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샘플데이타 제어시스템 설계를 위한 새로운 고전기법

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<요 약>

본 논문은 샘플데이타 제어시스템을 위한 새로운 직접설계방법(Direct Design method)을 제안한다. 그 제안된 기법은 아날로그 제어기설계 방법을 직접적으로 디지탈 제어기설계에 응용한 것으로 이는 w-Domain에서의 근궤적법을 이용한 하나의 고전기법이다. 고전기법으로 부터 얻은 아날로그 제어기설계 경험을 살려 샘플데이타 제어시스템을 설계하기 위해 우선 w-Domain이라는 새로운 복소수 영역과 안정성 판별기준을 정의하고 그 영역에서 상태방정식으로 표현된 샘플데이타 시스템을 제안한다. 또한 그 영역에서 근궤적법으로 설계된 제어기를 디지탈 제어기로 구현하기 위해 이득변환식(Gain Transformation Formulus)이 고안된다. 본 논문에서 제안된 방법을 안테나 방위각 제어문제에 적용해 보임으로써, 아날로그 제어기설계 경험과 기술이 디지탈 제어기 설계를 위한 직접설계방법에 응용될 수 있음을 보여준다.

New Classical Design Method for Sampled-Data Control System

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<Abstract>

This paper presents a new direct design method for sampled-data control systems. The proposed synthesis method utilizes a design method of analog control system to design a digital controller using direct design method, which is a classical root-locus method in the *w*-domain. To adopt the valuable experiences obtained from design of analog controllers in classical design methods to design of sampled-data control systems, first of all, in this paper a new complex domain, what we call w-domain, and stability criterion in the domain are defined. Then a sampled-data system in the w-domain is proposed, which is represented in state space. To realize a unique digital controller synthesized in the w-domain, exact gain transformation formulus is invented. Through an illustrative example of antenna azimuth control in the proposed approach, any experts to analog designs can easily take advantage of their valuable knowledge and skill to design sampled-data control systems in direct design method.

I. Introduction

As integrated circuit technology has been progressed, application of cheaper microprocessor to industrial automatic control has attracted control system designers. Recently, broad attention to sampled-data control systems has been paid. However, sampled-data control system is quite different from analog system so that expert of analog control systems can not directly utilize their valuable experiences to design of sampled-data systems. Since Ref.[1] has shown precious discretization of sampled-data system for linear quadratic design, a transformation method associated with a special mapping was proposed in Ref.[2]. They showed possibility of utilization of the mapping to direct design (i.e., opposite to emulation design) of digital control systems. Ref.[3] utilized the mapping to design a digital controller in Bode plot. Furthermore this method was extended to direct design of optimal digital controller for multivariable sampled-data systems in Ref.[4] and [5]. In Ref.[6], the method of optimal digital control design, called W-Synthesis methodology, successfully solved a typical problem resulted from digital implementation (i.e., computation time-delay). However, none of them did treat the similar concept with a systematic classical design method for sampled-data control systems. From the references mentioned above, it seems to be optimistic that there exists possibility of unifying both of the design approaches of direct digital design and analog design into one in the newly defined domain in this paper. Hence this paper proposes a unified synthesis approach applicable to direct design of digital control systems which is based on knowledge and skill obtained from analog designs. So far in the history of development of control system design methods, analog design methods have been established in the Laplace s-domain while digital design methods have in the z-domain, separately. Thus unifying the two different view points is meaningful. In order to do that, we introduce a well-known conformal mapping whose result is called the w-domain. Many experts of digital control designs recognized that the mapping as an approximation tool is very useful to implementation of already existing analog controllers. However, this approximation method has fatal limits; instability, performance degradation and so on [7]. So direct digital design methods purely in the z-domain has been favored by most digital designers. This implies value of the unified approach in the w-domain proposed in this paper for direct digital design. In section II, a typical sampled-data system is defined in state space. Section III mentions the w-domain and representation of the sampled-data system is given in the w-domain in section IV. In section V, in order to realize a unique digital controller, exact gain transformation is formulated. A unified design procedure is proposed in section VI and an example is illustrated in section VII. Finally conculsion is given in section VIII.

II. A Sampled-Data System in State-Space

Consider a typical sampled-data system consisting of a continuous-time linear time-invariant (LTI) plant model and a discrete-time linear shift-invariant (LSI) measurement model in state space.

