

A Note on the Reinvestment Rate in Capital Budgeting Analysis

Beom Joon Yu

Department of Management

(Received December 21, 1981)

〈Abstract〉

The purpose of this paper is to demonstrate that the correct assumption for reinvestment rate where financial capital is freely available at any point in time is not the firm's marginal cost of capital but the average rate of return on new investment.

If the profit maximization firm pushes investment to the point where the marginal cost of capital is equal to the marginal rate of return, then the optimal capital budget will be made. Since the rates of return (IRR & ARR) are always assumed to be negatively downward sloping and there is also an average internal rate of return schedule on new investment, the rate of return on reinvestment from intermediate cash flow must be equal to the average rate of return on new investment, not the marginal cost of capital.

It is also demonstrated that the internal rate of return method is superior to the net present value method if the marginal cost of capital lies to the left of Fisher's rate, while the net present value method will give the most valid ranking of mutually exclusive proposals if the average rate of return lies to the left of Fisher's rate.

資本豫算分析에 있어서 再投資利益率에 관한 研究

俞 凡 濬

經營學科

(1981.12.21 접수)

〈要 約〉

本 研究에서는 資本豫算에서 再投資利益率이 前提로 하는 올바른 假定은 限界資本費用이 아니라 平均資本收率이라는 論理를 전개하였다.

相互排他的인 投資率의 經濟性分析에서 純現價法과 内部收益率法이 投資優先順位에서 不一致하는 근원으로 再投資利益率의 暗示的 假定을 考察하였다.

어떤 市場形態를 表明하고 限界利潤이 존재하는 限 投資가 계속되고 限界收入과 限界費用이 일치하는 點에서 利潤極大點이 형성된다는 企業의 利潤極大化를 위한 行動準則에 의거 新規投資의 收率은 항상 投資額에 대한 負의 函數로 나타내므로 再投資의 收率은 新規投資의 平均資本收率과 일치하여야 한다는 논리가 성립될 수 있다. 또한 이 論理는 投資決定基準으로서 純現價法과 内部收益率法의 制限의 妥當性을 支持하여 준다.

I

It is well recognized that under certain circumstances the two most commonly preferred criteria in capital budgeting analysis—net present value method (NPV) and internal rate of return method (IRR)—can and occasionally do give conflicting signals as to investment choices. From a theoretical viewpoint, use of net present value method is considered to have fewer ambiguities than use of internal rate of return method in making investment decisions.¹⁾ But, from a practical viewpoint, internal rate of return method has a long standing and substantial popularity in use.

Many articles²⁾ have appeared noting the reinvestment assumption. The point made by Solomon[11] in his original paper was that:

"The valid comparison is not simply between two projects but between two alternative courses of action. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date of the longer-lived project. In order to make a fair comparison, an explicit and common assumption must be made regarding the rate at which funds released by either project can be reinvested up to terminal date."³⁾

There are two basic conditions under which the net present value method and internal rate of return method may give different rankings: (1) scale effects—the cost of one project is larger than that of other, (2) timing effects—the timing of cash flows from the two projects differs, with one producing high cash flows in later years. Although scale and timing are the basic conditions that lead to conflicts, the basic source of the conflicts can be often traced to the different implicit assump-

tions about the reinvestment rate of cash flow receipts from the alternative investments.⁴⁾

Dudley[5] has presented that while neither the NPV nor IRR criterion makes any assumption about the reinvestment of cash flows, the selection of an optimal criterion must be accompanied by the explicit reinvestment rate assumption if the two criteria yield conflicting rankings. And he demonstrates that if the rate of return in reinvestment is less than the Fisher's rate⁵⁾ the present value criterion will lead to the selection of that project which will result in the greatest value at the terminal date, while if reinvestment is assumed to be made at a rate of return higher than the Fisher's rate, the internal rate of return criterion will provide optimal decisions.⁶⁾

To illustrate the nature of the problem, let us consider two mutually exclusive proposals. In Figure 1, proposal I is preferred if we use the present value method, whereas proposal II is preferred if we use the internal rate of return method.

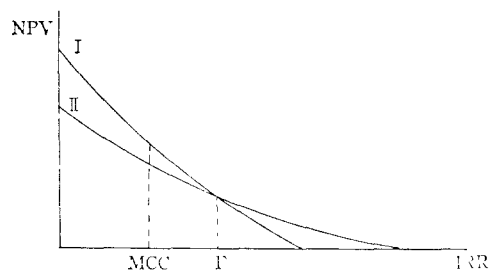


Fig.1 Two Alternative Options

MCC: firm's marginal cost of capital

F: Fisher's rate of return over cost

Whenever the NPV profiles of two projects cross one another, a conflict will exist if the cost of capital is below the cross-over rate

1) See Conrad Doenges [4], p.85.

