purpose of using the simulated data is to see what happens if the different model from the one used in generating the data is applied for estimation Next, the scanner panel data were used Unlike the simulated data, the true model could not be known from the real data In addition, estimators may be inconsistent if some omitted variables are correlated with the included variables in the model So, the sales response function to be estimated must reflect the real world In this paper, some consumer-specific variables were added to the model used in the simulated data

Literature Review

Since Palda (1964) applied lagged variable models to the marketing problem of the effect of advertising on sales, researchers have studied the cumulative effects of advertising on sales Also the effects of two or more marketing mix variables on sales have been studied. The major issues in these econometric models of sales and marketing mix variables can be summarized as follows

1) Whether current and lagged variables of advertising are included in the model?

2) Whether other marketing mix variables are included?
3) Whether variables other than advertising have lagged effects or lagged sales is used as a general term of lagged effects of marketing variables?
4) Whether non-marketing variables such as consumer-specific variables are included in the model?
5) Whether error terms are white noises with mean 0 and variance σ^2, that is, white noise or have serial correlations in the model?
6) Whether the model has additive form or product form?

The first five issues represent the elements of the sales response functions such as explanatory variables and error terms. The last issue represents the form of response functions. Each issue will be explained in detail next.

Inclusion of only current and lagged advertising: The following studies included advertising only as an explanatory variable Bass and Clarke (1972) tested six different distributed advertising lag models by using ordinary least squares (OLS) and maximum likelihood (ML) method. Their results indicate that the model with lag sales, current advertising, and lag advertising on the right hand side is preferred against other models in terms of significance of parameter estimates and analysis of residual variance. The relationship between models of sales and advertising was explained by Weiss and Windal (1980) in terms of values of lag advertisings coefficient, lag sales coefficient, and serial correlation coefficient They used the partial adjustment model in which error terms are white noise. They claimed that the autoregressive version of the partial adjustment model best explains the annual Pinkham data used in Palda's study among the six models compared. They also concluded that substantial cumulative effects and serial correlation appear to exist in that Pinkham data. Bass and Leone (1983), Weiss et al. (1983), Srinivasan and Weir (1988) all tried to recover the microparameters in aggregated data by using the Koyck sales-advertising model and their own approximation methods Their approximation methods have bias from the true value because the aggregated data are used to recover the microparameters. Other studies, however, did not consider the lag effect of advertising on sales, that is, the Current Effect model was tested

Vanhonacker (1984) tested a sales response model with data aggregated over time by using the autoregressive Current Effect model If serial correlation exists, OLS may have underestimated standard errors and result in the incorrect acceptance of some predictors. Kanetkar et al. (1986) used the autocorrelated Current Effects model and obtained a good recovery of microparameters with aggregated data from Monte Carlo experiment. In both cases, the lag sales is added to the right hand side after transformation Clark (1976) gave a summary of the cumulative effects of advertising in a survey of studies on the sales-advertising relationship and claimed that the lag effect exists

Inclusion of other marketing mix variables: The next issue is whether the other marketing mix variables are included in the model Hagerty et al. (1988) defined sales response function as a product form to estimate elasticities of marketing mix variables on sales. They took the logarithm of the equation. They related the elasticities of quality, price, advertising, promotion, sales force, and market size to industry/market attributes. The absolute magnitude of price elasticity on sales is around 0.9, elasticities of advertising and promotion are less than 0.2, quality and salesforces elasticities are around 0.3. So the price elasticity is largest in the absolute value They also showed that there is a significant positive correlation between elasticities of advertising and price.

Inclusion of lagged sales as a general lag effect in the non-reduced form: The next consideration is whether these marketing mix variables have lag effects Houston and Weiss (1975) introduced the Brand Loyal model in which the lagged sales is simply rationalized as being attributable to marketing carryover effects that are more general than just advertising. Therefore, current sales is a function of current advertising and lag sales in the model even without transformation They compared the performances between the Koyck model and the Brand Loyal model. The Koyck model is rejected in favor of the Brand Loyal model. Weiss et al (1983), and Bass and Leone (1986) used the Brand Loyal model in recovering the microparameters by using macro data

Inclusion of other variables: When variables other than advertising are used in the Koyck model, their lag variables are added to the right hand side of the reduced form Palda (1964) just added advertising, a dummy variable of advertising copy, trend, and disposal personal income to the reduced form of the Koyck model with lag sales and current advertising He assumed that error terms have first order serial correlation and that lag coefficient of carryover effect equals the coefficient of serial correlation So, no correlation is assumed between error terms and lag sales such that OLS can be used Peles (1971) used advertising, relative prices, and private consumption on the right hand side of the model And then he got the reduced form that has lag sales, current advertising, current and lag relative prices, and current and lag private consumption But he ran a regression by using just first order differencing of sales, prices, and consumption.