The continuous-time LTI plant model is given by

$$\Sigma_s : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ z(t) = Cx(t) + Du(t) \\ y(t) = Hx(t) \end{cases} \tag{1}$$

with initial conditions $x(0)=x_0$ where $x(t)$ is a state vector in $\Re^n$, $u(t)$ is an input vector in $\Re^m$, $w(t)$ is a disturbance vector in $\Re^{d_1}$, $z(t)$ is a criterion vector in $\Re^q$ and $y(t)$ is an output vector in $\Re^p$. The state matrices A, B, G, C, D and H are constant matrices of dimensions in $\Re^{n\times n}$, $\Re^{n\times m}$, $\Re^{n\times d_1}$, $\Re^{q\times n}$, $\Re^{q\times m}$ and $\Re^{p\times n}$ respectively. Note possibly several filters, if necessary, are included in the plant model. Here we introduce 'ideal' samplers at the plant input $u(t)$ and output $y(t)$. They are analog-to-digital converters (ADC) which sample the input signal $u(t)$ and the measurement output signal $y(t)$ at each sampling time T. A zero-order hold (ZOH) is a digital-to-analog converter (DAC) used to hold the signal $u(kT)$ over the sampling interval of $kT \le t < (k+1)T$. In this sampled-data system, we assume that the ADC and the DAC are synchronized, and there is no computation time-delay at control action.

For notation convenience, we define the following matrix functions,

$$\Phi(t) = e^{A\cdot t}, \quad \Psi(t) = \int_0^t e^{A\cdot\theta} B \, d\theta \tag{2}$$

and G_d is a linear transformation of $w(t) \in L_2$ $[0,T]$ given by

$$G_d \xi_k = \int_0^T e^{A(T-\theta)} G \, w(kT+\theta) \, d\theta \tag{3}$$

where ξ_k is a certain discrete-time signal vector in $\Re^n$ defined from the operator relation of Eq. (3).

$$G_d : L_2[0,T] \rightarrow \Re^n$$

Note that $\Phi(t)$ is nonsingular for $t \ge 0$.

A typical sampled-data system is then

defined by the system Σ_{sz} and depicted in Figure 1.

$$\Sigma_{sz}: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ z(t) = Cx(t) + Du(t) \\ y_k = Hx_k + D_2 v_k \end{cases} \quad (4)$$

Note that $x_k \equiv x(kT)$, $u_k \equiv u(kT)$ and $y_k \equiv y(kT)$. And the term $D_2 v_k$ is introduced into the measurement equation in Eq.(1) to represent discrete-time measurement noises where the measurement noise v_k is of dimension $\Re^{d_2}$ and the matrix D_2 is of dimension $\Re^{p \times p}$.

III. What is the *w*-Domain ?

Traditionally, methods developed in the frequency domain have been applied to control system design and anaysis; for instances, the Laplace *s*-domain for analog control system and the Jurry's *z*-domain for digital control system. Along the same line, we introduce a special frequency domain in which we will develop a new synthesis method for a sampled-data control system ; we call it the *w*-domain. Basically, the *w*-domain is defined from a well-known conformal mapping of the modified Tustin transformation; $w = \frac{2}{T}\frac{z-1}{z+1}$ [2] where the operator z is defined as e^{sT}. The *w*-domain has very special characteristics in a sense of stability and geometry solutions of the domain. Usually, the system stability is characterized by of the characteristic equation of the system concerned. When all of the solutions are located inside unit-circle of the *z*-domain (excluding the $|z|=1$) for a discrete-time system, we say the system is asymptotically stable. However, region of the same sampled-data system stability in the *z*-domain (i.e., asymptotic stability) is defined as the left-half plane of the *w*-domain (excluding the imaginary-axis of the *w*-domain), which is exactly the same as that for a continuous-time system. It will be proved later. Also geometry of the *w*-domain can be represented by damping ratio (ζ_{eq}) and natural frequency ($w_{n_{eq}}$) which are useful to design and anaysis of control systems. The parameters are a function of the sampling time T; if $|w_s|$ is less than $\frac{\pi}{4T}$, then v is approximately equal to w_s where $s=\sigma + jw_s$, $w=\eta + jv$ and $j=\sqrt{-1}$. A typical propetry of the *w*-domain is pictured in Figure 2.

