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Double Sample Test For the Signed Test

Lee, Hyun-Woo
Dept. of Mathematics
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〈Abstract〉

t-distribution and binomial distribution tend toward the normal distribution for large sample size N, as we know. Using this fact, We can apply the double sample t-test procedure for the mean of a population to the nonparametric signed test for the median of a symmetric population. The purpose of this note is to represent the double sampling signed test procedure.

Signed Test에 대한 중 표본 검사에 관하여

이 현 우
수 학 과
(1981. 12. 30 접수)

〈요 약〉

t 분포와 이항분포는 우리가 잘 아는 바와 같이 표본크기 N 이 커질수록 정규분포에 가까와진다는 사실을 이용하여 모집단의 평균에 대한 중 표본 t 검정과정을 대칭모집단의 중앙값에 대한 비매개적 signed test 에 적용시키는 것이 이 논문의 목적이다.

Ⅰ. Introduction

Double sample test for hypothesis concerning the mean μ of a normal population were suggested by D. Owen. Many authors have presented this method which can be used rather than the single sample t-test for testing hypothesis $H_0: \mu=\mu_0$. Now let us consider fairly large simple random sample that were drawn from a symmetric population. Since t-distribution and binomial distribution tend toward the normal distribution for large sample size N, we can apply the double sample t-test procedure for the mean of a population to the nonparametric signed test for the median of a symmetric population. The purpose of this note is to represent the double sampling signed test procedure.

Ⅱ. Tests

Suppose we are interested in testing at level α the hypothesis $H_0: M=M_0$ against $H_A: M>M_0$ where M is the median of a symmetric population. The usual single sample signed test based on N observations $X_1, \cdots, X_N$ would reject H_0 in favor of H_A whenever $Z_\alpha \leq Z_0$. Here Z_α and Z_0 are defined as

$$Z_\alpha = \frac{(k_\alpha - 0.5) - 0.5N}{0.5\sqrt{N}}$$

$$Z_0 = \frac{(k - 0.5) - 0.5N}{0.5\sqrt{N}}$$

where k is the number of observations above

M_0 and k_α is chosen to be the smallest integer which satisfies $\sum_{k=k_\alpha}^{N}\binom{N}{k}(0.5)^k(0.5)^{N-k}\leq\alpha$. For the double sample test, $N_1(N_1<N)$ observations $X_1,\cdots,X_{N_1}$ are taken and

$$Z_1=\frac{(k_1-0.5)-0.5N_1}{0.5\sqrt{N_1}}$$

is computed where k_1 is the number of observations above M_0 in N_1. If $B<Z_1$ reject H_0; if $Z_1<A$ accept H_0 and if $A\leq Z_1\leq B$ take N-N_1 additional observations. Then the second part of the test requires that for N observations in all

$$Z_2=\frac{(k-0.5)-0.5N}{0.5\sqrt{N}}$$

be computed.

If $Z_2>C$ reject H_0 and if $Z_2\leq C$ accept H_0. We will find the values for A and B by Owen's t-test method and C by Hewett's method. Let G be the distribution function of a standard normal variable and let h be the unique positive number such that $G(h)=1-\alpha$. We can choose A and B to be the unique solutions to the equations (1) and (2)

$$P(Z_1>B)=1-G\left(\sqrt{\frac{N_1}{N}}\cdot h+\theta\right) \qquad (1)$$

$$P(Z_1<A)=G\left(\sqrt{\frac{N_1}{N}}\cdot h-\theta\right) \qquad (2)$$

where θ is a positive constant.

If the type 1 error of the double sample test is to be equal to α, then $P(Z_1>B)+\alpha P(A\leq Z_1\leq B)=\alpha$.

Hence $$(1-\alpha)\left\{1-G\left(\sqrt{\frac{N_1}{N}}\cdot h+\theta\right)\right\}=\alpha\cdot G\left(\sqrt{\frac{N_1}{N}}\cdot h-\theta\right) \text{ holds. } \qquad (3)$$

By equation (3), for each given $\frac{N_1}{N}$ and α there exists one and only one θ. It can be easily shown.

For the double sample procedure C is chosen as the unique solution to the equation

$$P(Z_2>C)=\alpha\left\{G\left(\sqrt{\frac{N_1}{N}}\cdot h+\theta\right)-G\left(\sqrt{\frac{N_1}{N}}\cdot h-\theta\right)\right\}.$$

Ⅲ. Results

The values of h, θ, A, B and C for certain values of $\frac{N_1}{N}$ for each $\alpha=0.01$ and $\alpha=0.05$ can be calculated by (1), (2) and (3) as following table.

Table is a tabulation of the rejection and acceptance points for various values of $\frac{N_1}{N}$ and α, together with a tabulation of θ.

Table. 1

$\frac{N_1}{N}$	θ		A		B		C	
	$\alpha=.01$	$\alpha=.05$	$\alpha=.01$	$\alpha=.05$	$\alpha=.01$	$\alpha=.05$	$\alpha=.01$	$\alpha=.05$
$\frac{50}{100}$	0.755	0.621	0.899	0.542	2.400	1.784	2.922	2.231
$\frac{60}{100}$	0.563	0.457	1.239	0.817	2.365	1.731	3.101	2.395
$\frac{70}{100}$	0.400	0.319	1.547	1.057	2.347	1.695	3.269	2.575
$\frac{80}{100}$	0.255	0.201	1.826	1.270	2.336	1.672	3.487	2.777
$\frac{90}{100}$	0.123	0.095	2.085	1.466	2.331	1.656	3.891	3.048
h	2.327	1.645	2.327	1.645	2.327	1.645	2.327	1.645

If we find how to compute Z_2 with given values Z_1 and $k-k_1$, this procedure will be more useful than usual.

References

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