Treatment of error terms: Different treatments of error terms make the model and estimation method different. Houston and Weiss (1975) tested the role of serial correlation in the cumulative advertising effect They concluded that carryover effect still exists, which is distinct from serial correlation because carryover effect is statistically significant. Weiss et al. (1983) showed that the coefficient of serial correlation decreases to zero as the level of time aggregation increases in simulated data

Functional forms of sales response models: The sales response models may have different functional forms as additive or product form. The transformed model of product forms of independent variables has the additive form after taking logarithm. Nerlove and Waugh (1961) used the product form of sales, quantity of sales, income, and advertising; they took logarithms of the equation.

Estimation in Econometric Models

In relation to the first issue mentioned in literature review, this paper tested for the existence of lag effect by the significance of the coefficient of the lag variable of advertising in the Koyck model. With regard to the second issue, the model with advertising was extended with more marketing mix variables. In addition, the lagged effects of marketing variables as a general term were investigated by using the Brand Loyal model. Regarding the fourth issue, non-marketing mix variables such as family size, income, and female age are added to the models with real data. The fifth issue was investigated with the Koyck model and the Brand Loyal model having different error terms. The models in this paper have additive forms. From these, two models were compared 1) the Koyck model, and 2) the Brand Loyal model The Koyck model was used as the baseline model. A simple Koyck model (Johnston, 1984) is as follows

$$S_t = c + \beta A_t + \beta\lambda A_{t-1} + \beta\lambda^2 A_{t-2} + \ldots + e_t \quad (1)$$

Where e_t is identically and independently distributed error term with mean 0 and variance σ^2,
S_t is the sales at time t,
A_t is advertising expenditure at time t,
c is intercept,
β is the current effect of advertising, and
λ is the carryover effect of advertising.

The above model has a lot of explanatory variables on the right hand side of the model. Therefore, the reduced model is used instead of estimating the above model directly.

$$S_t = c(1-\lambda) + \lambda S_{t-1} + \beta A_t + v_t \quad (2)$$

Where $v_t = e_t - \lambda e_{t-1}$.

The Koyck model (1) with advertising is extended with more explanatory variables in this paper.
The following model is like this.

$$S_t = c + \beta A_t + \beta\lambda A_{t-1} + \beta\lambda^2 A_{t-2} + \ldots + \gamma X_t + e_t \quad (3)$$

Where X_t is a vector of variables other than advertising at time t, and γ is the coefficient vector of the X vector.
Then,

$$S_{t-1} = c + \beta A_{t-1} + \beta\lambda A_{t-2} + \beta\lambda^2 A_{t-3} + .. + \gamma X_{t-1} + e_{t-1} \quad (4)$$

So the reduced form is

$$S_t = c(1-\lambda) + \lambda S_{t-1} + \beta A_t + \gamma X_t - \gamma X_{t-1} + v_t \quad (5)$$

Where $v_t = e_t - \lambda e_{t-1}$.

The first interest is the significance of carryover (cumulative) effect of advertising So, the compared model is the one with no carrryover effect

$$S_t = c + \beta A_t + \gamma X_t + e_t \quad (6)$$

The significance of lag effect of advertising was investigated in this paper. Another model to be compared is the Brand Loyal model. The extended Brand Loyal model with other variables including advertising is as follows:

$$S_t = c + \lambda S_{t-1} + \beta A_t + \gamma X_t + e_t \quad (7)$$

Later, the performances of the Koyck model and the Brand Loyal model were compared in terms of parameter recovery and fit.