IV. The Sampled-Data System in the *w*-Domain

The sampled-data system Σ_{sz} given in Eq.(4) can not be described in terms of system transfer function matrix because it contains both continuous-time signals and discrete-time sequences; namely, the system is a linear, hybrid and periodically time-varying system. However if we consider the system at the sampling instants $t=kT$ ($k=0,1,...$), the sampled-data system can be described in the *w*-domain. As a result, the sampled-data system with zero initial conditions in the *w*-domain is defined as the following system represent-ation:

$$\Sigma_w : \begin{cases} wx_w(w) = A_w x_w(w) + B_w u(w) + G_w \xi(w) \\ z_w(w) = C_w x_w(w) + D_w u(w) + J_w \xi(w) \\ y_w(w) = H_w x_w(w) + \Gamma_w \xi(w) + D_2 v(w) \end{cases} \quad (5)$$

where $y_w(w)$ is newly defined in the w-domain as the output vector; $y_w(w) \equiv y_w(w) - L_w u(w)$. And $z_w(w)$ denotes a criterion vector of dimension of $\Re^{n+m}$, containing both the original state $x(w)$ and the control input $u(w)$; namely

$$z_w(w) = \begin{bmatrix} x(w) \\ u(w) \end{bmatrix} \quad (6)$$

where

$$A_w = \frac{2}{T}(\Phi(T) + I_n)^{-1}(\Phi(T) - I_n)$$
$$B_w = \frac{4}{T}(\Phi(T) + I_n)^{-2}\Psi(T)$$
$$G_w = \frac{4}{T}(\Phi(T) + I_n)^{-2}G_d$$
$$C_w = \begin{bmatrix} I_n \\ 0_{m \times n} \end{bmatrix}$$
$$D_w = \begin{bmatrix} -(\Phi(T) + I_n)^{-1}\Psi(T) \\ I_m \end{bmatrix}$$
$$J_w = \begin{bmatrix} (\Phi(T) + I_n)^{-1}G_d \\ 0_{m \times d_2} \end{bmatrix}$$
$$H_w = H$$
$$\Gamma_w = -H(\Phi(T) + I_n)^{-1}G_d$$
$$L_w = -H(\Phi(T) + I_n)^{-1}\Psi(T)$$

Now we define stability in the w-domain as follows.

Theorem IV.1 *The sampled-data system* Σ_{sz} *is (asymptotically) stable in the Jurry's stability sense* if and only if *the sampled-data system* Σ_w *is (asymptotically) stable in the sense of stability defined in the w-domain.*

Proof: Let λ_z and λ_w be eigenvalues of the system matrices $\Phi(T)$ and A_w respectively. It can be shown that $\lambda_w = \frac{2}{T}\frac{\lambda_z - 1}{\lambda_z + 1}$. From this relationship, we can deduce that the real part of the eigenvalue λ_w is negative (i.e., stable) if and only if the magnitude of the eigenvalue λ_z is less than one. This can be easily seen from the fact that $\eta = Re(\lambda_w) = \frac{2}{T}\frac{|\lambda_z|^2 - 1}{|\lambda_z|^2 + 2Re(\lambda_z) + 1} < 0$ implies $|\lambda_z| < 1$), where Re(λ) means the real part of λ.

There are some remarkable observations on the sampled-data system representation in the w-domain.

Remark IV.1 *When the sampling time T approaches zero, the w-operator becomes the Laplace operator s; that is,* $\lim_{T \to 0} w = s$.

Remark IV.2 *Even through the plant transfer function matrix* $G_s(s)$ *(or* $G_z(z)$*) between the inputs u(s) (or u(z)) and the outputs y(s) (or y (z)) is strictly proper, the corresponding transfer function matrix expressed in the w-domain will always be proper with a nonzero direct feedthrough matrix* L_w. *The transfer function matrix in the w-domain is defined by* $(G(w) = H_w(wI_n - A_w)^{-1}B_w + L_w$.

Now, we examine the 'infinite' structure in the w-domain. Usually, the 'infinite' structure of the Laplace-domain is characterized by magnitude of eigenvalues and zeros of a continuous-time system. However, the 'infinite' structure of the w-domain is interpreted that eigenvalues and zeros of the system Σ_w are located at $\pm\frac{2}{T}$. It is evident from Remark IV.1 that, as the sampling time T approaches zero, the 'infinite' location of $\pm\frac{2}{T}$ in the w-domain will map into $|s|\rightarrow\infty$ in the Laplace-domain.

Definition IV.1 *Infinite structure of the w-domain defines the location of* $\pm\frac{2}{T}$

It is noted that knowledge of the infinite structure in the s-domain plays an important role in problems of disturbance decoupling, optimal regulator (or estimator), closed-loop transfer recovery (CLTR), model reduction, and H_∞ control of a continuous-time system.

