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Economic Design of np-Control Chart Using a Chain Sampling.

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〈**Abstract**〉

This paper formulates the cost model for economic design of np-control chart by using the method of a chain sampling and it proposes an algorithm for determining its optimal parameters also. These parameters are the sample size, the number of defectives in sample determining the control limit, the sampling interval, and the number of preceding samplings to be considered when one defective is observed under chain sampling. Several examples show that this cost model by using a chain sampling gives more economic optimum than the other existing models.

체인샘플링을 이용한 np관리도의 경제적 설계

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〈요 약〉

본 논문은 체인샘플링을 이용하여 관리도의 경제적 설계를 위한 비용모형을 정립한 후 그 최적 모수들을 결정하는 해법을 제시하고 있다.

이 모수들은 표본의 크기, 관리 한계를 결정하는 샘플내의 불량품의 수, 샘플링 구간, 그리고 체인샘플링 하에서 불량품이 하나 발견될때 고려되어야 할 이전 샘플링의 횟수이다.

다수의 예들은 체인샘플링을 사용한 본 비용 모형의 최적해가 기존의 다른 비용 모형보다 더 경제적임을 보여준다.

I. Introduction

In recent years, considerable attention has been devoted to to the economic design of *np*-control charts. The economic design of an *np*-control chart involves the determination of the statistical parameters that minimize a relevant cost function. These parameters are the sample size, the intersample interval, and the factor that determines the spread of the cotrol limits.

Most recently the attention of a few authors has been focused on the economic design of *np*-control charts. Ladany (7) developed a model based on Duncan's model (2). He formulated a cost function consisting of the cost of sampling, the cost incurred by defective items, the cost due to false alarms, and the cost of adjusting the process after a shift has occurred. The author suggested minimizing the total cost function by computer enumeration; however, no numerical examples were given. Chiu (1) modified Duncan's model(2) by assuming that when the *np*-control chart signals the presence of an assignable cause of variation, the process is stopped and a search is undertaken. Two average costs were defined. One is associated with false alarms and the other with real alarms. They author suggested two methods for minimization of the formulated cost function: a two-dimensional Fibonacci search, and a simplified method based on a prescribed probability of 90 percent that the number of defective items found in a sample falls outside the control limits when the process is out of control.

Montgomery, Heikes, and Mance (9) developed a model for a process subject to the occurrence of several assignable causes of variation. They used the same approach developed by Knappenberger(6). The proposed cost function consists of the cost of sampling, the cost of investigating, and possibly correcting the process when an alarm is signaled by the chart, and the cost of producing defective product. Direct compter research techniques were used to optimize the expected cost function. Gibra (5) modified the models developed by Duncan (2), Ladany (7), and Chiu (1) by developing two models based on a realistic and more comprehensive cost function that included all possible costs that were likely to be incurred in practice. The first model assumes that the process is shut down during the search for the assignable cause. The second assumes that the process is kept running until the assignable cause is discovered. With the aid of these two models, the manufacturer can decide on the appropriate model to use if the search for the assignable cause can be undertaken when the process is either shut down or operative. In this paper we introduce the idea of a chain sampling plan into cost model developed by Gibra. (5) A chain sampling plan has been suggested by Harold F. Dodge that it might be a desirable substitute foro rdinary single sampling plans with zero acceptance numbers in that they have OC curves that are concave at the start. A chain sampling plan runs as follows. For each lot, a sample of n units is selected and each unit is tested for conformance to specifications. If a sample from a given lot contains zero defectives, the lot is also accepted.

The lot is also accepted if only one of the sample units is defective provided no defectives were found in the previous i samples. If the sample contains two or more defective items, the lot is rejected.

Finally several examples show that this cost model using a chain sampling gives more economic solution than other existing models.

Ⅱ. Assumptions and Operating Conditions

The assumptions and the nature of the operating conditions are summarized as follows:

(1) One or more quality characteristics are under the surveillance of an *np*-control chart. The expected fraction defective produced when the process is in a state of control is p_0 where p_0 is constant and known. Samples of size n are taken every v units of time. The process is continued without a search for the assignable cause whenever a sample contains zero defective, or only one of sample units is defective provided no defectives were found in the previous i samples.

