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Determination of Stochastic Layers of a Nonlinear Oscillator

Yoon, Byung Gook and Lee, Seong Jae
Department of Physics
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〈Abstract〉

Stochastic layers of a nonlinear oscillator under perturbing wave field are determined numerically. It is found that the dependences of the layer widths on the perturbing frequency and amplitude are in good agreement with theory and that dissipation diminishes the stochasticity.

비선형진동자의 Stochastic Layer의 결정

윤 병 국 · 이 성 재
물 리 학 과
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〈요　　약〉

섭동파가 있는 비선형 진동자의 Stochastic Layer를 수치계산으로 결정했다. 그것의 섭동파의 진폭과 파장에 대한 의존성은 이론과 잘 맞았으며 또 감쇠에 의해 Stochasticity가 줄어드는 것을 발견했다.

Ⅰ. Introduction

If a nonlinear oscillator is acted on by small periodic perturbations, under certain conditions it may oscillate as if random perturbations acted on it. This special type of motion is called stochasticity, which is peculiar to nonlinear oscillations. These phenomena are interesting because they may connect two different field of physics: nonlinear dynamics and statistical physics.

A semi-quantiative theory called the theory of stochasticity has been developed to study this type of motion.[1] It allows to obtain criteria and characteristics of instability of nonlinear oscillations.

In previous work[2], we introduced a nonlinear oscillator under perturbing wave field. Out of numerical calculations, phenomena of stochastic instability were observed. This oscillator has been studied both numerically and analytically[3]. In this work a method of determining the stochastic layers of the oscillator is developed and the observed layer widths are compared with theoretical predictions.

Ⅱ. Theoretical prediction of stochastic layers[4]

The equation of motion of interest is

$$\ddot{\varphi}+\sin\varphi=\varepsilon\sin(\lambda\varphi-\nu t) \qquad (1)$$

The unperturbed motion is that of a particle in the periodic potential well or of simple pendulum. When small perturbation is present so that $\varepsilon(\neq 0)\ll 1$, stochasticity appears under certain conditions.

To study the stochasticity of this oscillator the time dependent canonical transformation method[5] is used and the problem is formulated in terms of action-angle variables. If the perturbation is in the form of periodic pulse that depend only on the phase (angle variable θ), one method of characterizing the stochasticity is computing the dilatation rate[6] of phase difference

$$K_n = \frac{d\theta_{n+1}}{d\theta_n} - 1$$

where n is the serial number of canonical transformation step. When the average value of K_n, $K \gg 1$, the exponential divergence of the neighboring trajectories occurs.

The condition is applied to this problem and the stochastic boundary is determined by

$$1-\alpha < \varepsilon\nu e^{-\pi\nu/2} \quad \text{for } \nu \gg 1 \qquad (2)$$

and

$$1-\alpha < \varepsilon\nu \quad \text{for } \nu < 1 \qquad (3)$$

where α is initial energy of the oscillator.

Ⅲ. Numerical experiment and results

In order to solve Eq. (1) numerically, we adopted Runge-Kutta methods for first few steps and Adams-Moulton methods. Step size was $\Delta t = 0.01$ and the error was negligible after $10^4 - 3\times10^4$ iterations. Computer calculations were carried out with double precision variables.

We found that the stochasticity is characterized by the fact that the oscillator flings sufficiently to at least one next potential well from the original well. With this observation, given parameters ε, λ and ν, the stochastic layer was determined by binary searching method with varying initial energy α_i. The critical value $\alpha_{c,i}$ was obtained and the results were very different from one another, though they were obtained out of same parameters, depending on initial position φ_0. We calculated the trajectory and energy with the critical value $\alpha_{c,i}$ and found that stochasticity is due to the energy fluctuation caused by nonlinear resonance in the presence of periodic perturbation and that the theoretical values given by Eqs. (2) & (3) correspond to local minimum of energy just before stochasticity appears. The order of magnitude of this value is independent of initial positions and there is strong correlation between $\alpha_{c,i}$'s with same φ_0's and α_c in Eqs. (2) & (3) such that,

$$\alpha_{c,i} \approx (\text{numerical factor}) \times \alpha_c$$

where the numerical factors depend on initial position φ_0.

We compared determined values $1-\alpha_{c,i}$ with theoretical ones for some cases. In Fig. 1 the layer widths are plotted with varying ε when $\nu=10$, $\lambda=0.5$ $\varphi_0=3.0$. Linear dependence on ε is evident as in Eq. (2). The layers with varying $\nu (\gg 1)$ are in Fig. 2. The exponential dependence of $(1-\alpha_{c,i})/\nu$ on ν agrees with Eq. (3) when $\nu < 8$.

