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Economic Design of Attribute Control Chart with the Double Sampling Plan.

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〈**Abstract**〉

This paper treats an economic design of an np-control chart introducing the double sampling plan. It formulates the cost model and proposes an algorithm for determining optimal parameters. Several examples show that the cost model with double sampling plan gives more economic optimum than the conventional cost model when the acceptance number in the single sampling plan is greater than zero especially.

2회 샘플링방식을 갖는 계수형 관리도의 경제적 설계

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(85. 4. 30 접수)

〈**요 약**〉

본 논문은 2회 샘플링 방식을 도입한 np관리도의 경제적 설계를 다루고 있는바 비용모형을 정립한후 최적모수를 결정하기 위한 해법과정을 제시하고 있다.

다수의 예들은 2회 샘플링을 사용한 비용 모형의 최적해가 통상적인 비용 모형에 비해 특히 합격 판정갯수가 1이상일때 경제적임을 보여준다.

I. Introduction

Duncan[4] and Cowden[3] independently pioneered the study of the design of $\bar{X}$-charts by an economic approach. Duncan's model has received particular attention and a considerable amount of work has been developed from it since, e.g. Goel, et. al., [10] Duncan, [5] Gibra, [7] and Chiu and Wetherill. [2] A comprehensive survey of the recent developments in control chart techniques is given in Gibra. [8]

In recent years, considerable attention has been devoted to the economic design of attribute control charts. The economic design of an attribute control chart involves the determination of the parameters that minimize a relevant cost function. These parameters are the sample size, the intersample interval, and the factor that determines the spread of the control limits.

The attention of a few authors has been focused on the economic design of attribute control charts. Ladany[12] developed a model based on Duncan's model. [4] He formulated a cost function consisting of the cost of sampling, the cost incurred by defective items, the cost due to false alarms, and the cost of adjusting the process after a shift has occurred. The author suggested minimizing the total cost function by computer enumeration: however, no numerical examples were given. Chiu[1] modified Duncan's model[4] by assuming that when the np-control chart signals the presence of an assignable cause of variation, the process is stopped and a search is undertaken. Two average costs were defined. One is associated with false alarms and the other with real alarms. The author suggested two methods for minimization of the formulated cost function: a two-dimensional Fibonacci search, and a simplified method based on a prescribed probability of 90 percent that the number of defective items found in a sample falls outside the control limits when the process is out of control.

Montgomery, Heikes, and Mance[13] developed a model for a process subject to the occurrence of several assignable causes of variation. They used the same approach developed by Knappenberger.[11] The proposed cost function consists of the cost of sampling, the cost of investigating, and possibly correcting the process when an alarm is signaled by the chart, and the cost of producing defective product. Direct computer research techniques were used to optimize the expected cost function. Gibra[9] modified the models developed by Duncan,[4] Ladany,[12] and Chiu[1] by developing two models based on a realistic and more comprehensive cost function that included all possible costs that were likely to be incurred in practice. The first model assumes that the process is shut down during the search for the assignable cause. The second assumes that the process in kept running until the assignable cause is discovered. With the aid of these two models, the manufacturer can decide on the appropriate model to use if the search for the assignable cause can be undertaken when the process in either shut down or operative. Recently Ju[14] modified the models developed by Gibra[9] by using the method of a chain sampling. The author showed that these cost models were more economical than Gibra's by several examples.

In this paper we introduce the idea of a double sampling plan into cost models developed by Gibra.[9] A double sampling plan is described by the four numbers n_1, n_2, d_1, and d_2. The plan works as follows: If a first sample of size n_1 contains d_1 or less defective products, no search will be made for an assignable cause. If containing more than d_2 defective products, the search will be made. If the number of defective products is greater than d_1 but not more than d_2, a second sample of size n_2 is taken. If in the combined samples there are d_2 or less defect products, no search will be made. Otherwise, the search will be made.

We purpose an algorithm for determining optimal parameters, and show that these cost models are more economical than other conventional models through several examples.

Ⅱ. Assumptions and Operating Conditions

The assumptions and the nature of the operating conditions are summarized as follows:

(1) One or more quality characteristics are under the surveillance of an np-control chart. The expected fraction defective produced when the process is in a state of control is p_0. where p_0 is constant and known. Samples are taken every v hours. The process is continued without a search for the assignable cause whenever the number of defective products found in the first sample n_1 or in the combined samples n_1+n_2 does not exceed an acceptance number d_1 or d_2 respectively.

