Investigation of Color Effects in Lepton-Pair Collisions

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(Received June 10, 1980)

〈Abstract〉

The differential cross section of the high p_T jet production is calculated using the equivalent-photon approximation in the framework of the broken color gauge symmetry theory to investigate color effects. The cross section is found to have no parameter at all. And the results are compared with that of the exact color symmetry and it is found that the cross section of the broken color symmetry is larger than that of the excat color symmetry theory.

전자쌍 충돌에서의 색자효과의 연구

金聖奎 · 金 燁 · 李賢揆
應用物理學科
(1980. 6. 10 접수)

〈요 약〉

color의 효과를 실험과 비교하기 위해서 깨어진 color규격이론의 입장에서 등 광양자 근사법에 의하여 전자쌍 충돌에서의 고에너지 p_T jet 생성 미분 산란단면적을 계산했다. 이 산란 단면적은 Parameter에 관계 없었다. 또 이 결과를 정확한 color규격이론의 산란단면적과 비교한 결과 깨어진 쪽의 것이 정확한 쪽보다 상당히 큰 것을 알아냈다.

I. Introduction

For the gauge realization of the color symmetry(QCD) which is now believed to be responsible for the strong interaction, there are two alternative models depending upon the pattern of the spontaneous symmetry breaking of the color: the broken color symmetry theory[1] with the liberated integer charged quarks (gluons) and the confined fractionally charged quarks[2] (neutral gluons) with the exact color symmetry. These two alternatives have the same predictions, so far, for the various inclusive scattering processes below the color threshold; $e(\mu)N\to e(\mu)X^{(3)}$, $\nu N\to\nu(\mu)X^{(4)}$, $PP\to\mu^+\mu^-X^{(5)}$ and etc. Above the color threshold, the color excitation is expected to give rise to a sizable effect for the broken color symmetry model which is not expected for the exact color symmetry theory. However, unfortunately, for the most of the inclusive processes, the sizable effect of

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color is expected only for the threshold region of color because of the color suppression from the interference of photon (weak gauge boson) and color gluon propagators[6]. Moreover in the threshold energy region, the analysis becomes very complicated because of the unknown structure of the threshold behavior. There is another ambiguity in the analysis that comes from the parametrization of the gluon distribution functions for which we have not enough knowledge yet.

In contrast to the processes above mentioned, the two-photon(on shell) collision into hadrons is free from these difficulties. It is not needed to include the color suppression because of the on-shell photons. And the elementary-nature of the photons needs no informations on the distribution functions of the color gluons. Moreover, kinematically, the cross sections of the two-photon processes[7] become larger than those of one-photon processes so that this process becomes important in the high energy region.

The purpose of this paper is to investigate the color effect especially for the hard jet production in the process $e^+e^-\rightarrow e^-e^-\gamma\gamma\rightarrow e^+e^-jetX$. In section Ⅱ, the cross section for this process is calculated and discussed using the equivalent photon approximation[8] via the parton model and the results are given in section Ⅲ.

Ⅱ. Calculation of the Cross Section for the Hard Jet Production

In this section, we calculate the differential cross section for the large p_T jet production in the process $e^+e^-\rightarrow e^+e^-$ jet$X$ using the equivalent photon approximation(FIG. 1). The assumption for this appoximation is that the two photons

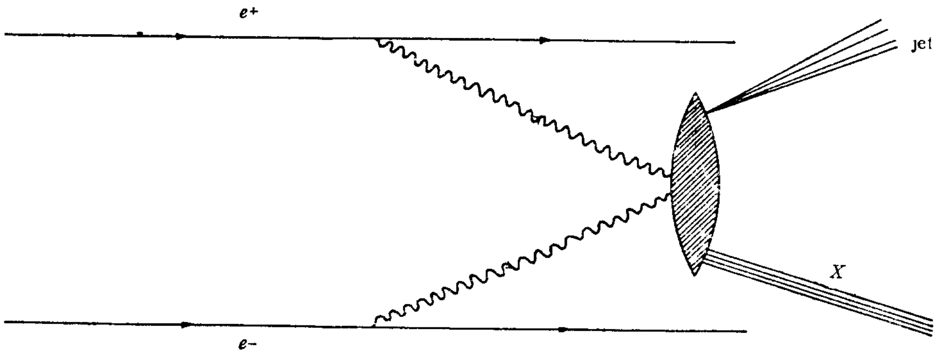

Fig. 1 Diagram for the process $e^+e^-\rightarrow e^+e^-\gamma\gamma\rightarrow e^+e^-jetX$