In the Koyck transformed model (2), $v_t = e_t - \lambda e_{t-1}$ The problem is that v_t is correlated with S_{t-1}, one of the explanatory variables.

$$E(v_t S_{t-1}) = E\{(e_t - \lambda e_{t-1})(c + \beta A_{t-1} + \beta\lambda A_{t-2} + \beta\lambda^2 A_{t-3} + \quad + e_{t-1})\}$$
$$= -\lambda\sigma_e^2$$

So OLS or Generalized least squares (GLS) estimates of the coefficients are inconsistent because error terms are correlated with the explanatory variable Two main methods of obtaining consistent estimates are Instrument Variables (IV) and Maximum likelihood method (Johnston, 1984).

Instrument variables are as follows (Johnston, 1984)
Sales is a function of Z

$$S_t = \alpha Z_t + v_t \quad (8)$$

Where S_t is sales at time t,
Z_t is a vector of explanatory variables,
α is the coefficient of Z,
v_t is error term

Suppose that there is another vector of variables W having the same dimension as Z such that

$Plim(1/n*W'Z)$ = positive definite,

$Plim(1/n*Wv) = 0$,

Then,

IV estimator $= (W'Z)^{-1}W'S$

$= \alpha$.

Also, IV estimation of $S_t = \alpha Z_t + v_t$ with instruments W is the same as replacing Z_t with predicted value of Z_t from OLS regression.

Then Z_t is expressed like the following:

$$Z_t = W_t \delta + e_t \qquad (9)$$

In the model (5) to be estimated with the simulated data in this paper, Z is a vector of lag sales, advertising, price, and lag price; W is not correlated with v. So, the following reduced Koyck model was used in the simulated data

$$S_t = c(1-\lambda) + \lambda S_{t-1} + \beta A_t + \gamma P_t - \gamma P_{t-1} + v_t \qquad (10)$$

Where $v_t = e_t - \lambda e_{t-1}$

In the present model, there is no need to replace advertising (A), price (P), and lag price because they are already assumed to be independent of the error terms, as instruments for the lag sales, the predicted value from OLS of lag sales on lag advertising, first order and second order lags of price will be used because the predicted valued is also independent from the error terms.

Estimation in serial correlation

If the coefficient of serial correlation (ρ) is known and λ equals ρ in the Koyck model, the following method can be used.

$$S_t = c(1-\lambda) + \lambda S_{t-1} + \beta A_t + \gamma P_t + u_t \qquad (11)$$

Where $u_t = \rho u_{t-1} + e_t$

e_t is a white noise.

Then,

$$S_t - \rho S_{t-1} = c(1-\rho) + \rho A_t + \gamma(P_t - \rho P_{t-1}) + e_t \qquad (12)$$

Where $e_t = u_t - u_{t-1}$

Now OLS can be applied for estimation because there is no correlation between explanatory variables and error terms.

If λ does not equal in ρ the Koyck model, then Cochrane-Orcutt procedure may be used by using ρ in the reduced Koyck model.

$S_t - \rho S_{t-1} = c(1-\lambda)(1-\rho) + \lambda(S_{t-1} - \rho S_{t-2}) + \beta(A_t - A_{t-1}) + \gamma(P_t - P_{t-1}) + \gamma\lambda(P_{t-1} - P_{t-2}) + q_t$ (13)
Where $q_t = v_t - v_{t-1}$,
ρ is estimated by using the formula, $\Sigma_{t-2} v_t * v_{t-1} / \Sigma_{t=2} v_t^2$

In the Brand Loyal model
$S_t = c + \lambda S_{t-1} + \beta A_t + \gamma P_t + e_t$ (14)
Where $e_t = e_{t-1} + r_t$,
r_t is a white noise.

Then S_{t-1} and e_t, the right hand side variables of the model, are correlated because e_t is a function of e_{t-1}, so the differencing of the first order (Cochrane-Oructt procedure) can be used such that OLS is applied
$S_t - \rho S_{t-1} = c(1-\rho) + \lambda(S_{t-1} - S_{t-2}) + \beta(A_t - \rho A_{t-1}) + \gamma(P_t - \rho P_{t-1}) + r_t$ (15)

