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On mappings with strongly semi-closed graphs

K. K. Dube* · Gyu Ihn Chae · O. S. Panwar*
Dept. of Mathematics
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〈Abstract〉

A mapping $f: X \to Y$ has a strongly semi-closed graph iff for each $(x, y) \notin G(f)$, there exist semi-open sets U and V containing x and y, respectively, such that $(U \times \text{scl } V) \cap G(f) = \phi$. This type of graph is weaker than a strongly-closed graph [12] and is stronger than a semi-closed graph [7]. However, it is independent of a closed graph. Several known mappings may fail to have a strongly semi-closed graph and similarly, a mapping having a strongly semi-closed graph need not be a certain type of mappings. Conditions under which a certain mapping has a strongly semi-closed graph and conditions under which range or/and domain of a mapping having a strongly semi-closed graph become semi-T_1 or/and semi-T_2 are obtained.

반폐그래프를 갖는 사상들에 관하여

K. K. Dube* · 채 규 인 · O. S. Panwar*
수 학 과
(1984. 4. 30 접수)

〈요 약〉

우리는 strongly 반폐그래프라는, 새로운 약화된 폐그래프를 정의하여, 그 성질과 이미 알려진 여러 약화된 폐그래프들의 성질과 비교하고, 새로 도입된 이 개념은 폐그래프와는 독립적임을 보인다.

여러 사상들이 strongly 반폐그래프를 가질때 성질을 알아보고 그 개념을 이용 Semi-T_1 공간, semi-T_2 공간들의 성질을 알아본다.

I. Introduction:

It is well known that a mapping f of a topological space X into a Hausdorff compact topological space Y is continuous iff the graph of f is closed in the product space $X \times Y$. In general, a continuous mapping fails to have a closed graph and similarly, a mapping with a closed graph need not be continuous. In the recent past, these considerations led to a search for conditions under which (1) a certain mapping has a graph of requisite nature and (2) a mapping having a certain graph is of requisite nature. The graph condition of a mapping f is linked, in a natural way, with the separation conditions on its domain or its range space. This can be justified by the known results, viz., a space X is Hausdorff iff, for $i: X \to X$, the graph is closed in the product space $X \times X$. In 1975, Herrington and Long [12] introduced the notion of strongly-

* Professor of Dept. of Math., University of Saugar, India.

closed graph of a mapping. There, they showed that if the graph of a mapping is strongly-closed, then it is closed also, but not conversely. Also, mappings with semi-closed graphs have been discussed in [7].

In this note, a condition with which the graph is termed to be strongly semi-closed is introduced. It is shown that the strongly semi-closed graph is weaker than a strongly-closed graph and is stronger than a semi-closed graph. However, it is independent of a closed graph. Also, it is shown that some types of known mappings may fail to have a strongly semi-closed graph and that a mapping having a strongly semi-closed graph need not be a certain type of mappings. Furthermore, some conditions are discussed under which certain types of mappings have strongly semi-closed graphs, and also, some conditions under which range or/and domain of a mapping having a strongly semi-closed graph become semi-T_1 or/and semi-T_2 are investigated.

Ⅱ. Terminology

Throughout this note, a space means a topological space. A set A in a space X is semi-open [16] iff there exists an open set 0 in X such that $0 \subset A \subset \text{cl } 0$, where cl0 denotes the closure of 0 in X. Every open set is semi-open [16]. The complement of a semi-open set is a semi-closed set [3]. The intersection of all the semi-closed sets containing A is the semi-closure of A and is denoted by scl A [3]; also, $A \subset \text{scl } A \subset \text{cl } A$ [3], and $A \subset B$ implies scl $A \subset$ scl B. A subset M of a space X is a semi-neighbourhood [2] of a point x of X iff there exists a semi-open set A in X such that $x \in A \subset M$. A subset A of X is semi-open iff it is a semi-neighbourhood of each of its points [2]. $f: X \to Y$ is semi-continuous [16] iff $f^{-1}(V)$ is semi-open in X for each open V in Y, equivalently, iff for each $x \in X$ and each neighbourhood V of $f(x)$, there exists a semi-neighbourhood U of x such that $f(U) \subset V$ [2]. $f: X \to Y$ is irresolute [4] iff $f^{-1}(V)$ is semi-open in X for each semi-open V in Y, equivalently, iff for each $x \in X$ and each semi-neighborhood V of $f(x)$, there exists a semi-neighborhood U of x such that $f(U) \subset V$. Also in [4], $f: X \to Y$ is irresolute iff for all $B \subset Y$, scl $f^{-1}(B) \subset f^{-1}$(scl B). $f: X \to Y$ is pre-semi-open [4] iff $f(A)$ is semi-open in Y for each semi open A of X. A space X is semi-T_1 [19] iff for each pair of distinct points x, y of X, there exists a semi-open set A containing x but not y and a semi-open set B containing y but not x. A space X is semi-T_2 [19] iff to each pair of distinct points x, y of X, there exist disjoint semi-open sets U and V containing x and y, respectively, equivalently, iff for each pair of distinct points x, y of X, there exists a semiopen set V containing y such that $x \notin$ scl V. Also in [19], every semi-T_2 space is semi-T_1. By a semi-clopen set, we shall mean a set which is both semi-open and semi-closed.