V. Realization of A Digital Feed-back Controller

In general, digital controllers are implemented in microprocessors in terms of difference equations to industrial applications. A typical closed-loop sampled-data system is depicted in Figure 1 where Σ_s and $K_z(z)$ denote the usual continuous-time plant model shown in Eq.(1) and a typical transfer function matrix of the discrete-time controller given in Eq.(9), respectively. As discussed before, the w-domain is not a physical domain but rather it is a mathematical frequency-domain. In order to implement a discrete-time controller synthesized in this domain, the equivalent discrete-time controller in the z-domain must be constructed *uniquely*. The conversion process to the equivalent in the z-domain must also preserve the overall closed-loop system stability and performance associated with the sampled-data system Σ_w and an 'analog-type' controller designed in the w-domain. In other words, if a stabilizing controller in the w-domain is found and possesses the desired performance for the system Σ_w given in Eq.(5), then the equivalent discrete-time controller obtained uniquely under such a gain transformation to be found in this section must also stabilize the sampled-data system Σ_{sz} defined in Eq.(4) and have the same level of performance as that is evaluated in the w-domain. One kind of performance criterion is, for example, an appropriately-defined norm. What to follow is about the gain transformation for a well-posed sampled-data system Σ_w.

To begin with, let us define a general type of digital controllers in the w-domain with dimension of r ($r\leq n$) in terms of a LTI state model of the form,

$$\Sigma_{cw}\ :\ \begin{cases} wx_\tau(w) = A_\tau x_\tau(w) + B_\tau y_w(w) \\ u(w) = C_\tau x_\tau(w) + D_\tau y_w(w) \end{cases} \tag{7}$$

where $y_w(w)$ denotes the design output vector defined in Eq.(5) and we assume zero initial conditions of the controller state (i.e., $x_\tau(0)=0$). The controller formulation in Eq.(7) is general and it permits a complete freedom in selecting design parameters among the controller state matrices A_τ, B_τ, C_τ and D_τ.

For simplicity, one can represent the controller state equations given in Eq.(7) by a single constant gain matrix K_τ in a compact manner,

$$K_\tau = \begin{bmatrix} D_\tau & C_\tau \\ B_\tau & A_\tau \end{bmatrix} \tag{8}$$

As a counterpart of the controller defined in the w-domain as given in Eq.(7), one can also define a general LSI discrete-time controller in the discrete-time domain,

$$\Sigma_{ck} : \begin{cases} x_{c_{k+1}} = A_c x_{c_k} + B_c y_k \\ u_k = C_c x_{c_k} + D_c y_k \end{cases} \tag{9}$$

and initial conditions of the controller states are assumed to be zero, i.e., $x_{c_0}=0$. The controller state matrix A_c is assumed to be of order r (i.e., $r \leq n$). The controller structure in Eq.(9) is general, and the design parameters in the controller state matrices A_c, B_c, C_c and D_c can be selected as design variables. Furthermore, structure and order r of the controller are arbitrary and can be set up to represent any desired configuration, e.g., integral control, a controller of lead-lag network and a low-pass controller (where D_c is zero). For simplicity in notation, again, we define the quadruple $\{A_c, B_c, C_c, D_c\}$ in Eq.(9) by a single constant gain matrix K as follows,

$$K = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \tag{10}$$

Suppose that a stabilizing gain K_τ has been found in the w-domain, then the next step is to reconstruct from K_τ the equivalent feedback gain matrix K as given in Eq.(10) that keeps the same closed-loop stability and performance as those obtained in the w-domain. So we have the following theorem.

Theorem V.1 *Let K_τ be a stabilizing feedback controller defined in Eq.(8) for the system Σ_w. The equivalent discrete-time feedback controller is obtained by the following transformation.*

$$K = \tilde{C}\tilde{A}\tilde{B} + \tilde{D} \tag{11}$$

where

$$\tilde{A} = \begin{bmatrix} (\frac{2}{T} I_r - \bar{A})^{-1} & I_r \\ \frac{4}{T}(\frac{2}{T} I_r - \bar{A})^{-2} & (\frac{2}{T} I_r - \bar{A})^{-1} (\frac{2}{T} I_r + \bar{A}) \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} \bar{B} & 0 \\ 0 & I_r \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} \bar{C} & 0 \\ 0 & I_r \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} \bar{D} & 0 \\ 0 & 0_{r\times r} \end{bmatrix}$$

and

$$\begin{aligned} \bar{A} &= A_\tau - B_\tau L_w (I_m + D_\tau L_w)^{-1} C_\tau \\ \bar{B} &= B_\tau (I_p + L_w D_\tau)^{-1} \\ \bar{C} &= (I_m + D_\tau L_w)^{-1} C_\tau \\ \bar{D} &= D_\tau (I_p + L_w D_\tau)^{-1} \end{aligned} \tag{12}$$

The resulting controller also stabilizes the sampled-data system Σ_{sz} in Eq.(4).