Results and Discussion

For the purpose of model specification, the simulated data were generated by using the following Koyck model.
$S_t = c(1-\lambda) + \lambda S_{t-1} + \beta A_t + \gamma P_t - \gamma P_{t-1} + v_t$ (16)
Where $v_t = e_t - \lambda e_{t-1}$.
(Price was used as the other variable for simplification at this time.)
1200 values were generated for each variable with the following parameters
The values set[1] were $c(1-\lambda) = 1000$, $\beta = 1.2$, Var(A) = 10,000, σ_e^2 = 10,000, and E(e) = 0 And $\lambda = 0.6$, $\gamma = 3$, E(P) = 5, and Var(P) = 1
The matrix of correlation is like this

<Table 1>
Correlations

	S_{t-1}	v_t	e_t
v_t	-0.2097		
e_t	0.0396	0.8534	
e_{t-1}	0.4681	-0.4956	0.0298

There is a significant negative correlation between lag sales S_{t-1} and error terms v_t. So OLS and GLS can not be applied for estimation. Therefore, IV method is used for Koyck model. When the same model is fitted, the parameters have to be recovered. In this case, the similar parameters have to reproduced when the Koyck model is fitted

[1] The same values as in the Weiss et al (1983)'s study were used for comparison

<Table 2>
Estimates of Parameters

	λ	β	γ	$\gamma * \lambda$	$c(1-\lambda)$	R^2
True model	0.6	1.2	3.0	-1.8	1000	
Koyck model	0.61*	1.2*	2.7	-1.3	944*	0.64
Brand Loyal model	0.49*	1.18*	1.74		1640*	0.66

* means that the coefficient is significant at the0.05 level of p

All coefficients in Table 2 are significant except those of price and lag price. Values of price and lag price variables are small in comparison to those of other variables, price and lag price have a little portion in explaining sales The carryover effect λ is significant, so the Current Effect model is rejected in this simulated data. There is a difference between the Kovck model and the Brand Loyal model in recovering the parameters. All coefficients except β are much different from the true parameters in the Brand Loyal model, that is, the estimators are not consistent So, if error terms have the first order of moving average, the Brand Loyal model with white noise is not good in recovering the true parameters.

In addition, the scanner panel data were used to test the sales response models in the disaggregated data Generally, sales response models use marketing mix variables at the company level because data about price, advertising expenditure and sales can be obtained in the first place from the company. So, the macroeffects of marketing mix variables at the company level have been studied. On the other hand, the effects of marketing mix variables to consumers can be investigated by testing the sales response models at the consumer level as opposed to the company level. The sample product is laundry detergent frequently purchased Information Resource Incorporated (IRI) gathered the data over 88 weeks period in Eau Claire, Wisconsin from January 24, 1983 to September 17, 1984 via single source collection, that is, panelists buying behavior was traced over time by using optical scanners at the check-out counter of supermarkets and then supplementing this information with panelists profiles, advertisements, promotions[2] Volume of each purchase was used as sales. Advertising exposure was measured by the total seconds of viewing commercials Price per unit was used as a price variable. Deal, feature, display were 1 if they existed and were 0 if they did not exist Deal is the price promotion such as price cut Feature is the retailer advertising of the product in the local newspaper. Display means the special display of the product at the beginning, middle, or the end of the aisle, or at times in other places. Also, non-marketing mix variables such as family size, income, and

[2] This data set is one of the few data sets provided by IRI for academic research and was used for several studies

female age were added to the reduced Koyck model and the Brand Loyal model because omitted variable bias is caused if omitted variables are correlated with the variables included in the model. The magnitudes of correlation coefficients between marketing mix variables such as price, advertising, deal, feature in media, display in store, and demographic variables such as family size, income, and female age range between 0.02 and 0.05 (see appendix 1). Total size of observation used is 1,095.

In this scanner panel data, the coefficient of the lag sales is significant such that carryover effect of advertising exists. Therefore, Current Effect model representing no carryover effect is rejected. The effect of advertising exposure is small. Definitely, the portion of advertising in explaining sales may be smaller if other variables are included in the model. However, carryover effect of advertising is significant such that lag advertising of higher order may affect sales. Price has positive effect in opposition to the expectation. Lag effect of price is negative. It may suggest that some omitted variables are correlated with price. The model also may not represent the data although R^2 is high. Among promotions, feature and display have positive effect. Demographic variables have negative effects. These negative effects are not clear to interpret. The frequency of purchase may compound the effects. In comparison between the

