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Optimal Burn-in Time for Warranted Products with Two Types of Failures

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〈**Abstract**〉

For products with a bathtub-shape harzard rate, burn-in can be used to reduce the warranty cost. This paper examines the optimal burn-in time for products with two types of failures: Type 1 is removed by minimal repair, but type 2 can not be removed.

두가지 유형의 고장이 있는 보증부 생산품의 최적 시험가동기간

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〈요 약〉

浴槽型 고장율을 가진 保證附 생산품에는 보증비용을 감소시키기 위해 시험 가동이 사용될 수가 있다. 본 논문에서는 두가지 유형의 고장이 발생하는 보증부 생산품의 최적 시험가동기간을 결정한다. 고장유형 1은 응급수리에 의해 수리되나 고장유형 2는 수리될 수 있다고 가정하였다.

I. Introduction

Warranty for consumer durable products is becoming an important factor in marketplace, particularly when the product is complex and expensive. The warranty cost may drastically affect the profitability of the manufacturer. Usually the warranty cost involves the failure cost of the product during the initially high harzard rate period (infant mortality). Since the failure cost during the production process is cheaper than the failure cost during the warranty period, burn-in is considered as a means to reduce the warranty cost.

The effect of a burn-in program is to eliminate the infant mortality region so that the finished product will be operating in the region of near constant harzard rate. Such a burn-in program, however, adds directly to the product manufacturing cost.

Warranty cost modeling has been considered by many authors. The product is assumed to have a specified failuretime distribution. For a given warranty policy, the warranty cost per unit has been estimated by Amato et al. (1, 2) for nonrepairable products and by Karmarkar(8) for repairable products. Blischke and Scheuer(5) compared the longrun expected profit with and without warranty. The optimal price and warranty period that maximize the total expected profit have been developed by Anderson(3) and Glickman and Berger(7).

In early models, burn-in has been used as a method of increasing reliability for products with monotonically decreasing harzard rate without considering the problem of costs. The burn-in policy has been analyzed by Weiss and Dishon(11) for nonrepairable products and by Dishon and Weiss(6) for repairable products. For products with a bathtub-shpe harzard rate, the optimal burn-in and replacement times that minimize the expected cost per unit time of service are studied by Plesser and Field(10) and by Nguyen and Murthy(9) under various warranty policies.

However, every product can not be dichotomized into "repairable" and "nonrepairable". In practice, during the running of a product, two types of failures can happen:

Type 1 (slight or repairable) failures are removed by minimal repair.

Type 2 (critical or nonrepairable) failures occur total breakdowns.

This classification is suggested by Beichelt and Fisher(4).

This paper determines the amount of time to burn-in a product with two types of failures before selling it to the consumer. The optimal burn-in time is selected to minimize the total cost (i.e., manufacturing plus warranty costs) under the given warranty policy.

Ⅱ. A General Failure Model

Without burn-in, the failure characteristic of the product with two types of failures is described by:

X : a r.v. denoting the time to the first type 1 failure without an intervening type 2 failure.

$f(x), F(x)$: p.d.f. and c.d.f. of X.

$\bar{F}(x)$: the survival function of X, i.e., $1-F(x)$.

$h_1(x)$: the harzard rate function of X, i.e., $f(x)/\bar{F}(x)$.

Y : a r.v. denoting the time to the first type 2 failure.

$g(y), G(y)$: p.d.f. and c.d.f. of Y.

$\bar{G}(y), h_2(y)$: the survival function and harzard rate function of Y, i.e., $1-G(y)$ and $g(y)/\bar{G}(y)$.

The failure of the product can either be type 1 or type 2. For type 1 failures, it is assumed that the harzard rate of the product remains unchanged after a repair, i.e., "minimal repair" or "bad as old" model. This is a reasonable assumption for complex and expensive products, since the repair involves only a small part of the product.

It is assumed that two types are stochastically independent. From Beichelt and Fischer (4), the expected number of type 1 failures during the time interval $[0, y]$, $N(y)$, is given by

$$N(y)=\int h_1(x)dx \tag{1}$$

After a burn-in time τ, the product is age of τ. Let subscript τ denote the corresponding failure functions after a burn-in time τ (e.g., $F_\tau(x)$ is the failure c.d.f. after a burn-in time τ). For products with burn-in time τ, the expected number of type 1 failures in $[0, y]$ is given by

$$\begin{aligned} N\tau(y) &= \int_0^y h_{1,\tau}(x)dx \\ &= \int_0^y h_1(\tau+x)dx. \end{aligned} \tag{2}$$

Ⅲ. Manufacturing Cost Model

The manufacturing model contains four costs:

C_0 is the manufacturing cost per unit with-out burn-in.

C_1 is the fixed setup cost of burn-in per unit.

C_2 is the cost per unit time of burn-in per unit.

C_3 is the minimal repair cost per type 1 failure.

Let $v(\tau)$ be the expected cost per unit for products with burn-in time τ. The burn-in cost per unit is

C_1+C_2y if $y<\tau$,

$C_1+C_2\tau$ if $y\geq\tau$.