(2) The process is subject to the occurrence of a single assignable cause that takes the form of a shift in the process fraction defective to a value p_1, $(p_1>p_0)$, where p_1 is constant and known.

(3) The time occurrence of the assignable cause exponentially distributed with parameter λ per unit of operating time.

(4) A study of many industrial operations reveals that the following situations are common in practice:

(a) the process is allowed to continue in operation during the search for the assignable cause.

(b) the process is shut down during the search for the assignable cause.

(c) the process is shut down for repair.

Consequently, two models will be formulated. The first model to be studied will be designated as Model I. It assumes that whenever an alarm is signaled, the process is shut down during the search for the assignable cause. The search time is assumed to be a random variable with an expected value for false or real alarms. If the alarm is found to be false, the process will be allowed to continue in operation as if there had been no interruption. If, however, the search indicates the presence of an assignable cause, the process will remain inoperative and necessary repair will be made. Repair time is assumed to be a random variable with an expected value. The process will start in state of control after the repair has been made. The second model to be studied will be designated as Model Ⅱ. It assumes that the process remains operative during the search for the assignable cause. However, if the search indicates the presence of an assignable cause, the process will be shut down and necessary repair will be made.

Ⅲ. Formulation of Cost Function

Before we proceed to formulate the cost function, the following characteristics shall be derived:

(1) If the assignable cause occurs between the jth and $j+1$st interval, then the average time of occurrence of the assignable cause within a sampling interval(v) is given by

$$\tau=\int_{jv}^{(j+1)v}(t-jv)\,\lambda e^{-\lambda t}dt\Big/\int_{jv}^{(j+1)v}\lambda e^{-\lambda t}dt$$
$$=\frac{1}{\lambda}-\frac{v}{e^{\lambda v}-1} \qquad (1)$$

(2) Let α be the probability of a false alam when the process is in control, and let P be the probability that the assignable cause is detected when the process is out of control.

$$\alpha=1-\{P(c=0|p_0)+P(c=1|p_0)\cdot P(c=0|p_0)^i\} \qquad (2)$$

$$P=1-\{P(c=0|p_1)+P(c=1|p_1)\cdot P(c=0|p_1)^i\} \qquad (3)$$

, where $P(c=x|p_j)=\sum_{k=0}^{x}\binom{n}{k}p_j^k(1-p_j)^{n-k}$

for $j=0$ or 1

(3) The expected number of sampling intervals to detect the assignable cause on the jth inspected sample is

$$\sum_{j=1}^{\infty}jP(1-P)^{j-1}=1/P \qquad (4)$$

, where P is given by equation(3).

Therefore, the expected time that the process is out of control before the search for the assignable cause is instituted is given by

$$v/P-\tau+ng, \qquad (5)$$

where g is the expected inspection and charting time per unit sampled.

(4) The total expected search time for false alarms prior to the occurrence of an assignable cause($=T_1$ times the expected number of false alarms) is

$$\alpha T_1\sum_{j=0}^{\infty}\int_{jv}^{(j+1)v}j\lambda e^{-\lambda t}dt=\frac{\alpha T_1}{e^{\lambda v}-1}$$
$$=\alpha T_1(1/\lambda-\tau)/v, \qquad (6)$$

since from equation(1),

$$(e^{\lambda v}-1)^{-1}=(1/\lambda-\tau)/v.$$

(5) We define the expected quality cycle time, denoted by L_i, as the expeceted interval between two successive periods of statistical stability. Therefore,

$$L_1=1/\lambda+v/P-\tau+ng+T_1+\alpha T_1(1/\lambda-\tau)/v+T_2 \text{ (Model I)} \qquad (7)$$

and

$$L_2=1/\lambda+v/P-\tau+ng+T_1+T_2 \text{ (Model II)} \qquad (8)$$

The cost function under these assumptions is the same expression that formulated by Gibra(5). Therefore we omit the procedure formulating the cost function.

