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The (s, q) System With Ordering Quantity Dependent Leadtime.

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〈Abstract〉

This paper examines the reorder-point-lot-size system of which the leadtime depends on the quantity ordered. By prescribing one of two decision variables, the optimal values of the other decision variable are obtained. Probabilistic demand, backordering of shortage and continuous reviewing of inventory level are assumed.

주문량에 비례하는 인도 기간을 가진 (s, q) 체계

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〈요 약〉

본 논문은 주문된 제품의 인도 기간이 주문량에 비례할 때 (s, q) 체계의 최적화 방안을 모색한다. 체계의 한 정책변수의 값을 명시함으로써 다른 정책변수의 최적치를 구한다. 이때 확률적 수요, 재고 고갈분의 사후 보충 그리고 재고수준의 계속적인 검사를 가정한다.

I. Introduction

The studies of inventory system deal with two classes of leadtime model. The one is the constant leadtime model and the other is the probabilistic one. The inventory systems with constant leadtime find many applications where variation of leadtime is not significant and where rough estimate can be used for prior analysis. The endeavour to apply the probabilistic leadtime model meets some difficulties for the fact that the probability density function (*pdf*) of on-order quantity is required to formulate the cost function. But, recently, many results of the studies of the probabilistic leadtime model(1, 2, 3) involving dynamic inventory model(6, 7) are being published. However, the studies of leadtime commonly disregards the relationship between ordering quantity and leadtime(5). In general it is the fact that leadtime, the time lag between the placing of an order and its arrival, is determined by the ordering quantity. There exists a relationship between leadtime and ordering quantity. Thus it is a realistic assumption that leadtime is a function of the ordering quantity. In this paper, it is treated with a linear function with constant additive. But it can be easily extended to general function if it is satisfied that the function is increasing one.

One of the endeavour to apply this leadtime model was carried out on the scheduling-period-order-level system(5). In this paper, this ordering quantity dependent leadtime model is applied to reorder-point-lot-size system under continuous reviewing, where the orders are placed by the quantity of a lot at the time the inventory level involving on-order quantity reaches the reorder point. The cost function will be derived and optimal reorder point and lot size will be able to be obtained from this.

Ⅱ. Assumptions.

The assumptions are summarized as follows:

(1) shortages are backordered;

(2) the demand distribution is continuous and demand pattern for a scheduling period is uniform(4);

(3) the leadtime(L) is a linear function of lot size (q) such as: $L=a+b\cdot q$ where a and b are constants;

(4) there exist three cost elements, inventory holding cost (C_1), shortage cost (C_2) and replenishment cost(C_3);

(5) the system considered is reorder-point-lot-size system((s, q)system) under continuous reviewing;

(6) cost functions are formulated in terms of expected average cost.

Ⅲ. Model

The progress will be made in three phases. First, the cost function will be derived for the lot size system where the reorder point is prescribed. And then the reorder point system of which the lot size is prescribed, will be tried. Finally, the extension to the reorder-point-lot-size system will be made from the above two resuts.

1. The lot size system ((s_p, q) system)

When the reorder point is a prescribed constant, s_p, then expected average amount in inventory can be expressed as

$$I_1=s_p+\frac{q}{2}-r\cdot L$$

where q, r and L are lot size, demand rate and leadtime, respectively(4). Since inventory reviewing is continuous, then ;

$$s_p=V_{max}=U_{max}\cdot L=U_{max}(a+bq)$$

where V_{max} and U_{max} are the maximum demands during the leadtime and during unit time.

Hence, expected average amount in inventory can be rewritten as

$$I_1=U_{max}\cdot(a+b\cdot q)+\frac{q}{2}-r(a+b\cdot q)$$

The expected number of replenishment per unit time is

$$I_3=\frac{1}{\bar{t}}=\frac{r}{q}$$

where $\bar{t}$ is the average length of a scheduling period.

Thus, the expected total cost of the lot size system with continuous reviewing is then

$$C(q)=c_1\cdot\left[U_{max}\cdot(a+b\cdot q)+\frac{q}{2}-r\cdot(a+b\cdot q)\right]+c_3\frac{r}{q}$$

The optimal lot size is then

$$q_0=\sqrt{c_1\cdot r/\left\{c_1\cdot\left[(U_{max}-r)b+\frac{1}{2}\right]\right\}}$$

and the corresponding minimum expected total cost is

$$C_0=c_1\cdot a(U_{max}-r)+2\sqrt{c_1\cdot c_3\cdot r\left[(U_{max}-r)\cdot b+\frac{1}{2}\right]}$$

2. The reorder point system ((s, q_p) system)

Now, let's try the reorder point system of which the lot size is a prescribed constant, q_p.