The expected burn-in cost per unit is

$$C_1+\int_0^\tau [C_2y+C_3N(y)]\,dG(y)$$
$$=C_1+C_2\int_0^\tau \bar{G}(y)dy+C_3\int_0^\tau \bar{G}(y)h_1(y)dy.$$

The probability that a unit survives a burn-in time τ is $\bar{G}(\tau)$. Thus the expected manufacturing cost per unit is

$$v(\tau)=[C_1+C_2\int_0^\tau \bar{G}(y)dy + C_3\int_0^\tau \bar{G}(y)h_1(y)dy]/\bar{G}(\tau). \quad (3)$$

Differentiating (3) with respect to τ yields

$$v'(\tau)=C_2+C_3h_1(\tau)+v(\tau)h_2(\tau)>0. \quad (4)$$

Thus the manufacturing cost per unit increases with the burn-in time.

Ⅳ. Warranty Policy and Warranty Cost Model

In this section, we derive the expected warranty cost per unit, $w(T,\tau)$, for products with warranty period T and burn-in time τ. It is assumed that all product failures give rise to valid claims and all rights to claim are exercised.

The warranty policy considered in this paper is the rebate policy. In the rebate policy, the consumer is refunded some proportion of the sales prices P if a type 2 failure occurs during the warranty period $[0,T]$. The amount of rebate, $R(y)$, is a function of the type 2 failure time, y.

$$R(y)=\begin{cases} kP(1-\alpha y/T), & \text{for } 0\leq y\leq T, \\ 0 & \text{, for } y>T. \end{cases} \quad (5)$$

where $0<y\leq 1$, $0\leq\alpha\leq 1$. Two special forms of condition (5) are the lump sum rebate policy ($\alpha=0$) and the pro rata rebate policy ($\alpha=1$, $k=1$). Note that various warranty policies may be considered like Nguyen and Murthy(9).

For a rebate policy, we have

$$w(T,\tau)=\int_0^T [R(y)+(C_3+C_4)N_\tau(y)]\,dG_\tau(y)$$
$$=[T\bar{G}(\tau)-(1-\alpha)T\bar{G}(T+\tau)-\alpha\int_\tau^{T+\tau}\bar{G}(y)dy]\,kP/T\bar{G}(\tau)$$
$$+(C_3+C_4)[\int_\tau^{T+\tau}h_1(y)\bar{G}(y)dy-G(T+\tau)\int_\tau^{T+\tau}h_1(x)dx]/\bar{G}(\tau), \quad (6)$$

where C_4 is the extra cost that arises when a failure occurs during the warranty period. Differentiating (6) with respect to τ yields

$$w'(T,\tau)=\{(1-\alpha)TG(T+\tau)[h_2(T+\tau)-h_2(\tau)]+\alpha[G(T+\tau)-G(\tau)-h_2(y)\int_\tau^{T+\tau}\bar{G}(y)dy]\}kP/T\bar{G}(\tau)$$
$$+(C_3+C_4)\Big\{g(T+\tau)\int_\tau^{T+\tau}h_1(x)dx+h_1(\tau)[\bar{G}(T+\tau)-\bar{G}(\tau)]+h_2(\tau)[\int_\tau^{T+\tau}h_1(y)\bar{G}(y)dy-\bar{G}(T+\tau)\int_\tau^{T+\tau}h_1(x)dx]\Big\}/\bar{G}(\tau). \quad (7)$$

Ⅴ. Optimization Model

Let $C(T,\tau)$ be the expected total cost per unit for a product with burn-in time τ and warranty period T, and $\bar{C}(T)$be the corresponding without burn-in. We have

$$C(T,\tau)=v(\tau)+w(T,\tau). \quad (8)$$

Note that

$$\bar{C}(T)<\lim_{\tau\to 0^+}C(T,\tau).$$

This is due to the fixed burn-in cost $C_1>0$. If $C_1=0$, then

$$\bar{C}(T)=\lim_{\tau\to 0^+}C(T,\tau).$$

Thus for a specified warranty period T, the objective of the manufacturer is

(1) Determine the optimal burn-in time τ^* to minimize $C(T,\tau)$ when burn-in is used.

(2) Compare $C(T,\tau^*)$ with $\bar{C}(T)$. If $C(T,\tau^*)>\bar{C}(T)$, then the optimal policy is to have no burn-in. On the other hand, if $C(T,\tau^*)\leq\bar{C}(T)$, then the optimal burn-in is given by τ^*.

Our analysis deals with the optimal burn-in time τ^*, when burn-in is used. Differentiating

(8) with respect to τ and equating to zero yields a necessary condition for minimum cost:

$$C'(T,\tau)=v'(\tau)+w'(T,\tau)=0. \quad (9)$$

Sufficient conditions for τ^* to be optimal are that it satisfies (9) and that

$$C''(T,\tau^*)>0. \quad (10)$$

Since $C(T,\tau)\to\infty$ as $\tau\to\infty$, by (4), τ^* is always finite. If (9) has no solution then $\tau^*=0$ (i.e., no burn-in).

The optimal τ^* can be found by solving (9) or by directly minimizing (8) using some numerical methods.

Ⅶ. Remarks

In this paper we have studied the problem of optimal burn-in to minimize the expected total cost per unit for products with two types of failures. The considered warranty policy is the rebate policy. Therefore other warranty policies can be investigated. For an example, if type 2 failure occurs during the warranty period, the manufacturer is responsible for all replacement costs.

It is worth stressing that a burn-in policy not only reduces the cost but also increases the goodwill associated with the product, since less failures will occur during the warranty period. Occasionally the latter can be more important.

In practice, the sales of a product is an increasing function of the warranty period. The profit per unit, however, decreases as the warranty period increases. Thus, if the sales responce function is known, the model can be used to simultaneously determine the warranty period and burn-in time to maximize the total profit.

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