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A Study on The Strength Estimation of Stiffened Cylindrical Members

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<Abstract>

In spite of their frequent uses as submarine pressure hulls and major structural members of various offshore structures, the complicated structural behaviours of cylindrical shells make their design not to depend on analytical or numerical method but on semi-empirical method. The basic concept of the latter is that the ultimate strength of the member can be thought as linear elastic buckling loads multiplied by reduction factors derived from the test data. So the most important thing in making the semi-empirical formulae is to derive the reduction factors efficiently from the limited number of test data which can not cover the whole range of the cylinder gemetries.

In this paper worldwidely used semi-empirical formulae, API code and BS5500, are investigated and finally measures are proposed to improve them.

보강 원통부재의 강도해석에 관한 연구

박치모
조선 및 해양공학과

<요 약>

보강원통쉘은 잠수함의 선각, 각종 해양구조물의 주요 부재등으로 활용도가 대단히 크나 구조적 거동이 아주 복잡해서 그 강도추정이 해석적인 방법이나 유한요소법등의 수치

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해법에 의해 합리적으로 이루어지지 못하고, 선형탄성 좌굴하중에 실험자료들로 부터 유도된 감소계수를 곱해서 최종강도를 추정하는, 반경험적 방법에 전적으로 의존하고 있다. 이와 같은 반경험식의 정확도는 제한된 실험자료로 부터 얻어지는 감소계수의 정확도에 따라 결정된다고 볼 수 있다.

본 논문에서는 이와 같은 원통부재의 강도해석에 세계적으로 널리 사용되는 API 설계규정과 BS5500 등의 반경험식들을 조사하고 유한요소해석법을 활용하여 반경험식을 개선하는 방안을 제시한다.

INTRODUCTION

Cylindrical shells having been used as submarine pressure hulls and major structural members of various offshore structures are not flat but curved in their shapes and are usually subject to various types of stiffening (unstiffened, ring-stiffened, stringer-stiffened, both ring- and stringer-stiffened) and load cases(axial load, bending load, external pressure, combined load) and also contains initial imperfections such as shape imperfections and residual stresses introduced during the fabrication processes.

Due to these complex factors to be considered, it is known to be almost impossible to predict the exact strength of these members by analytical methods alone.

And even the finite element method(FEM), which is very powerful in almost every other field of structural analysis, has not given satisfctory answer to this field for practical use.

In this context the ultimate strength estimation of these members usually depends on semi-empirical method which has its own shortcomings in many aspects.

The basic concept of the semi-empirical method is that the ultimate strength of the member can be obtained by multiplying the linear elastic buckling loads by reduction factors which account for the effects of initial imperfection, geometric and material nonlinearity, uncertainties in boundary condition, etc. which are not considered in the linear elastic buckling theory. In most codes or recommendations[1-3] reduction factors were obtained by comparing the linear elastic buckling loads with the lower bounds of test results. The choice of lower bounds was very deliberate, since generally too few test data were avail able on which to base a statistical interpretation for the formulae[7]. However, the gap between the real structural behaviours and the linear elastic analysis results seems too big to be properly filled only with the reduction factors directly derived from the limited number of test results, and it seems that any possible nonlinear finite element analyses could be stepping-stones across this gap.

Adopting these assumptions, in this paper, the semi-empirical method and the finite element method are applied to the models tested by Miller and Kinra[5], and these two kinds of evaluated results are compared with each other and also with the test results to derive useful informations in improving the semi-empirical method. As examples of widely used semi-empirical formulae, API code[1] and BS5500[2] are chosen.

FAILURE MODES

The collapse of cylindrical shell under various loads are caused by yielding, or elastic buckling or inelastic buckling depending on the sturdiness of the shell. While yielding is a very simple mode which keeps the axisymmetricity of the shell, buckling shows various modes as follows depending on the type of stiffening and the relative proportion of the shell plating to the stiffeners.