2) Akhan[1], Conrad Doenges[4], Dudley [5], Hirschleifer[7], Solomon[11].

3) Solomon [11], p.127.

4) Brigham[3], p.304.

5) Irving Fisher[6] defined the rate of return over cost or the rate of return on sacrifice as the rate of discount that will cause the present values of two optional income streams to be equal. This rate of discount is referred to as "the Fisher's rate."

6) Dudley[5], p.913.

(Fisher's rate of return on sacrifice between these two options). Basically, the conflict has to do with the investment of cash flows—the NPV method implicitly assumes reinvestment at the marginal cost of capital (MCC), while the IRR method implicitly assumes reinvestment at the internal rate of return (IRR).⁷⁾ This apparent inconsistency in ranking can be traced to the differing assumptions that the two ranking criteria make concerning reinvestment rates. If the investment projects are mutually exclusive, a choice of the correct ranking method must be done.

II

The key problem lies in determining the rate of return to be used for the reinvestment of intermediate cash flows.

Weston and Brigham[13] show that the assumption of reinvestment at the cost of capital, which is implied in the case of the NPV method, is the better one and that this is, in turn, at least for firms willing and able to obtain capital at a cost reasonably close to their current cost of capital. This means, in effect, that intermediate cash flows are reinvested to yield the cost of capital.

Dudley [5] suggests that if financial capital is freely available at any point in time, the investment is always up to the point where the marginal project yields the marginal cost of capital and that the appropriate reinvestment rate must be the firm's marginal cost of the alternative funds, i.e., the firm's cost of capital. In such a case, if the marginal cost of capital lies to the left of Fisher's rate, the present value criterion is to be preferred.

If capital rationing is assumed, the choice becomes more difficult. If the problem is restricted to a single investment in the present and a single future cash return, the appropriate

rate to assume for reinvestment is the return expected on the marginal project in the future period. This marginal rate of return could easily vary from one cash flow period to another and probably would do so.

If Weston and Brigham's and Dudley's statements are true and the marginal cost of capital lies to the left of Fisher's rate, then the NPV approach will give the optimal result.

But this conclusion about the reinvestment rate may be inconsistent with the general assumptions about the firm's investment opportunity schedule.⁸⁾

III

The rates of return on new investment are typically assumed to be negative functions of the amount of new investment which is sloping in all periods. The assumption is perhaps reasonable on a continuing basis because the investment opportunity to earn marginal profit may always exist if technology development occurs.

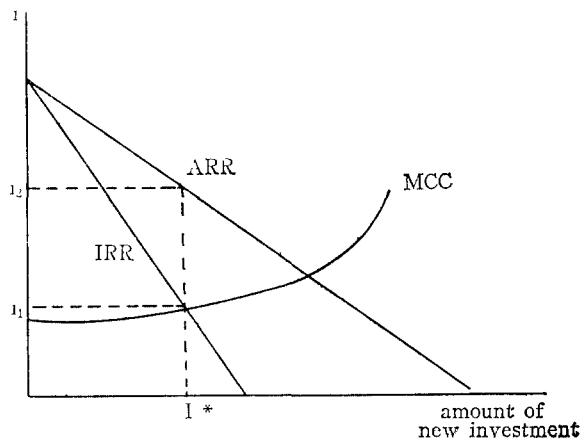


Fig. 2 Optimal Capital Budget

If the firm pursues profit maximization, it will expand to the point where the marginal of cost capital (MCC) is equal to the internal rate of return which is really a marginal rate of

7) Weston & Brigham [13], p. 231.

8) Meyer [10], p. 1251.

return curve as shown by IRR in Figure 2. The optimal capital budget is denoted by I^* . But if the marginal internal rate of return curve is negatively sloped, then there is also an average internal rate of return schedule on new investment as denoted by ARR in Figure 2. If cash flows from previous years' investments are reinvested in the firm, the rate of return earned on these cash flows must be equal to the average rate of return on new investment (i_2 in Figure 2) and not the marginal cost of capital (i_1).

Such an observation is based on the behavioral rules for the profit maximization firm⁹⁾:

Rule 1: A firm should not produce at all if the average revenue from its product does not equal or exceed its average variable cost.

Rule 2: Assuming that it pays the firm to produce at all, it will be profitable for the firm to expand output whenever marginal revenue is greater than marginal cost; expansion must thus continue until marginal revenue equals marginal cost.

It is entirely possible then that when we are selecting among mutually exclusive proposals the internal rate of return method of ranking is superior to net present value even if the marginal cost of capital lies to the left of Fisher's rate. Here we assume that capital rationing does not exist and the firm pushes investment up to the point where the marginal revenue on new investment equals the marginal cost of capital. In Figure 3, we see that the marginal cost of capital lies to the left of Fisher's rate F . But because the firm has a downward sloping investment opportunity curve on new investment, the average rate of return in the optimal capital budget (ARR) lies to the right of F .