When the lot size is prescribed, the leadtime is then

$$L=a+b\cdot q_p=\text{constant}$$

Thus this system reduces to the reorder point system with constant leadtime. Naddor (4) formulated the cost function of this system as follows

$$C(s)=c_1\left(s+\frac{q_p}{2}-\bar{v}\right)+(c_1+c_2)\cdot\frac{A(s)-A(s+q_p)}{q_p}$$

where

$$A(p)=\begin{cases} 0, & p\geq V_{max} \\ \frac{\bar{v}}{2}\int_p^{V_{max}}\frac{(v-p)^2}{v}e(v)dv, & 0\leq p\leq V_{max} \\ \frac{(\bar{v}-p)^2}{2} & p\leq 0 \end{cases}$$

In the above formulation, v is the demand during the leadtime, $e(v)$ is the *pdf* of v, $\bar{v}$ is the mean of v and V_{max} is the maximum demand during the leadtime.

The optimal reorder point, s_0 can be found from

$$D(s_0)=\frac{c_1}{c_1+c_2}$$

where $q_p\geq V_{max}$, the function $D(s)$ is given by

$$D(s)=\begin{cases} 0, & s\geq V_{max} \\ \frac{\bar{v}}{q_p}\int_s^{V_{max}}\frac{v-s}{v}e(v)dv, & 0\leq s\leq V_{max} \\ \frac{\bar{v}}{q_p}\left(1-\frac{s}{\bar{v}}\right), & -q_p-V_{max})\leq s\leq 0 \\ \frac{\bar{v}}{q_p}\left[1-\frac{s}{\bar{v}}-\int_{s+q_p}^{V_{max}}\frac{(v-s-q_p)}{v}e(v)dv\right] & -q_p\leq s\leq -(q_p-V_{max}) \\ 1, & s\leq -q_p \end{cases}$$

But, where $q_p\leq V_{max}$, it is given by

$$D(s)=\begin{cases} 0, & s\geq V_{max} \\ \frac{\bar{v}}{q_p}\int_s^{V_{max}}\frac{v-s}{v}e(v)dv, & V_{max}-q_p\leq s\leq V_{max} \\ \frac{\bar{v}}{q_p}\left[\int_s^{V_{max}}\frac{v-s}{v}e(v)dv-\int_{s+q_p}^{V_{max}}\frac{(v-s-q_p)}{v}e(v)dv\right], & 0\leq s\leq V_{max}-q_p \\ \frac{\bar{v}}{q_p}\left[1-\frac{s}{\bar{v}}-\int_{s+q_p}^{V_{max}}\frac{(v-s-q_p)}{v}e(v)dv\right], & -q_p\leq s\leq 0 \\ 1 & s\leq -q_p \end{cases}$$

In either case the function $D(s)$ is an increasing function in s. Hence, for any given values c_1, c_2, q_p, and $\bar{v}$, it is relatively simple to establish the range within which s_0 ought to be and then it can be computed(4).

3. The reorder-point-lot-size system ((s, q)system)

The cost function of the reorder-point-lot-size system can be extended from the above two results as follows

$$C(s,\ q)=c_1\cdot\left(s+\frac{q}{2}-\bar{v}\right)+(c_1+c_2)\cdot\frac{A(s)-A(s+q)}{q}+c_3\cdot\frac{r}{q}$$

where

$$A(p)=\begin{cases} 0 & p\geq V_{max} \\ \frac{\bar{v}}{2}\int_p^{V_{max}}\frac{(v-p)^2}{v}e(v)dv, & 0\leq p\leq V_{max} \\ \frac{(\bar{v}-p)^2}{2}, & p\leq 0 \end{cases}$$

In this formulation, v is also the function of ordering quantity. Therefore, it is not easy to obtain the closed form representation of *p d f* of v, $e(v)$. Hence it will be left as a topic for further study to obtain the optimality condition to optimize s and q simultaneously.

Ⅳ. Discussion

This paper tried to derive the cost function of the reorder-point-lot-size system with ordering quantity dependent leadtime under continuous reviewing. Leadtime was treated with linear function of ordering quantity. When one of two decision variables was prescribed, we could represent the cost function in closed form and the optimal solution or the optimality condition obtained satisfactorily. But there existed some difficulties in optimizing two decision variables simultaneously. Thus it will be left as a further study material. And the extension to periodic reviewing system will be able to be another topic for further study.

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