In Fig. 3 & 4 are the cases of $\nu < 1$. Linear dependence of layer widths on ε is shown in Fig. 3. The width increases with increasing frequency in Fig. 4 although there are scatterings from the straight line.

Finally we introduced a term $\gamma\dot{\varphi}$ in lefthand side of Eq. (1) and observed the dissipation effect. It has been found that dissipation diminishes the stochasticity and the layer width decreases. The details about dissipation effect

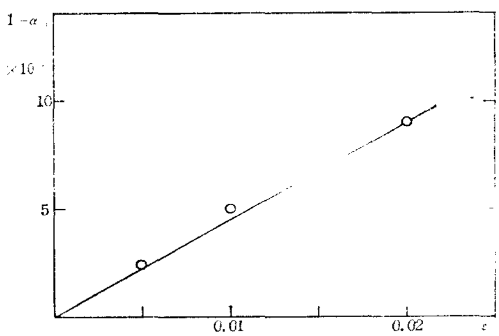

Fig. 1 stochastic layer width vs. perturbing amplitude ($\lambda=0.5$, $\nu=10$, $\varphi_0=3.0$)

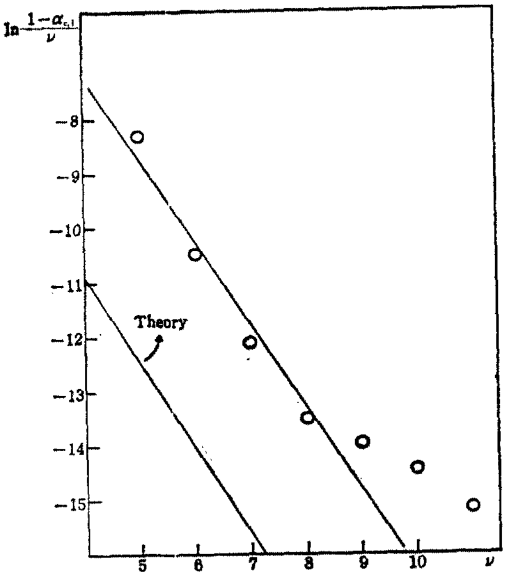

Fig. 2 stochastic layer width vs. perturbing frequency ($\nu>1$) ($\lambda=0.5$, $\varepsilon=0.01$, $\varphi_0=3.0$)

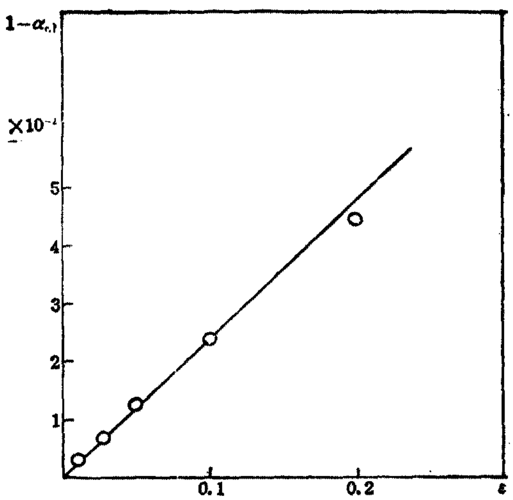

Fig. 3 stochastic layer width vs. perturbing amplitude ($\lambda=0.5$, $\nu=0.1$, $\varphi_0=2.5$)

will be the subject of next work.

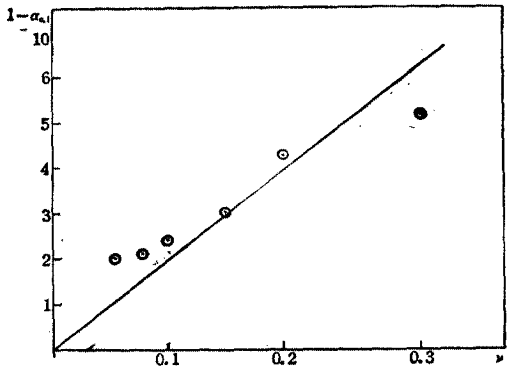

Fig. 4 stochastic layer width vs. perturbing frequency($\nu<1$) ($\lambda=0.5$, $\varepsilon=0.01$, $\varphi_0=2.5$)

Ⅳ. Conclusions

We determined the stochastic layers of a nonlinear oscillator with a simple numerical method. The dependence on parameters ε and ν are in good agreement with theory. Meanwhile, we found that stochasticity is due to energy fluctuation out of nonlinear resonance. Dissipation has the effect of decreasing stochasticity regardless of the perturbing frequency.

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