produced by the incoming electrons are nearly on the mass-shell. This technique is a very useful method for obtaining the leading high energy behaviour and is well examined by comparing with the exact calculations[9]. In this approximation the energy spectrum of the equivalent photon, $N(x)$, is well known as

$$N(x)=[(\alpha/2\pi)\ln(s/4m_e^2)](1+(1-x)^2/x) \quad (1)$$

where s is the square of the total c.m. energy of e^+e^- system and x is the energy fraction of photon produced by the electron (i.e., $x=E_\gamma/(\sqrt{s}/2)$). The next ingredient in this approximation is the differential cross section of the process (FIG. 2a), $\gamma\gamma\rightarrow$jet$X$, $d\sigma/dt(\gamma\gamma\rightarrow$ jet$X)$.

Then the cross section of the process $e^+e^-\rightarrow e^-e^-$jet X in the equivalent photon approximation can be written as

$$Ed\sigma/d^3p=\int_0^1\int_0^1 dx_1dx_2N(x_1)N(x_2)d\sigma d\hat{t}(\gamma\gamma\rightarrow \text{jet}X)\ (s/\pi)\delta(s+\hat{t}+\hat{u}) \quad (2)$$

where $s\sim x_1x_2s$, $\hat{t}\sim x_1t$, and $\hat{u}\sim x_2u$ in the high energy limit and x_i's are the energy fractions

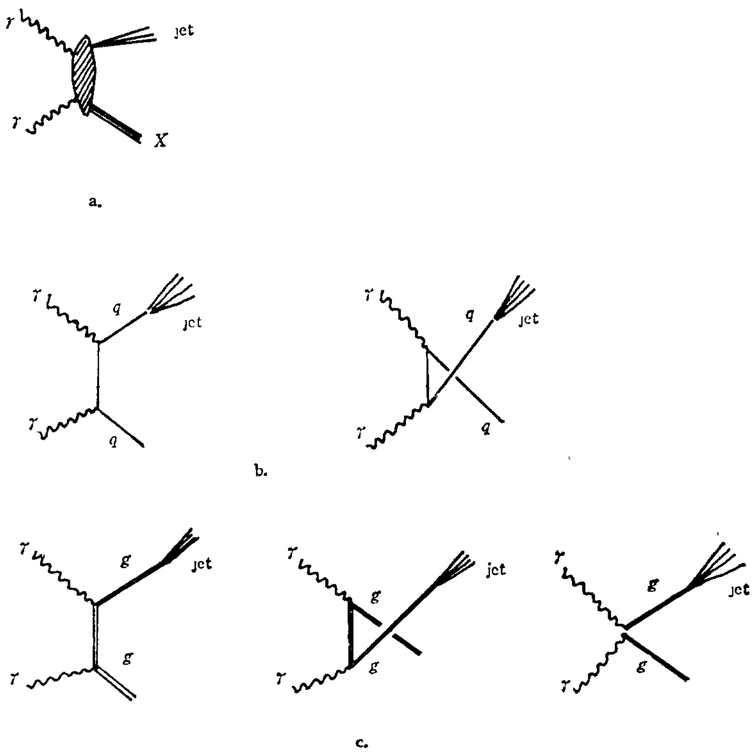

Fig. 2 a. Diagram for $\gamma\gamma$ annihilation into jets and hadrons
b. parton model diagrams leading order for quark jet production
c. parton model diagrams leading order for gluon jet production

of photons.

The subprocesses $\gamma\gamma\rightarrow$jet X with high p_T jet can be calculated via parton model as inthe process $e^+e^-\rightarrow$ jet X; jet can be produced by the high energy quarks and gluons produced by the annihilation of two photons as shown in FIG. 2*b* and 2*c*. The jet production cross section by quark production is well known as (FIG. 2*b*)

$$\frac{d\sigma^{\text{quark}}}{dt}=2\sum_i e_i^4\left(\frac{2\pi\alpha^2}{s^2}\right)\left(\frac{t}{u}+\frac{u}{t}\right) \qquad (3)$$

where $\sum_i e_i^4$ is the sum over the colors and flavors(u, d, s, c) which is 6 in the broken color gauge symmetry theory. The jet production cross section by the color gluon (FIG. 2*c*) is calculated as

$$\frac{d\sigma^{\text{gluon}}}{dt}=2\sum_g e_g^4\left(\frac{4\pi\alpha^2}{s^2}\right)\left[9-2\left(\frac{t}{u}+\frac{u}{t}\right)+\frac{(u-t)^3}{M^2tu}\right] \qquad (4)$$