(2) When a second sampling is necessary, it is immediately followed by the first sampling with curtailed inspection. And let the second

sample size n_2 be equal to the first one n_1, i. e. $n_2=n_1$. Let us designate these two notation n_1 and n_2 as n.

(3) The process is subject to the occurrence of a single asignable cause that takes the form of a shift in the process fraction defective to a value p_1, $(p_1>p_0)$, where p_1 is constant and known.

(4) The time occurrence of the assignable cause exponentially distributed with parameter λ per unit of operating time.

(5) A study of many industrial operations reveals that two following situations are common in practice:

(a) the process is shut down during the search for the assignable cause and during necessary repair.

(b) the process continues in operation during the search for the assignable cause but is shut down during repair.

Let these two situations be designated as Model I and Model II respectively.

III. Formulation of a Cost Function

Before we proceed to formulate the cost function, the following characteristics shall be derived:

(1) If the assignable cause occurs between the j th and $j+1$st interval, then the average time of occurrence of the assignable cause within a sampling interval(v) is given by

$$\tau=\int_{jv}^{(j+1)v}(t-jv)\lambda e^{-\lambda t}dt\Big/\int_{jv}^{(j+1)v}\lambda e^{-\lambda t}dt$$
$$=\frac{1}{\lambda}-\frac{v}{e^{\lambda v}-1} \qquad (1)$$

(2) For double sampling plan(n_1, n_2, d_1, d_2) with curtailed inspection the average sample size $\bar{n}$ is

$$\bar{n}=n_1+\sum_{k=d_1+1}^{d_2}P(n_1:k)\Big[n_2P''(n_2:d_2-k)$$
$$+\frac{d_2-k+1}{p}P'(n_2+1:d_2-k+2)\Big] \qquad (2)$$

where

$P(n:x)$: probability of exactly x defective products out of n.

$P'(n:x)$: probability of x or more defective products out of n.

$P''(n:x)$: probability of x or less defective products out of n.

p: proportion of defective products.

Also under the same condition let α be the probability of a false alarm when the process is in control, and let P be the probability that the assignable cause is detected when the process is out of control.

$$\alpha=P(X_1\geq d_2+1|p_0)+\sum_{k=d_1+1}^{d_2}P(X_1=k|p_0)$$
$$P(X_2\geq d_2-k+1|p_0) \qquad (3)$$

$$P=P(X_1\geq d_2+1|p_1)+\sum_{k=d_1+1}^{d^2}P(X_1=k|p_1)$$
$$P(X_2\geq d_2-k+1|p_1) \qquad (4)$$

where

X_i: number of defective products found in the i th sample, $i=1$ or 2.

$$P(X_i=k|p_j)=\binom{n_i}{k}p_j^k(1-p_j)^{n_i-k}$$

for $i=0$ or 1.

$$P(X_i\geq k|p_j)=\sum_{m=k}^{n_i}P(X_i=m|p_j)$$

(3) The expected number of sampling intervals to detect the assignable cause on the j th inspected sample is

$$\sum_{j=1}^{\infty}jP(1-P)^{j-1}=1/P \qquad (5)$$

where P is given by equation(4).

Therefore, the expected time that the process is out of control before the search for the assignable cause is instituted is given by

$$v/P-\tau+\bar{n}g$$

where g is the expected inspection and charting time per unit sampled.

(4) The total expected search time for false alarms prior to the occurrence or an assignable cause($=T_1$ time the expected number of false alarms) is

$$\alpha T_1\sum_{j=0}^{\infty}\int_{jv}^{(j+1)v}j\lambda e^{-\lambda t}dt=\frac{\alpha T_1}{e^{\lambda v}-1}$$