Proof: see the reference [4].

There are some observations from the above results as summarized in the following.

Remark V.1 *Theorem V.1 is a generalization of the gain transformation associated with a* *static output-feedback design. When* $A_\tau = B_\tau = C_\tau = 0$, *then* $K=\bar{D}$ *where* $D_c=\bar{D}=(I_m + D_\tau L_w)^{-1} D_\tau$. *Hence* $K_z=\bar{D}$.

Remark V.2 *Results of Theorem V.1 provide us with a framework to first design discrete-time controllers in the w-domain and then later reconstruct them in the z-domain for design implementation. Note that under the relation of Eq.(11), closed-loop stability and performance is preserved. This result plays a key role in the development of a new digital control synthesis methods directly in the w-domain instead of the z-domain.*

Remark V.3 *When we apply the gain transformation to the gain matrix* K_τ *designed in the w-domain, structure of the controllers defined in the w-domain may not be preserved. In other words, for a given structure of the digital controller K in the discrete time-domain, a quite different structure may be defined in the w-domain and then can be achieved using nonlinear constraints. Nontheless, results of the gain transformation theorem are useful when no specific structure needs to be specified defined for the discrete-time controller except its order.*

In next section, we would like to propose an alternative approach of designing digital compensators. The approach is based on root-locus in the w-domain with the sampled-data system properly defined in the w-domain.

VI. Procedure of a Direct Digital Controller Synthesis

In the previous sections, we present some fundamental results for a new synthesis procedure. They are summarized below:

- State model representation in the w-domain; the system Σ_w represents a discretized sampled-data system in the w-domain.
- Stability definition in the w-domain; characteristic roots of the system Σ_w should locate in the open left-half of the w-plane for stability.
- Structure of the w-domain; constant damping- and frequency-line in the w-domain which are a function of a sampling time and equivalent to those in the s-domain.
- Exact gain transformation relation between a feedback gain matrix in the w-domain and a gain matrix in the discrete-time domain; closed-loop system stability and performance level achieved in the w-domain are preserved under the gain transformation.

Among these results, knowledge of the geometry of the w-domain and the exact gain transformation allow us to extend the capability of the w-domain to development of a systematic synthesis procedure in the w-domain for multivariable sampled-data systems Σ_{sz}. As pointed out, the w-domain has only been applied to design and analysis of using a Bode frequency-response for a single-input-single-output (SISO) systems [2]. The w-mapping has also been used as a mathematical tool to approximate an analog filter to the 'well-emulated' discrete filter.

Here, however, we develop a systematic procedure of synthesizing a multivariable sampled-data control system Σ_{sz} as a direct digital design method. A general framework for direct digital control synthesis in the w-domain is proposed. It differs from early work in the w-domain [3] by the fact that this approach is basically framed with a new vector $y_w(w)$ called here as the design output vector. The vector $y_w(w)$ is derived from the output $y(w)$ by removing the direct feed-through term L_w from the control input $u(w)$. Effects of the direct feedthrough term L_w is accounted for in the realization of the 'equivalent' discrete-time controller. The synthesis procedure is stated as follows.

1. Build the sampled-data system Σ_w in state space.
2. Design an analog-type controller directly in the w-domain to stabilize the system Σ_w and to satisfy design specifications represented in the w-domain.
3. Reconstruct the equivalent digital controller via the Gain Transformation.

Schematic of the synthesis procedure is depicted in Figure 3 where $K_w(w)$ denotes the transfer function matrix of a controller defined in the w-domain as given in Eq.(7).

Within the procedure of the systematic synthesis, what to do first of all is to determine how to design the analog-type controller defined in the w-domain. Among various possible methods, one can utilize a *classical design technique*, applicable to an analog controller design in the s-domain; successive-loop-closure of the root-locus method. It is noted that, to some problems of a digital controller design associated with a specific structure in the discrete-time domain, application of the synthesis procedure proposed in this paper needs to be cautious.