Ⅲ. Mappings with Strongly Semi-Closed Graphs

If $f: X \to Y$, then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $X \times Y$ is called the graph of f.

Definition 1: The mapping $f: X \to Y$ has a strongly semi-closed graph iff for each $(x, y) \notin G(f)$, there exist semi-open sets U and V containing x and y, respectively, such that $(U \times \text{scl } V) \cap G(f) = \phi$.

A useful characterization of mappings with strongly semi-closed graphs is given below.

Theorem 1: The mapping $f: X \to Y$ has a strongly semi-closed graph iff for each $x \in X$ and $y \in Y$ such that $y \neq f(x)$, there exist semi-open sets U and V containing x and y, respectively, such that $f(U) \cap \text{scl } V = \phi$.

Proof: Immediately follows from Definition 1.

The mapping $f: X \to Y$ has a strongly-closed graph [12] iff for each $(x, y) \notin G(f)$, there exist open sets U and V containing x and y, respectively, such that $(U \times \mathrm{cl}\ V) \cap G(f) = \phi$. Also, $f: X \to Y$ has a semi-closed graph [7] if for each $(x, y) \notin G(f)$, there exist semi-open sets U and V containing x and y, respectively, such that $(U \times V) \cap G(f) = \phi$.

Evidently, every strongly-closed graph is strongly semi-closed and every strongly semi-closed graph is semi-closed, but the converses are not true, in general. It may be seen by the following examples.

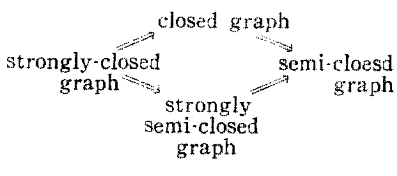
Example 1: Let $X = \{a, b\}$ with topology $\mathscr{T}_1 = \{\phi, X, \{a\}, \{b\}\}$ and $Y = \{a, b, c\}$ with topology $\mathscr{T}_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Then, clearly, the graph of the identity mapping $i: X \to Y$ is semi-closed and strongly semi-closed but is neither closed nor strongly-closed.

Example 2: Let X be the space given in Example 1, and let $Y = \{a, b, c\}$ with topology $\mathscr{T}_2 = \{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$. Then, obviously, the graph of the identity mapping $i: X \to Y$ is semi-closed and, also, closed but is not strongly semi-closed.

From Examples 1 and 2, it is also clear that a mapping having a strongly semi-closed graph need not have a closed graph and a mapping having a closed graph may fail to have a strongly semi-closed graph.

Again, every mapping with a strongly closed graph also has a closed graph, but the converse, however, fails [12]. Also, a mapping having a closed graph also has a semi-closed graph but the converse may not be true [7].

Hence, we have the following implications diagram:



A mapping $f: X \to Y$ is set-s-connected [5] iff the inverse image under f of every semi-clopen subset of $f(X)$ is semi-clopen in X. A mapping $f: X \to Y$ is C-monotone [1] iff $f^{-1}(V)$ is a connected subset of X for every connected set V of Y.

Remark 1: Several types of known mappings, viz., a continuous mapping, a θ-continuous mapping [8], a weakly continuous mapping [15], a mapping almost continuous in the sense of Frolik [9], a semi-continuous mapping [16], a mapping almost continuous in the sense of Husain [11], a mapping almost continuous in the sense of Singal and Singal [23], a c-continuous mapping [10], a c^*-continuous mapping [22], an H-continuous mapping [18], an s-continuous mapping [14], a set-connected mapping [13], a set-s-connected mapping [5], an irresolute mapping [4], a connected mapping [21], a C-monotone [1], a semi-connected mapping [17], and a weak semi-connected mapping [14], may fail to have a strongly semi-closed graph. It is shown by the following example:

Example 3: Let X be a space, containing more than one point, with the indiscrete topology, and let $i: X \to X$ be the identity mapping. Here, $G(i)$ is not strongly semi-closed.