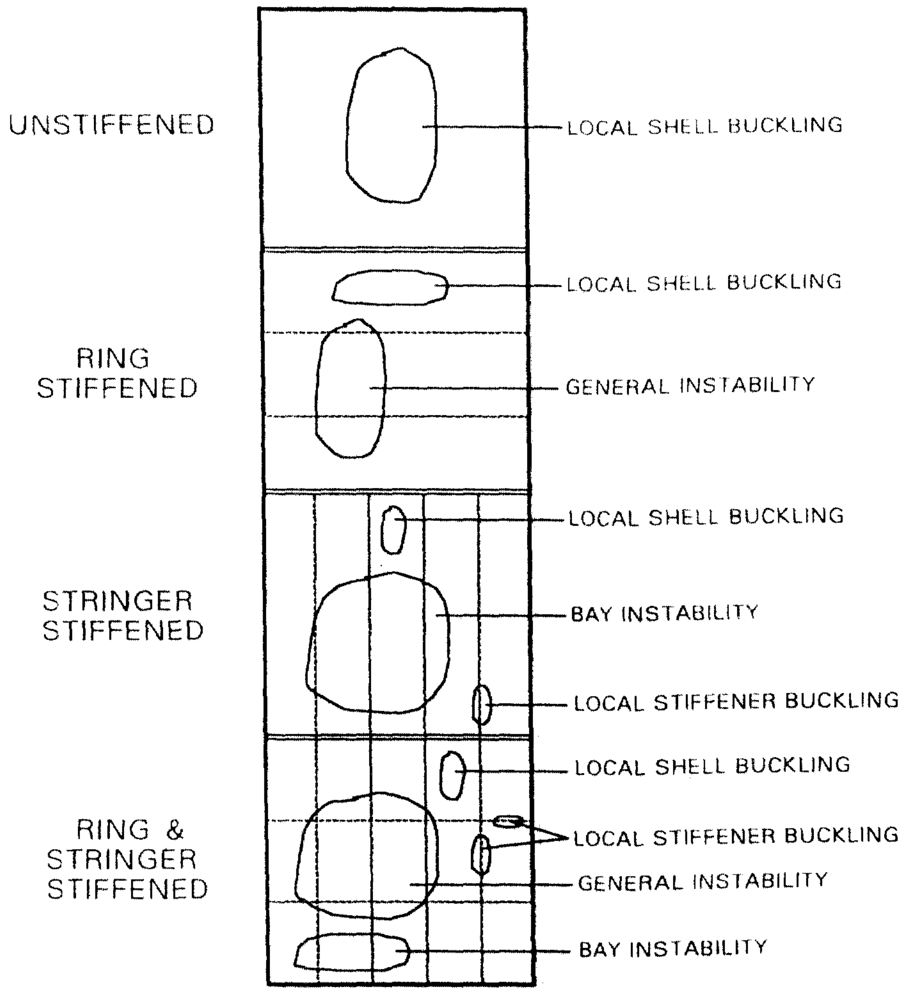

FIGURE 1. Shell buckling modes for cylinders[1]

a. Local shell buckling; buckling of the shell plate between stiffeners. The stringers remain straight and the rings remain round.
b. Bay Instability; buckling of the stringers together with the attached shell plate between rings(or the ends of the cylinders for stringer stiffened cylinders). The rings and the ends of the cylinders remain round.
c. General Instability; buckling of one or more rings together with the attached shell (shell plus stringers for ring- and stringer-stiffened cylinders).
d. Local Stiffener Buckling; buckling of the stiffener elements.

FIGURE 1 shows the buckling modes of cylinders together with several types of stiffening.

Most of rules recommend that the stiffeners be sized so that the first mode of failure will be local buckling of the shell between stiffeners. If the failure loads for the bay or general instability modes are equal to the local buckling load, there may be an interaction of buckling modes which will result in a lower buckling load than predicted. To avoid mode interation with local shell buckling it is recommended that the elastic failure loads for the bay and general instability modes of failure be equal to or greater than 1.2 times the elastic local shell buckling load.

APPLIED METHODS FOR STRENGTH ESTIMATION

Semi-Empirical Method

Most of rules or recommendations for the design of circular cylinders are based on this method. In this paper, API code[1] and BS5500[2] are selected as examples of semi-empirical method. Both cover the wide range of stiffening and load types, but here investigations are limited to the case of ring stiffened cylinders under hydrostatic pressure. Strength formulae of API code for the ring-stiffened cylindrical shell under hydrostatic pressure are briefly summarized in APPENDIX.