In next section, we would like to illustrate a simple design example using the classical design method proposed within the unified framework of the synthesis procedure; Generalized Root-Locus in the w-domain.

VII. A Design Example in the Classical Design Method

In the following, we illustrate the procedure of a new synthesis methodology proposed in this paper. Since the synthesis procedure generalizes in the w-domain the well-known classsical design method of the root-locus in the s-domain, it is called 'Generalized Root-Locus Method in the w-domain'. As an illustration, a simple design example of an antenna azimuth control is considered, which is given in Ref.[3]. The linear model for the azimuth control is given as $J\ddot{\theta}(t)+B\dot{\theta}(t)=T_c(t)+T_d(t)$ where J implies a moment of inertia of the antenna and drive parts, B denotes the bearing and aerodynamic friction and the back emf of the DC-drive motor, and $T_c(t)$ and $T_d(t)$ mean the applied net torque and the disturbance torque due to wind. After nondimensionalization in B (i.e., B/J=0.1, $T_c(t)/B=u(t)$, $T_d(t)/B=w_d(t)$), we get a single input single output (SISO) transfer function $G_s(t)$ between input $u(t)$ and output $\theta(t)$

$$G_s(s) = \frac{1}{s\,(10s + 1)} \qquad (13)$$

For a sampling time T=1 *sec* given, the sampled-data control design must stabilize the antenna azimuth ($\theta(t)$) with equivalent damping ratio (ζ_{eq}) greater than 0.6. From Eq.(5) the transfer function between $u(w)$ and $y_w(w)$ in the system Σ_w.

$$G_w(w) = \frac{(-\ 0.049084)\ (w\ -\ 2.0365)}{w(w\ +\ 0.099917)} \qquad (14)$$

Let us define a phase-lead compensator directly in the w-domain as follows,

$$K_w(w) = \frac{K_p\,(w + 0.2)}{(w + 3)} \qquad (15)$$

Note that the compensator pole and zero are selected in trial-and-error. Now, the gain K_p is obtained by means of the Generalized Root-Locus in the w-domain.

After the trial-and-error process, the gain K_p of -13.0385 is selected from the root-locus, shown in Figure 4. The resulted compensator with K_p selected stabilizes the system $G_w(w)$ with desired damping ratios. The closed-loop eigenvalues in the w-domain are $-3.9461 \times 10^{-1} \pm j1.5294 \times 10^{-2}$ and -1.6707, which satisfy the required damping ratio greater than 0.6.

Once the compensator $K_w(w)$ is synthesized within the procedure depicted in Figure 3, then the equivalent (unique) digital compensator is obtained through the Gain Transformation given in TheoremV.1.

$$K_z(z) = \frac{(-\ 7.1603)\ (\ z\ -\ 0.94175)}{(z\ +\ 0.19660)} \qquad (16)$$

As result, the compensator $K_z(z)$ reconstructed is also phase-lead, minimum-phase and stable so it is easily implementable to microprocessor. In the closed-loop sampled-data system with the discrete-time compensator $K_z(z)$, the desired damping ratio is also satisfied. The closed-loop eigenvalues in the z-domain are 8.9710×10^{-2} (ζ_{eq}=1.0) and $6.7035 \times 10^{-1} \pm j1.0668 \times 10^{-2}$ (ζ_{eq}=9.9921×10^{-1}), which shows the uniquely reconstructed digital controller stabilizes the sampled-data system Σ_{sz} with the requirement satisfied. Time response $\theta(t)$ to initial con-dition $\theta(0)$ =1 *degree* is depicted in Figure 5, where intersample response is shown. Also time response to gust wind ($w_d(t)$=e^{-2t} is depicted in Figure 6. In time response to the gust wind, intersample response is dominant in transient time. So we should be careful of evaluation of performance of sampled-data systems because sometimes intersample response is critical.

VIII. Conclusion

In this paper, we proposed a unified synthesis procedure of direct digital design for sampled-data control systems. This new synthesis methodology proposed shows that valuable experience and knowledge obtained from analog control system designs can be utilized directly to digital control system designs in classical design approach. As an illustrative example, we demonstrated an antenna azimuth control problem in the proposed method. By using the Generalized Root-Locus proposed as a tool in classical design approach in the *w*-domain, a phase-lead digital compensator was realized from a predetermined phase-lead compensator in the *w*-domain which resulted from experiences obtained in analog compensator design. Also the resulted digital compensator stabilized the sampled-data antenna azimuth control system, which proved the important statement that if any compensator determined in the *w*-domain stabilizes the system Σ_w, then the sampled-data system Σ_{sz} is also stabilized by the unique digital controller exactly reconstructed from the 'analog-type' compensator through the gain transformation. However, the gain trans-formation does not conserve compensator structure; the digital compensator recon-structed in discrete-time domain via the gain transformation may not have the 'required' structure for implementation even though the 'analog-type' compensator has the 'required' structure in the *w*-domain. To verify the azimuth control system, we analyzed time responses to an initial condition ($\theta(0)=1^\circ$) and to wind gust ($w_d(t)=e^{-2t}$). From the analysis, sometimes intersample response should be carefully considered in performance evaluation.