<Table3>
Estimates of Parameters

Variables	Koyck model	Brand Loyal model
Lag sales	0.38*	0.41*
Family size	-0.29*	-0.32*
Income	-0.03	-0.05*
Female age	-0.25*	-0.34*
Price	1.39*	0.95*
Deal	-0.17*	-0.14*
Feature	0.37	0.12
Display	0.34*	0.11
Advertising exposure	0.004	0.005*
Lag price	-0.55*	
Lag deal	-0.002	
Lag feature	-0.11	
Lag display	-0.73*	
Lag advertising exposure	0.003	
Adjusted R^2	0.77	0.70
Root mean square error	0.98	1.15

* means that the coefficient is significant at the 0.05 level of p

Koyck model and the Brand Loyal model in terms of R^2 and root mean square error, the Koyck model is better but is not much different from the Brand Loyal model.

The first order serial correlation in the Koyck model was checked by using the Breusch and Godfrey method. The result[3] showed that the serial correlation coefficient is statistically significant. But it is small (0.085) when Cochrane-Orcutt procedure is used, the result is not much different from the one without that method (see appendix 2).

Conclusion

Model specification is the first step to identify the sales response model. In marketing studies, there is a tendency that only marketing mix variables are included in the model. Omitted variables cause the estimators to be inconsistent if those variables are correlated with the variables included in the model. Therefore, those non-marketing mix variables have to be identified and included in the model. Especially model specification is important in using real data because the true model is not known and has to be identified Therefore, this paper included some consumer-specific variables in the model.

The next step is the estimation by using the model. Each estimation method has its own assumptions. If those assumptions are violated, then that estimation method should not be used. If the sales response models have lag sales as one of the regressors, there is a correlation between the lag sales and error terms. Therefore, OLS and GLS should not be applied in those cases ML or IV methods can be used to avoid the inconsistency of the estimates caused by the correlation between the lag sales and error terms. In this paper, the IV method was used to avoid the correlation between explanatory variables and error terms Then the model selection among alternative models has to be based on the theory and testing In the data used in this paper, the Koyck model is better. This paper compared the recovery of the parameters of the models by using the simulated data and then applied the model by using the real data. But the result with the real data showed that the Koyck model is not much better. In addition, some signs of variables are different from the expected ones. Therefore, more studies need to be done to clarify the effects of marketing mix variables and non-marketing variables

Moreover, some other relevant variables affecting the sales may be omitted in this paper. For example, competition may affect sales, so the effect of competition may be considered in the sales response functions. Therefore, the comprehensive study has to include all relevant variables that affect the sales, including non-marketing mix variables including competition

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3 $T*R^2$ follows the X^2 distribution where T is the order of serial correlation and R^2 is the statistical fit from the regression $T*R^2$ is greater than the critical value, 3.84 with degrees of freedom 1

Appendix 1
Correlation Matrix

	Family size	Income	Female age
Price	-0.11*	0.25*	0.13*
Deal	0.09*	-0.07*	0.08*
Feature	-0.02	-0.02	-0.04
Display	-0.14*	-0.18*	0.23*
Advertising exposure	0.02	-0.03	-0.01
Lag volume	-0.13*	0.20*	0.04
Lag price	-0.11*	0.25*	0.14*
Lag deal	0.09	-0.07	0.08
Lag feature	-0.03	0.02	-0.03
Lag display	-0.14*	-0.18*	0.23*
Lag Advertising exposure	0.02	-0.03	-0.02

* means that the coefficient is significant at the 0.05 level of p

Appendix 2
Comparison of the Koyck models between without and with C-O procedure

Variables	Koyck model without C-O	Koyck model with C-O
Lag sales	0.38*	0.47*
Family size	-0.29*	-0.23*
Income	-0.03	-0.02
Female age	-0.25*	-0.19*
Price	1.39*	1.37*
Deal	-0.17*	-0.20*
Feature	0.37	0.42*
Display	0.34*	0.40*
Advertising exposure	0.004	0.003
Lag price	-0.55*	-0.66*
Lag deal	-0.002	-0.003
Lag feature	0.11	0.09
Lag display	-0.73*	-0.78*
Lag advertising exposure	0.003	0.001
Adjusted R2	0.77	0.74
Root mean square error	0.98	0.97

* means that the coefficient is significant at the 0.05 level of p

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