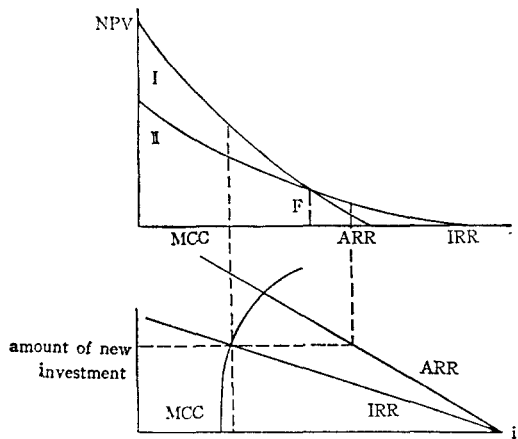


Fig. 3 Location of the Reinvestment Rate

In this illustration, we can see that the internal rate of return would be most appropriate for selecting among mutually exclusive proposals even though there is no capital rationing and the firm invests to the point where marginal revenue equals marginal cost, because the reinvestment rate is equal to the average rate of return on new investment.¹⁰⁾

However, the fact that the use of the internal rate of return leads to the correct decision in a particular case or a particular class of cases does not mean that it is correct in principle. If the average rate of return lies to the left of Fisher's rate, the NPV method will give the most appropriate ranking of mutually exclusive proposals.

IV

This discussion has been limited to a subject that the correct assumption for reinvestment rates without capital rationing is not the firm's marginal cost of capital but the average of rate of return on new investment.

9) These two rules can be restated as three necessary conditions for a firm to be maximizing its profits:

(a) Price is at least as great as average variable cost.

(b) Marginal revenue equals marginal cost, and

(c) The marginal cost curve cuts the marginal revenue curve from below.

Lipsey[8], p.212.

10) Meyer[10], p.1253

The selection of a particular rate of return criterion to be used, however, may imply a weak assumption about the relative rate of return on reinvestment if the two criteria yield conflicting recommendations as to the optimal choice. The consideration of uncertainty in the analysis of a proposed investment may be one of the most important factors making the reinvestment question indecisive. With capital rationing, it should reflect the return that would be lost if an alternative investment of comparable risk were not taken. A more precise determination of an optimal capital investment strategy should make specific assumptions about the rate of return to be earned on the reinvestment of cash flows. If many alternative combinations are possible, a sophisticated programming solution to the resulting problem must be pursued to select the optimal investment set.

References

1. Armen A. Archain, "The rate of Interest, Fisher's Rate of Return over Costs and Keynes's Internal Rate of Return," *The American Economic Review*, Vol. XLV, No. 5, (December 1955), pp. 938-42.
2. Harold Bierman Jr., & Seymour Smidt, *The Capital Budgeting Decision*, Fourth Edition, (New York: The Macmillan Publishing Co., Inc., 1975) pp. 41-68.
3. Eugene F. Brigham, *Fundamentals of Financial Management*, Second Edition, (Hinsdale, Illinois: The Dryden Press, 1980), pp. 216-317.
4. R. Conrad Doenges, "The 'Reinvestment Problem' in a Practical Perspective," *Financial Management*, (Spring 1972), pp. 85-91.
5. Carlton L. Dudley, Jr., "A Note on Reinvestment Assumptions in Choosing Between Net Present Value and Internal Rate of Return," *The Journal of Finance*, Vol. XXVII, No. 4, (September 1972) pp. 907-15.
6. Irving Fisher, *The Theory of Interest*, Reprinted, (New York: The Macmillan Co., 1967)
7. Jack Hirshleifer, "On the Theory of Optimal Investment Decision," *The Journal Political Economy*, Vol. LXVI, No. 4, (August, 1958), pp. 329-52.
8. Richard G. Lipsey & Peter O. Steiner, *Economics*, Fifth Edition, (New York: Harper & Row, Publishers, 1978), pp. 212.
9. James C. T. Mao, *Quantitative Analysis of Financial Decisions*, (New York: The Macmillan Publishing Co., 1969).
10. Richard L. Meyer, "A Note on Capital Budgeting Techniques and the Reinvestment Rate," *The Journal of Finance*, Vol. XXXI, No. 5, (December 1976), pp. 1251-54.
11. Ezra Solomon, "The Arithmetic of Capital Budgeting Decisions," *The Journal of Business*, Vol. XXIX, No. 2, pp. 124-29.
12. James C. Van Horne, *Financial Management and Policy*, Third Edition, (Englewood, New Jersey: Prentice Hall, Inc., 1974)
13. J. Fred Weston & Eugene F. Brigham, *Essentials of Managerial Finance*, (Hinsdale, Illinois: The Dryden Press, 1977), pp. 221-44.