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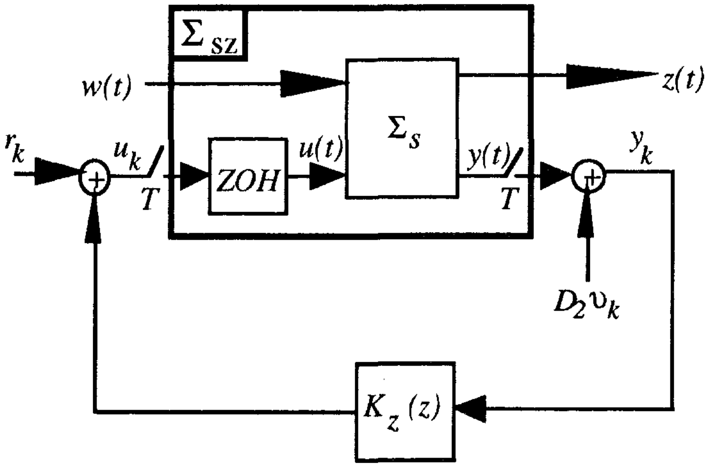

Figure 1: A typical sampled-data control system

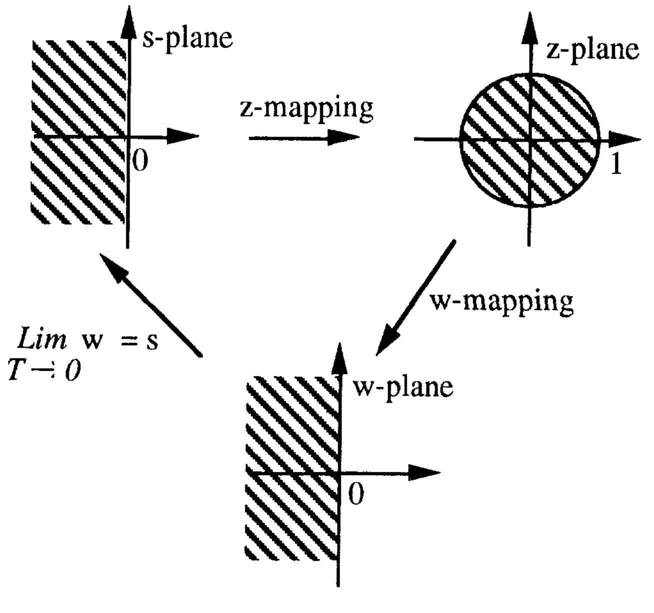

Figure 2: A characteristics of the w-domain

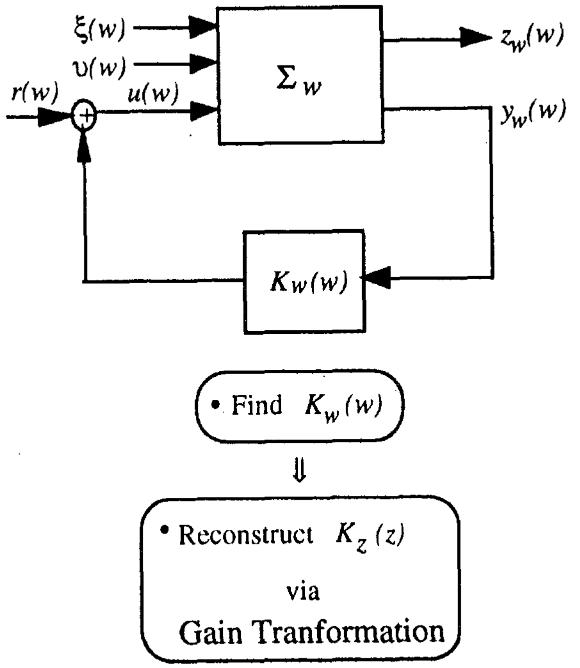

Figure 3: The systematic design procedure