Remark 2: A mapping having a strongly semi-closed graph need not be continuous, θ-continuous [8], weakly continuous [15], almost continuous in the sense of Frolik [9], semi-continuous [16], almost continuous in the sense of Husain [11], almost continuous in the sense of Singal and Singal [23], c-continuous [10], c^*-continuous [22], H-continuous [18], s-continuous [14], set-connected [13], set-s-connected [5], irresolute [4], connected [21], semi-connected [17], weak semi-connected [14]. It may be seen by the following example:

Example 4: Let $X = \{a, b, c\}$ with topologies $\mathscr{T}_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathscr{T}_2 =$ a discrete topology. Then, obviously, the identity mapping $i: (X, \mathscr{T}_1) \to (X, \mathscr{T}_2)$ has a strongly semi-closed graph but is none of the mappings mentioned in Remark 2.

Theorem 2: Let $f: X \to Y$ be a semi-continuous mapping with Y as T_2. Then $G(f)$ is strongly semi-closed.

Proof: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, Y being T_2, there is an open set V containing y such that $f(x) \notin \mathrm{cl}\ V$. By semicontinuity of f, there exists a semi-open set U containing x such that $f(U) \subset Y-\mathrm{cl}\ V$. Thus $f(U) \cap \mathrm{scl}\ V = \phi$. Consequently, $G(f)$ is strongly semi-closed.

The converse of Theorem 2, however, fails as is clear from Example 4.

Theorem 3: If $f: X \to Y$, where Y is semi-T_2, is an irresolute mapping, then $G(f)$ is strongly semi-closed.

Proof: Similar to that of Theorem 2, using the definitions of a semi-T_2 space and an irresolute mapping

In view of Example 4, the converse of Theorem 3 does not hold.

Definition 2 [6]: A space X is extremally s-disconnected iff the semiclosure of every semi-open set is semi-open.

Theorem 4: Let $f: X \to Y$ be a set-s-connected surjection and Y be an extremally s-disconnected, semi-T_2 space. Then $G(f)$ is strongly semiclosed.

Proof: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, y being semi-T_2, there is a semi-open set N containing y such that $f(x) \notin \mathrm{scl}\ N = V$, say. Since Y is extremally s-disconnected, V is semi-clopen in Y not containing $f(x)$. Therefore, f being set-s-connected surjection, $f^{-1}(V)$ is semi-clopen in X and $x \notin f^{-1}(V)$. Taking $U = X - f^{-1}(V)$, U is semi-open in X containing x and then $f(U) \cap V = \phi$, i.e., $f(U) \cap \mathrm{scl} N = \phi$. Consequently, $G(f)$ is strongly semi-closed.

The converse of Theorem 4 is not necessarily true as the Example 4 shows.

Definition 3 [20]: A space X is locally s-connected iff for each $x \in X$ and each open set 0 containing x, there exists an open s-connected set G such that $x \in G \subset 0$.

Definition 4 [20]: Two subsets A and B of the space X are termed semi-separated iff $A \cap \mathrm{scl}\ B = \phi = \mathrm{scl}\ A \cap B$.

A set A in the space X is s-connected [20] iff A is s-connected as a subspace of X.

Lemma 1 [20]: A space X is not s-connected iff it is the union of two nonempty disjoint semi-closed sets.

Lemma 2 [20]: Semi-closure of an open s-connected set in a space X is s-connected.

Lemma 3 [20]: If A, B are open s-connected and non-semi-separated sets in the space X, then $A \cup B$ is s-connected.

A space X is Urysohn iff for each pair of distinct points x, y of X, there exist open sets U and V containing x and y, respectively, such that $\mathrm{cl}\ U \cap \mathrm{cl}\ V = \phi$.

Definition 5 [7]: A mapping $f: X \to Y$ is almost irresolute on X iff for each $x \in X$ and each semi-neighbourhood V of $f(x)$, $\mathrm{scl}\ f^{-1}(V)$ is a semi-neighbourhood of x.

Evidently, every irresolute is almost irresolute but not conversely.