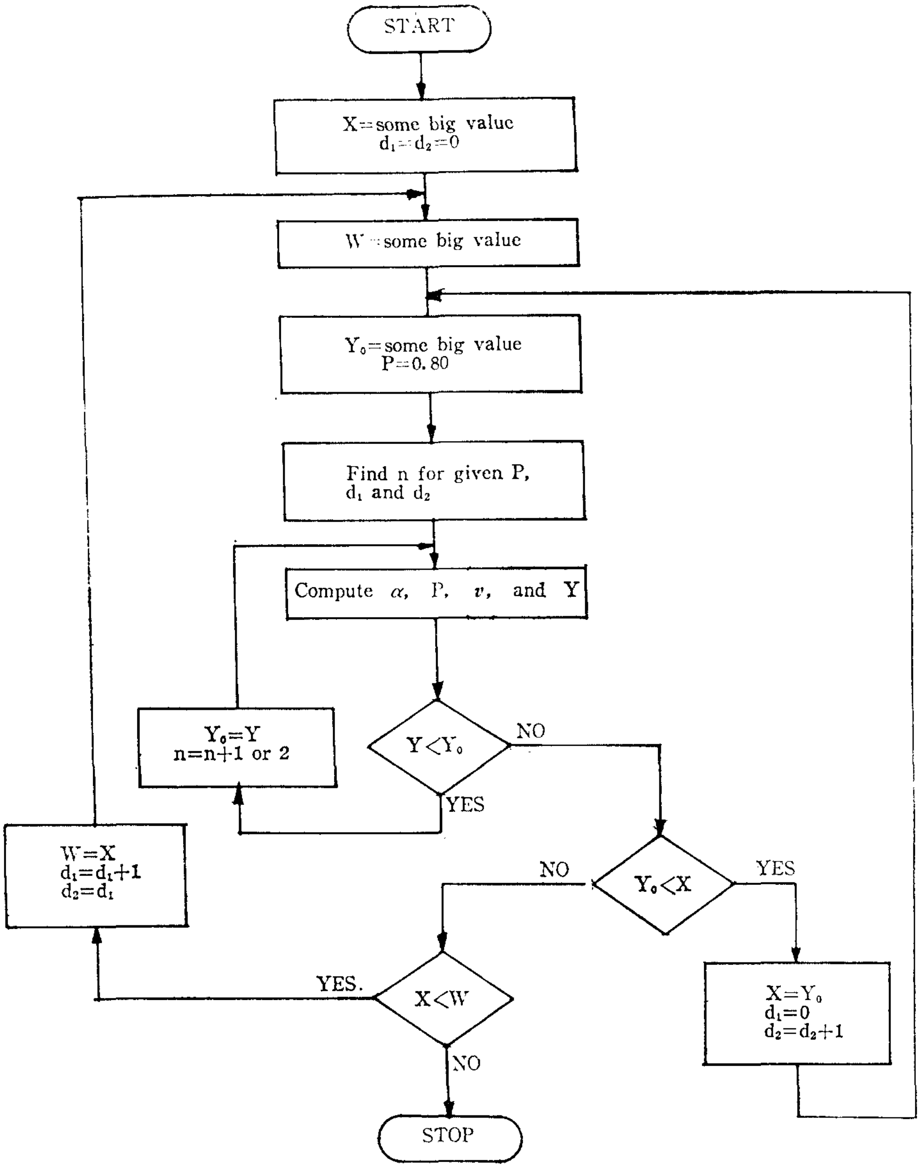

Fig. 1. Flow Chart for Search Procedure

$$=\alpha T_1(1/\lambda-\tau)/v, \qquad (6)$$

since from equation(1), $(e^{\lambda v}-1)^{-1}=(1/\lambda-T)/v$.

(5) We define the expected quality cycle time, denoted by L_i, as the expected interval between two successive periods of statistical stability. Therefore,

$$L_1=1/\lambda-v/P-\tau+\bar{n}g+\alpha T_1(1/\lambda-T)/v-T_1-T_2 \qquad \text{(Model I) (7)}$$

and

$$L_2=1/\lambda+v/P-\tau+\bar{n}g+T_1+T_2 \qquad \text{(Model II) (8)}$$

As the cost function under these assumptions is the same expression as Gibra except the sample size $\bar{n}$, we omit the procedure formulating the cost function. The notations used in the cost function are as follows:

r: production rate per unit time

C_1: cost of search for the assignable cause per unit time

C_2: cost of downtime per unit time

C_3: cost of repair per unit time

u: penalty incurred per defective item

b: cost of inspection and charting per unit sampled

h: overhead cost for maintaining the control chart per inspected sample.

