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"IVB non-locality in μ^-meson decay"

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〈**Abstract**〉

The invariant amplitude of μ^-meson decay is caculated in terms of the IVB(Intermediate Vector Boson). The result is similar to that of the famous 4-point V-A type Fermi interaction calculation with a correction term of order of m^2/M^2, where M is the mass of the IVB and m the masses of the involved particles, e^- and μ^-.

μ중간자 붕괴에 있어서 IVB에 의한 非局所性

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〈요 약〉

μ붕괴의 불변진폭을 IVB 모델로써 계산하였다. 결과는 전통적인 Fermi의 4-point V-A이론과 흡사하나 m^2/M^2의 작은율로 수정항이 나타났다. 이는 관측된 μ^-—붕괴의 전자분포에서의 미세한 assymmetry와 관계있을 것이다.

The 4-point V-A type weak interaction theory of Fermi[1] has known to be an incomplete theory, even though it describes very well the μ^--decay and the nuclear β decay, because of its non-renormalizability (i.e., impossible to calculate higher order graphs). So a renormalizable theory has been searched for and the IVB theory was proposed by Lee and Yang[2]. Later it was developed into the famous Weinberg-Salam model where the QED and the weak interaction was unified using the SU(2) ⊗U(1) gauge symmetry[3]. In that model there appear four gauge bosons, the photon, $W^{\pm}$, and Z respectively. The latter three have acquired masses by the so-called Higgs mechanism[4], in contrast with the massless photon. This means that the electromagnetic interaction is of infinite range, whereas the weak interaction is finite(in fact, nearly pointlike due to the large mass of the bosons). Soon the W-S model was accepted to be the reality, with the proof of its renormalizability by t'Hooft[5] and the believed discovery of the neutrl current (Z processes).

In this article, the μ-meson decay is calculated in terms of the established IVB. The importance of the μ^--decay is that this is the only process of weak interaction where no hardrons are involved, so we need not think of strong interaction[6].

Theμ^-—decay can be pictorialized as figure.

In (a), the Fermi picture, the process is described by the amplitude

$$\mathfrak{M}=G/\sqrt{2}\,[\bar{u}_{\nu'}(k)\gamma^{\mu}(1-\gamma_5)u_{\mu}(P)]\,[\bar{u}_e(p)\gamma_{\mu}(1-\gamma_5)v_{\bar{\nu}}(k)]$$

In (b), the IVB picture, the process is described by

$$\mathfrak{M}=ig_w^2[\bar{u}_{\nu'}(k)\gamma^{\mu}(1-\gamma_5)u_{\mu}(P)]\times\frac{g_{\mu\nu}-q_{\mu}q_{\nu}/M_w^2}{q^2+M_w^2}[\bar{u}_e(p)\gamma^{\nu}(1-\gamma_5)v_{\bar{\nu}}(k)]$$

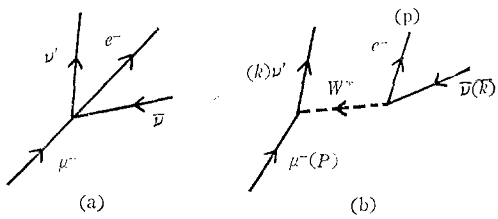

Fig. 1

Here, we can see that for low k (or heavy M) we have (a)=(b) provided

$$g_w^2/M_w^2=G/\sqrt{2}$$

Here,

u, v=Dirac spinors

q=4-momentum transfer

M_W=mass of W boson

g_W=lepton-W^+ vertex factor

G=Fermi coupling constant

For proceeding further, we must calculate

$$\sum_{\text{spins}} |\mathfrak{M}|^2=\frac{g_w^2}{4\pi^4}\left(\frac{1}{q^2+M^2_w}\right)^2\times$$

$$\sum_{\text{spins}} \left|\bar{u}_{\nu'}\left(\gamma^\mu-\frac{m_\mu}{M_W}\right)(1-\gamma_5)u_\mu\right|^2\left|\bar{u}_e\left(\gamma_\mu-\frac{m_e}{M_W}\right)\times(1-\gamma_5)v_{\bar{\nu}}\right|^2$$

where we used

$$\gamma^\mu g_{\mu\nu}\gamma^\nu=\gamma^\mu\gamma_\mu$$

and the Dirac equation$(\not{p}+m)u=0$.

The first of two squares gives (let it A)

$$A=\frac{1}{4}\sum_{\text{spins}}\left|\bar{u}_{\nu'}\left(\gamma^\mu-\frac{m_\mu}{M_W}\right)(1-\gamma_5)u_\mu\bar{u}_\mu\gamma^0 (1-\gamma_5{}^+)\left(\gamma_\nu{}^+-\frac{m_\mu}{M_W}\right)\gamma^0u_{\nu'}\right|$$

Introducing traces, we get after a short calculation

$$A=(2m_\mu)^{-1}\operatorname{Tr}\left[\left(\gamma^\mu-\frac{m_\mu}{M_W}\right)(1-\gamma_5)(\not{P}+m)(1-\gamma_5)\left(\gamma^\mu-\frac{m_\mu}{M_W}\right)\not{k}\right]$$

A long but tedious calculation of trace gives the $\mu-\nu'$ trace as

$$A=m_\mu^{-1}[(P\cdot k)(m_\mu^2/M_W^2+g^{\mu\nu})-p^\mu k^\nu-k^\mu p^\nu]$$

The second square gives the $e-\nu$ trace (let it B) in exactly the same manner as following;

$$B=\frac{1}{4}\sum_{\text{spins}}|\bar{u}_e(\gamma_\mu-m_e/M_W)(1-\gamma_5)v_{\bar{\nu}}|^2$$

$$=(2m_e)^{-1}\operatorname{Tr}[(\gamma'_\mu-m_e/M_W)(1-\gamma_5)(\not{p}+m_e)(1-\gamma_5)(\gamma_\nu-m_e/M_W)\bar{\not{k}}]$$

$$=m_e^{-1}[(p\cdot k)(m_e^2/M_W^2+g_{\mu\nu})-p_\mu\bar{k}_\nu-\bar{k}_\mu p_\nu]$$

So

$$\sum_{\text{spins}}|\mathfrak{M}|^2$$

$$=\left(\frac{g_W}{2\pi^2(q^2+M_W^2)}\right)^2\times A\times B$$

$$=\left(\frac{g_W}{2\pi^2(q^2+M_W^2)}\right)^2(m_em_\mu)^{-1}\{2[(k\cdot\bar{k})(P\cdot p)+(k\cdot p)(P\cdot\bar{k})]-4(m_e^2/M_W^2)(p\cdot\bar{k})E_\mu E_\nu-4(m_\mu^2/M_W^2)(P\cdot k)E_e E_{\bar{\nu}}\}$$

, where we used $g^{\mu\nu}\delta_{\mu\nu}=-2$, and $g^{\mu\nu}g_{\mu\nu}=4$ and neglected the $m_e^2m_\mu^2/M_W^4$ since

$$m_e^2m_\mu^2/M_W^4\ll m_e^2/M_W^2<m_\mu^2/M_W^2\ll 1$$

So the result looks similar to the conventional one[7] with a correction term

$$(m_e^2/M_W^2)(4(p.\bar{k})E_\mu E_\nu)+(m_\mu^2/M_W^2)(4(P\cdot k)E_eE)$$

If we choose the value of M_W as about~10Gev[8], the leading correction term (m_μ^2/M_W^2) is of order of $\sim 10^{-4}$, and this may be the non-local property[9] which caused the observed assymmetry of the electron spectrum in μ^--meson decay.

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