In view of Example 3, an almost irresolute mapping may fail to have a strongly semi-closed graph, and in view of Example 4, a mapping having a strongly semi-closed graph need not be almost irresolute. However, we have

Theorem 5: Let $f: X \to Y$ be an almost irresolute mapping, where Y is a locally s-connected Urysohn space. If f maps open s-connected sets onto s-connected sets and f^{-1} maps s-connected sets onto open s-connected sets, then $G(f)$ is strongly semi-closed.

Proof: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, Y being Urysohn locally s-connected, there exist open s-connected sets U and V containing y and $f(x)$, respectively, such that $\mathrm{cl}\ U \cap \mathrm{cl}\ V = \phi$. This gives $\mathrm{scl}\ U \cap \mathrm{scl}\ V = \phi$. Therefore $f^{-1}(\mathrm{scl}\ U) \cap f^{-1}(\mathrm{scl}\ V) = \phi$. Further, we claim that $f^{-1}(\text{scl } U) \cap \text{scl } f^{-1}(\text{scl } V)=\phi$. For, if not, then $f^{-1}(\text{scl } U)$ and $f^{-1}(\text{scl } V)$ are non-semi-separated (by Definition 4), and open s-connected (by Lemma 2 and hypothesis) subsets of X, and so, by Lemma 3, $f^{-1}(\text{scl } U) \cup f^{-1}(\text{scl } V)$ is open s-connected. But, again by hypothesis, $f[f^{-1}(\text{scl } U) \cup f^{-1}(\text{scl } V)]=\text{scl } U \cup \text{scl } V$ is s-connected whichis a contradiction in view of Lemma 1. Therefore, it follows that $f^{-1}(\text{scl } U) \cap \cap \text{scl } f^{-1}(\text{scl } V)=\phi$. Now, f being almost irresolute, $\text{scl } f^{-1}(V)$ and hence $\text{scl } f^{-1}(\text{scl } V)$ is a semi-neighbourhood of x and so there exists a semi-open set $T \subset \text{scl } f^{-1}(\text{scl } V)$ containing x. Therefore, $f^{-1}(\text{scl } U) \cap \cap T=\phi$ which gives $f(T) \cap \text{scl } U=\phi$. Consequently, $G(f)$ is strongly semi-closed.

Definition 6: A mapping $f: X \rightarrow Y$ is weakly irresolute iff for each $x \in X$ and each semi-neighbourhood V of $f(x)$, there exists a semi-neighbourhood U of x such that $f(U) \subset \text{scl } V$.

Evidently, every irresolute is weakly irresolute but not conversely.

A weakly irresolute mapping may not have a strongly semi-closed graph. For, the identity mapping, in Example 3, is weakly irresolute but its graph is not strongly semi-closed. However,

Theorem 6: If $f: X \rightarrow Y$ is weakly irresolute and Y is a Urysohn space, then $G(f)$ is strongly semi-closed.

Proof: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, since Y is Urysohn, there exist open sets V_1 and V_2 containing $f(x)$ and y, respectively, such that $\text{cl } V_1 \cap \text{cl } V_2=\phi$ which gives $\text{scl } V_1 \cap \text{scl } V_2=\phi$. Since f is weakly irresolute, there exists a semi-open set U containing x such that $f(U) \subset \text{scl } V_1$. Consequently, $f(U) \cap \text{scl } V_2=\phi$ and so $G(f)$ is strongly semi-closed.

The converse to Theorem 6 does not hold, in general, as is clear from Example 4.

Definition 7: A mapping $f: X \rightarrow Y$ is θ-irresolute iff for each $x \in X$ and each semi-neighbourhood V of $f(x)$, there is a semi-neighbourhood U of x such that $f(\text{scl } U) \subset \text{scl } V$.

Obviously, every θ-irresolute is weakly irresolute but converse may fail. Also, every irresolute is θ-irresolute but not conversely.

Hence we have the following implications diagram.

$$\begin{array}{c}\text{Irresolute} \\ \text{mapping}\end{array} \Longrightarrow \begin{array}{c}\theta\text{-irresolute} \\ \text{mapping}\end{array} \Longrightarrow \begin{array}{c}\text{weakly irresolute} \\ \text{mapping}\end{array}$$

In view of Example 3, a θ-irresolute mapping need not have a strongly semi-closed graph. However, in view of Theorem 6 and the fact that every θ-irresolute is weakly irresolute, we have

Corollary 1: If $f: X \rightarrow Y$ is θ-irresolute where Y is Urysohn, then $G(f)$ is strongly semi-closed.

The converse to Corollary 1, however, fails. It is clear by Example 4.