The following notation shall be used in the

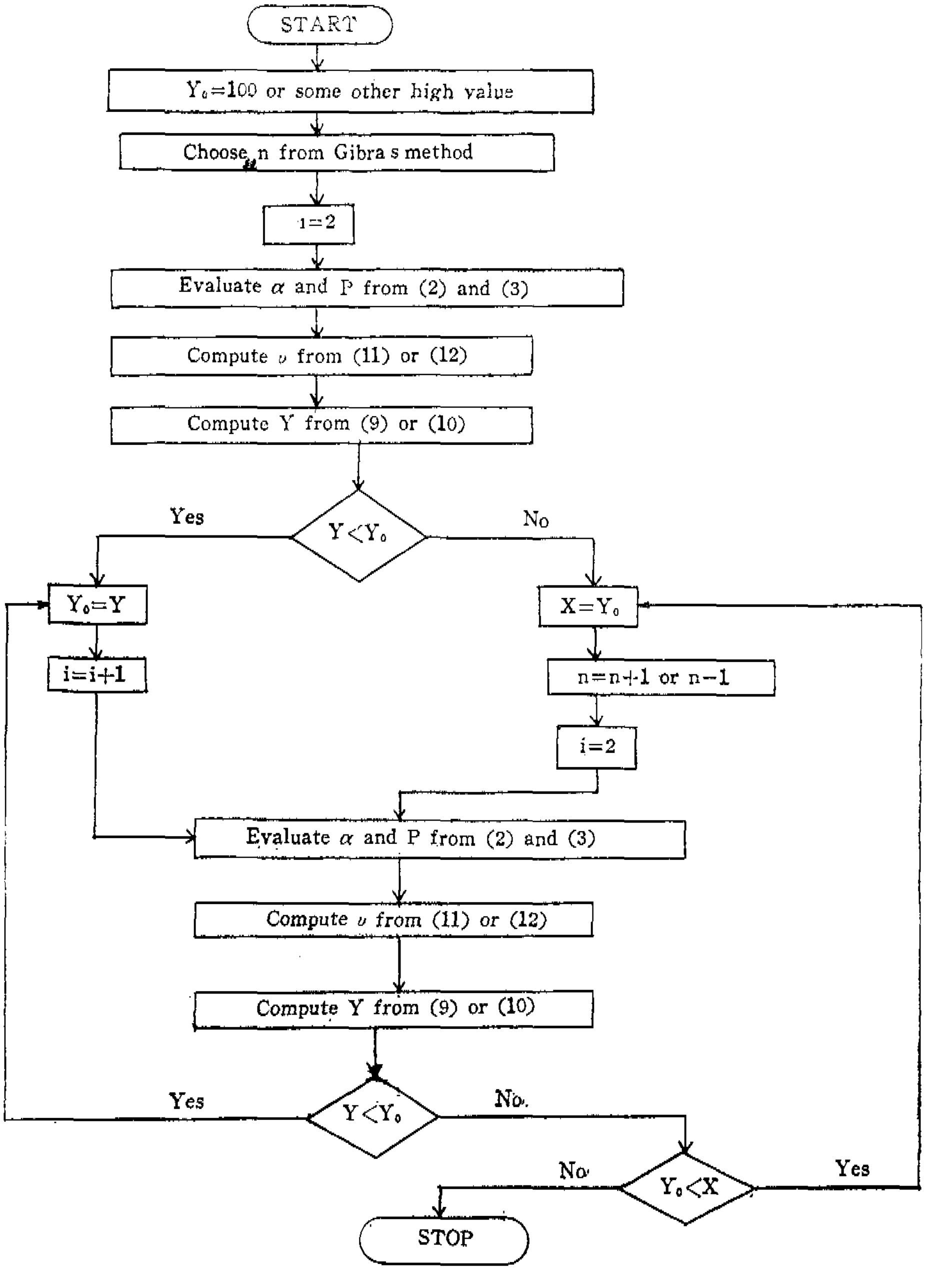

Fig. 1. Flow Chart for search procedure.

formulation of the cost function.

r: production rate per unit time

c_1: cost of search for the assignable cause per unit time

c_2: cost of downtime per unit time

c_3: cost of repair per unit time

u: penalty incurred per defective item

b: cost of inspection and charting per unit sampled

h: everhead cost for maintaining np-control chart per inspected sample.

i: number of preceding samplings to be considered when one defective is observed in sample.