For Model Ⅰ and Model Ⅱ the expected total costs per unit time, Y_1 and Y_2 respectively, are

$$Y_1=[(h+b\bar{n})(1/\lambda+v/P-\tau+\bar{n}g)/v+(C_1+C_2)\alpha T_1(1/\lambda-\tau)/v+(C_1+C_2)T_1+(C_2+C_3)T_2+ur(p_1-p_0)(\bar{n}g+v/P-\tau)]/L_1 \qquad \text{(Model Ⅰ) (9)}$$

and

$$Y_2=[(h+b\bar{n})(1/\lambda+v/P-\tau+\bar{n}g+T_1)/v+C_1\alpha T_1(1/\lambda-\tau)/v+C_1T_1+(C_2+C_3)T_2+ur(p_1-p_0)(\bar{n}g+v/P-\tau+T_1)]/L_2 \qquad \text{(Model Ⅱ) (10)}$$

where L_1 and L_2 is given by equation(7) and(8)

Ⅳ. Determination of Optimal Parameters

Now we will develope a procedure for the optimal determination of parameters n, d_1, d_2, and v. Fortunately, among these parameters the optimal intersample interval can be obtained by solving the equation $\frac{\partial Y}{\partial v}=0$ for specified values of (n, d_1, d_2), cost, and risk parameters. Then the root is found to be well approximated by

$$v\cong\left[\frac{(h+b\bar{n})+\alpha T_1(C_1+C_2)}{\lambda ru(p_1-p_0)(1/P-1/2)}\right]^{\frac{1}{2}} \qquad \text{(Model Ⅰ) (11)}$$

and

$$v\cong\left[\frac{(h+bn)+C_1T_1\alpha}{\lambda ru(p_1-p_0)(1/P-1/2)}\right]^{\frac{1}{2}} \qquad \text{(Model Ⅱ) (12)}$$

The search procedure can be performed with the aid of a personal or mini computer. But it is not as cumbersome as it might appear. Figure 1 shows a flow chart for the search procedure. Now we shall compare this cost model using double sampling plan with the one using single sampling play by several numeric examples given on Table 1. Notice that the single sampling plan can be regarded as the special double sampling plan with $d_1=d_2$.

The comparisons of the optimal parameters and cost to the examples on Table 1 lie in Table 2. They show that the optimum of this cost model is more economical than the optimum of the conventional cost model when the acceptance number d in the single sampling plan is greater than zero. For many other numeric examples we could get the same conclusion.

Table 1. Values of Cost and Risk Parameters Used in Examples

Example Number	p_0	p_1	λ	r	T_1	T_2	g	C_1	C_2	C_3	h	b	u
1[1]	.02	.10	.0125/hr	2,500/hr	.2hr	2.0hr	.005hr	\$10.0	\$15.0	\$20.0	\$2.0	\$.10	\$3.0
2	.01	.10	.0125	2,500/	.2	2.0	.005	10.0	90.0	20.0	2.0	.01	.5
3	.02	.10	.0125	2,500/	.2	2.0	.005	10.0	90.0	20.0	2.0	.01	.5
4	.02	.05	.0125	2,500/	.2	2.0	.005	10.0	90.0	20.0	2.0	.01	.5
5	.02	.10	.0125	2,500/	.2	2.0	.005	10.0	90.0	20.0	2.0	.10	3.0
6	.01	.05	.0125	2,500/	.2	2.0	.005	10.0	90.0	20.0	2.0	.01	3.0
7	.01	.05	.0125	2,500/	.2	2.0	.001	10.0	90.0	20.0	2.0	.01	3.0

1) These values of parameters are the same as in Example 1 of Gibra[9]

Table 2. Optimal Design for Numerical Examples of Table 1.

Example Number		Conventional Optumum				Optimum using Double Sampling				
		n	d	v	cost Y	n	d_1	d_2	v	Cost Y
Model I	1	16	0	.96	10.9610	16	0	0	.96	10.9610
	2	53	2	1.84	6.2108	48	1	4	1.80	5.9813
	3	77	4	2.02	6.3199	58	2	6	1.93	6.0338
	4	181	6	3.89	5.7266	122	3	8	3.54	5.4897
	5	27	1	1.06	15.8564	22	0	2	1.02	15.0944
	6	80	2	1.12	10.6752	43	0	2	1.09	10.1173
	7	124	3	1.27	9.2832	97	1	5	1.24	8.7440
Model II	1	16	0	.87	12.1272	16	0	0	.87	12.1272
	2	26	0	1.84	5.9610	27	0	1	1.75	5.8859
	3	41	1	1.97	5.8977	30	0	2	1.90	5.8312
	4	48	0	3.66	4.7616	48	0	0	3.66	4.7616
	5	16	0	.87	13.9363	16	0	0	.87	13.9363
	6	39	0	1.11	9.3903	39	0	0	1.11	9.3903
	7	50	0	1.23	8.7780	51	0	1	1.17	8.7048

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