where M is the mass of the color gluon and $\sum_g e_g^4$ is 2 which is zero in the theory of the exact color symmetry. This gluon jet production cross section is calculated in unitary gauge. Substituting eq. (3) and (4) into eq. (2), we can calculate the differential cross section[10] for the process $e^+e^-\rightarrow e^+e^-$jet$X$ as

$$E\frac{d\sigma^{\text{quark}}}{d^3p}=\int_0^1\int_0^1 dx_1\,dx_2\left[\frac{\alpha}{2\pi}\ln\left(\frac{s}{4m_e^2}\right)\right]^2\left[\frac{1+(1-x_1)^2}{x_1}\right]\left[\frac{1+(1-x_2)^2}{x_2}\right]$$
$$\times\frac{24\pi\alpha^2}{s^2}\left(\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right)\left(\frac{s}{\pi}\right)\delta\,(s+\hat{t}+\hat{u}) \qquad (5)$$

$$E\frac{d\sigma^{\text{gluon}}}{d^3p}=\int_0^1\int_0^1 dx_1\,dx_2\left[\frac{\alpha}{2\pi}\ln\left(\frac{s}{4m_e^2}\right)\right]^2\left[\frac{1+(1-x_1)^2}{x_1}\right]$$
$$\left[\frac{1+(1-x_2)^2}{x_2}\right]\left[\frac{16\pi\alpha^2}{s^2}\left[9-2\left(\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{t}\right)+\frac{(\hat{u}-\hat{t})^3}{M^2\hat{t}\hat{u}}\right]\right.$$
$$\left.\left(\frac{s}{\pi}\right)\delta(s+\hat{t}+\hat{u})\right. \qquad (6)$$

Since we are interested in the hard jet production in which one of the jets carries half of the energy which means $x_R=\frac{2E_{jet}}{\sqrt{s}}\to 1$, eq. (5) and (6) are evaluated in the limit $\epsilon\to 0$ where ϵ is defined as $1-x_R$: The results of evaluations are

$$E\frac{d\sigma^{quark}}{d^3p}=\frac{3\alpha^4}{2\pi}\left[\ln\left(\frac{s}{4m_e^2}\right)\right]^2\frac{(1-x_R)}{p_T^4} \qquad (7)$$

$$E\frac{d\sigma^{gluon}}{d^3p}=\frac{5\alpha^4}{\pi}\left[\ln\left(\frac{s}{4m_e^2}\right)\right]^2\frac{(1-x_R)}{p_T^4} \qquad (8)$$

In eq. (7) and (8), we can easily see the p_T behaviour for the high p_T production. One of the interesting feature is that the cross section by gluon jet production is independent of the gluon mass M. Therefore this production contains no ambiguous parameters at all. Another thing to notice is the p_T and x_R dependences are the same for the quark and gluon jets production. The total high p_T jet procuction cross section is the sum of eq. (7) and (8),

$$E\frac{d\sigma}{d^3p}=\frac{13\alpha^4}{2\pi}\left[\ln\left(\frac{s}{4m^2_e}\right)\right]^2\frac{(1-x_R)}{p^4_T} \qquad (9)$$

Ⅲ. Discussion and Result

We have calculated the high p_T jet production cross section for the process $e^+e^-\to e^+e^-$jet$X$ using the equivalent-photon approximation via parton model in the framework of the broken color gauge theory. It is found that the quark and gluon contributions have the same p_T and x_R behaviour in the limit $x_R\to 1$: $E\frac{d\sigma}{d^3p}\sim(1-x_R)$ p_T^{-4}. And the cross section is found to have no ambiguous parameters at all. Comparing with the cross section in the framework of the exact color symmetry in which only quarks can contribute in the leading order approximation with fractional charges,

$$E\frac{d\sigma}{d^3p}=\frac{17}{54}\frac{\alpha^4}{\pi}\left[\ln\left(\frac{s}{4m_e^2}\right)\right]^2\frac{(1-x_R)}{p_T^4}, \qquad (10)$$

we can easily see that the cross section of the broken color symmetry theory is 20 times as large as that of the exact color symmetry theory. The similar process is the one photon process $e^+e^-\to$jet$X$. However the color effect in this process is suppressed greatly so that there can be negligible differencies between the predictions from the two alternative theories even for the region above the color threshold.

Therefore, we can see that the process $e^+e^-\to e^+e^-$jet$X$ is the process in which we appreciate the color effect apparently when compared with the experiment[11].

Acknowledgement

This work was supported by the Asan Foundation

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