The expected total costs per unit time, Y_1 and Y_2 respectively, for Model I and II are

$$Y_1=[(h+bn)(1/\lambda+v/P-\tau+ng)/v +(c_1+c_2)\alpha T_1(1/\lambda-\tau)/v+(c_1+c_1)T_1 +(c_2+c_3)T_2+ur(p_1-p_0)(ng +v/P-\tau)]/L_1 \text{ (Model I)} \quad (9)$$

and

$$Y_2=[(h+bn)(1/\lambda+v/P-\tau+ng+T_1)/v +c_1\alpha T_1(1/\lambda-\tau)/v+c_1T_1+(c_2+c_3)T_2 +ur(p_1-p_0)(ng+v/P-\tau +T_1)]/L_2 \text{ (Model II)} \quad (10)$$

V. Determination of Optimal Parameters

Since we deal with the case that the optimal value of c_0 is zero, we have only to determine the optimal values of three parameters i, n and v that are left over. Fortunately, the optimal intersample interval v_0 can be accomplshed as follows: The expected total cost function is partially differentiated with respect to v, and the derivative is set equal to zero. The equation $\frac{\partial Y}{\partial v}=0$ was solved by Newton's method for specified values of (P,c), cost, and risk parameters.

The true root is found to be well approximated by

Table 1. Values of Cost and Risk Parameters Used in Examples

Example Number	p_0	p_1	λ	r	T_1	T_2	g	c_1	c_2	c_3	h	b	u
1[1)]	.02	.10	.0125/hr	2,500/hr	.2hr	2.0	.005hr	$10.00	$15.00	$20.00	$2.00	$.10	$3.00
2	.01	.05	.0125	2,500	.2	2.0	.005	10.00	15.00	20.00	2.00	.10	3.00
3	.02	.10	.0125	2,500	.2	2.0	.005	10.00	15.00	20.00	5.00	.50	3.00
4	.02	.10	.0125	2,500	.2	2.0	.010	10.00	15.00	20.00	2.00	.10	3.00

1) These values of parameters are the same as in Example 1 of Gibra(5)

Table 2. Optimum Design for Numerical Examples of Table 1.

Example number		Existing optimum				Optimum using chain sampling				
		n	c	v	cost Y	n	c	v	i	cost Y
Moddel I	1	16	0	0.96	10.9610	17	0	0.92	2	10.6835
	2	27	0	1.37	9.3528	27	0	1.31	3	9.1927
	3	15	0	1.56	17.9948	15	0	1.54	4	17.9452
	4	14	0	0.88	11.4729	14	0	0.83	3	11.2224
Model II	1	16	0	0.87	12.1272	17	0	0.87	2	11.9439
	2	27	0	1.29	9.8504	28	0	1.29	3	9.7569
	3	15	0	1.51	19.5121	15	0	1.50	4	19.4392
	4	14	0	0.81	12.6763	15	0	0.82	3	12.4979

$$\upsilon \cong \left[\frac{(h+bn)+\alpha T_1(c_1+c_2)}{\lambda ru(p_1-p_0)(1/P-1/2)} \right]^{\frac{1}{2}} \text{(Model I)} \quad (11)$$

and

$$\upsilon \cong \left[\frac{(h+bn)+c_1 T_1 \alpha}{\lambda ru(p_1-p_0)(1/P-1/2)} \right]^{\frac{1}{2}} \text{(Model II)} \quad (12)$$

Varying n in steps of 1 or 2, we can find the value of n that minimizes the expected total cost function.

To summarize, the search proceduce works as follows:

(1) Choose a value of n, from the solution method of Gibra(5).

(2) Given n, evalute α and p for $i=2,3,4,\cdots$ from equation (2) and (3).

(3) Compute υ from equation (11) or (12).

(4) Compute Y_1 or Y_2 from equation(9) or (10) respectively.

(5) Repeat steps(1) through(4) for various values of n.

(6) Choose the pair(i,n) that yields the minimum Y_1 or Y_2.

Figure 1 shows a flow chart for the search procedure.

Now we shall compare this cost model using a chain sampling wih the other existing one by a few numerical examples, on Table 1, which $c_0=0$.

The results of these comparisons are in Table 2, and they show that the optimum from this cost model is more economical than the optimum of the other in every instance. For many other numerical examples we could get the same conclusion.

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