Recall that a closed graph may fail to be strongly semi-closed and a strongly semi-closed graph may fail to be closed. However,

Theorem 7: Let $f: X \rightarrow Y$ be pre-semi-open and have a closed graph $G(f)$. Then $G(f)$ is strongly semi-closed.

Proof: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, $G(f)$ being closed, there exist open sets U and V containing x and y, respectively, such that $f(U) \cap V=\phi$. Now, since f is pre-semi-open, $f(U)$ is semi-open. Therefore, $f(U) \cap \text{scl } V=\phi$. Consequently, $G(f)$ is strongly semi-closed.

Theorem 8: Let $f: X \rightarrow Y$ be a surjection with strongly semi-closed $G(f)$. Then Y is semi-T_2, and hence semi-T_1.

Proof: Let y_1, y_2 be distinct points of Y. Then, f being surjective, there exists an $x_1 \in X$ such that $f(x_1)=y_1$. Thus, $(x_1, y_2) \notin G(f)$. Therefore, $G(f)$ being strongly semi-closed, there exist semi-open sets U and V containing x_1 and y_2, respectively, such that $f(U) \cap \text{scl } V=\phi$. Consequently, $y_1 \notin \text{scl } V$. This shows Y is semi-T_2. Since every semi-T_2 space is semi-T_1, Y is semi-T_1.

Theorem 9: The space X is semi-T_2 iff the identity mapping $i: X \to X$ has a strongly semi-closed graph.

Proof: Immediately it follows from Theorem 3 and Theorem 8.

Theorem 10: Let $f: X \to Y$ be injective with strongly semi-closed $G(f)$. Then X is semi-T_1.

Proof: Let x_1, x_2 be distinct points of X. Then, f being injective, $f(x_1) \neq f(x_2)$. Thus $(x_1, f(x_2)) \notin G(f)$. Therefore, there exist semi-open sets U and V containing x_1 and $f(x_2)$, respectively, such that $f(U) \cap \text{scl } V = \phi$. Thus $x_2 \notin U$. Consequently, X is semi-T_1.

Theorem 11: Let $f: X \to Y$ be bijective with strongly semi-closed graph. Then both X and Y are semi-T_1.

Proof: Theorem 8 and Theorem 10.

Theorem 12: Let $f: X \to Y$ be an injective weakly irresolute with strongly semi-closed graph $G(f)$. Then X is semi-T_2

Proof: Let x_1, x_2 be two distinct points of X. Then $f(x_1) \neq f(x_2)$. Thus $(x_1, f(x_2)) \notin G(f)$. And so, there exist semi-open sets U and V containing x_1 and $f(x_2)$, respectively, such that $f(U) \cap \text{scl } V = \phi$. Thus $f^{-1}(\text{scl } V) \subset X\text{-}U$. Now, since f is weakly irresolute, there exists a semi-open set N containing x_2 such that $N \subset f^{-1}(\text{scl } V) \subset X\text{-}U$. But then $x_1 \in U$ and $x_2 \in N \subset f^{-1}(\text{scl } V) \subset X\text{-}U$ so that X is semi-T_2.

Corollary 2: If $f: X \to Y$ is an injective θ-irresolute with strongly semi-closed graph $G(f)$, then X is semi-T_2.

Proof: Theorem 12 and the fact that every θ-irresolute is weakly irresolute.

Corollary 3: Let $f: X \to Y$ be an injective irresolute with strongly semi-closed $G(f)$. Then X is semi-T_2.

Proof: Corollary 2 and the fact that every irresolute is θ-irresolute

Theorem 13: Let $f: X \to Y$ be an injective almost irresolute with strongly semi-closed graph $G(f)$. Then X is semi-T_2.

Proof: Proceeding as in Theorem 12 and then using Definition 5, scl $f^{-1}(V)$ and hence scl $f^{-1}(\text{scl } V)$ is a semi-neighbourhood of x_2. Since $X\text{-}U$ is semi-closed, scl $f^{-1}(\text{scl } V) \subset X\text{-} U$. But then $x_1 \in U$ and $x_2 \in N$ (a semi-open set) $\subset$ scl $f^{-1}(\text{scl } V) \subset X\text{-} U$ (by definition of a semi-neighbourhood), so that X is semi-T_2.

Theorem 14: Let $f: X \to Y$ be a bijective irresolute (almost irresolute, θ-irresolute or weakly irresolute) with strongly semi-closed $G(f)$. Then X and Y both are semi-T_2.

Proof: Theorem 8 and Corollary 3 (Theorem 13, Corollary 2 or Theorem 12).

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