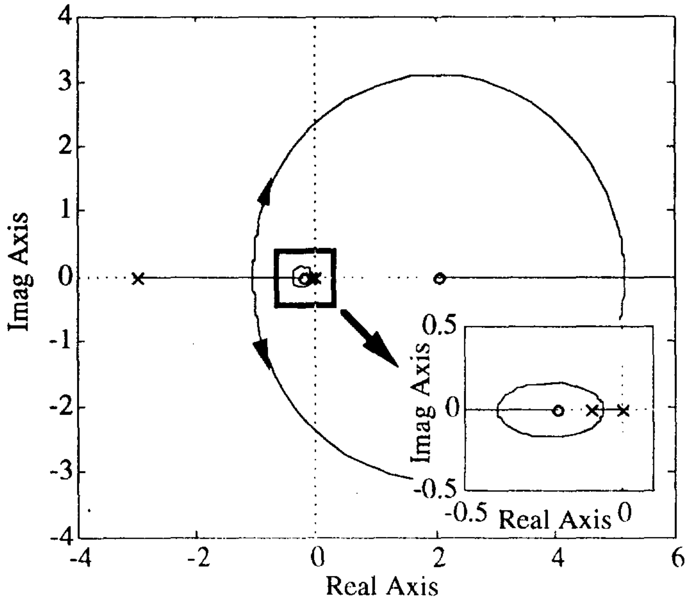

Figure 4: Root-locus in terms of K_p in the w-domain

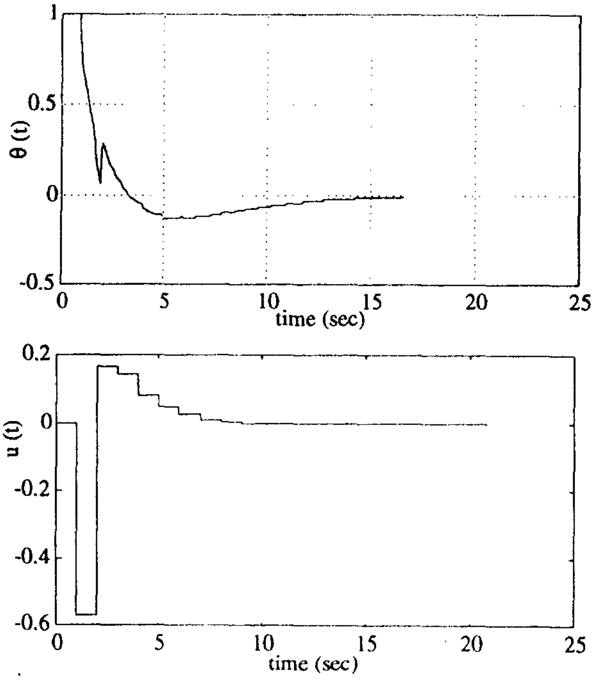

Figure 5: Time response to initial condition $\theta(0) = 1^o$

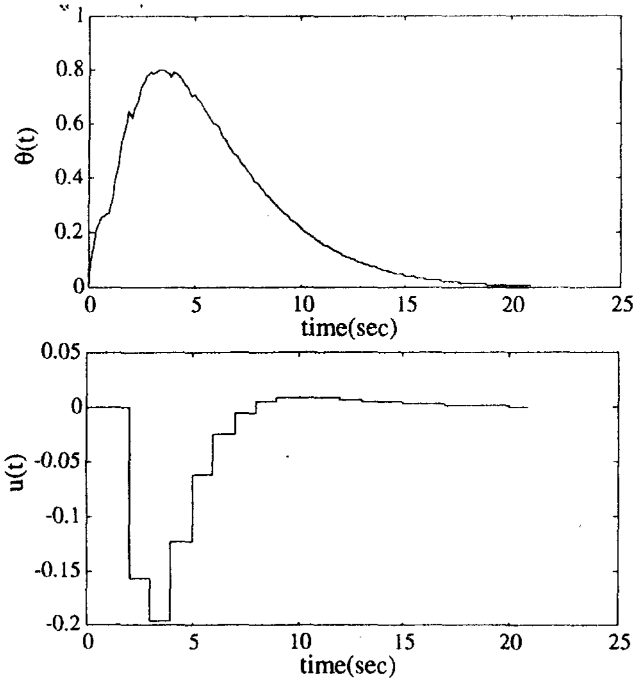

Figure 6: Time response to wind gust of e^{-2t}