Finite Element Method

There are two ways of finite element analysis in the field of axi-symmetric shell problems, that is, one-dimensional axi-symmetric shell analysis and three-dimensional general shell analysis. The latter is of wide application but too time-consuming for parametric studies. Therefore, more concerns have been focused on the former. But it had had a limitation that it can not treat the elasto-plastic behaviour which is essential in the ultimate strength analysis. In the meantime, this limitation was overcome by Park[6] by introducing a Gauss quadrature integration instead of analytical integration in constructing stiffness matrix of one-dimensional axi-symmetric shell element. In this paper, this new concept is applied to calculate the ultimate strength of the chosen models.

RESULTS OF ANALYSES AND DISCUSSION

To investigate the validity and accuracy of the semi-empirical method, API code[1] and BS5500[2] formulae are applied to the 14 models tested by Miller & Kinra[5], all of which are ring-stiffened and subjected to hydro- static pressure and having shape im- perfection data well measured after fa- brication. And also the finite element analyses are incorporated for com- parisons. Table 1 shows the geometric and material properties of the test models, and Table 2 gives the com- parison of the ultimate strengths esti- mated by API code, BS5500 formulae and FEM.

Table 2 shows the followings.

1. Agreement between the two semi-empirical formulae, i.e. API code and BS5500, is very poor. Comparing the two formulae aforementioned, BS5500 strength formula is in better agreement with the test results but it is too early to say that BS5500 is better than API code just with this extremely limited comparison.

2. FEM is in best agreement with the test results among the three approaches.

One of the important reasons why the agreements of various approaches with the test data are poor is that circular cylindrical shells are so sensitive to initial imperfection that their collapse load values are apt to vary at every test. FIGURE 2 shows the shape im-perfection sensitivities of some models selected. In principle, experimental data, therefore, should be treated statistically, but it is almost impossible to perform enough experiments to meet the st-atistical treatment.

So this paper proposes that, prior to application to the strength formulae, test data be modified by the ratios of coll-apse load values of models correspond-ing to tolerance values of the codes for initial imperfection to the ones corres-ponding to actual initial imperfection.

Table 1. Geometric and material properties of test specimens[5]

Model No.	L (mm)	R (mm)	t (mm)	L_r/R	R/t	stiffener(mm) h_w *	t_w	E [GPa]	σ_y [MPa]	e_{act} (mm)	e_{al} (mm)
1	4877	197.2	12.57	4.1	15	-78.7	13.28	204	272	4.12	3.00
2	4877	196.2	13.08	4.1	14	-78.5	13.13	204	408	5.23	3.02
3	4877	196.9	13.13	6.2	14	-77.2	13.03	194	242	5.99	4.05
4	4877	197.0	13.13	8.3	15	-77.7	13.03	194	242	6.15	4.05
5	4877	197.3	11.02	8.2	17	-78.2	11.13	196	260	4.52	3.97
6	4877	197.7	9.75	3.1	20	-76.5	9.91	202	289	2.67	2.87
7	4877	198.8	9.60	8.2	20	-79.5	9.78	200	231	3.96	3.96
8	4877	198.2	8.28	8.2	23	-80.0	8.18	201	287	2.74	3.89
9	4877	199.4	6.60	2.0	30	-65.8	6.96	194	278	1.93	1.95
10	4877	198.9	6.83	2.2	29	-69.9	6.88	206	371	2.13	2.23
11	4877	198.7	6.83	3.1	29	-71.1	7.04	206	371	3.00	2.23
12	4877	199.8	6.45	8.1	30	-63.2	6.68	197	276	6.66	2.73
13	2438	603.1	9.63	1.0	62	101.6	9.52	190	259	3.05	4.14
14	4877	301.8	4.98	4.0	60	63.5	4.75	189	273	2.36	3.04
NOTE; Negative sign in h_w means external ring-stiffener											

Table 2. Comparison of collapse load values derived from various approaches[Test, API code, BS5500 and FEM]

Model No.	Imperf. e_{act}/e_{al}	M_x $[=L_r/(Rt)^{0.5}]$	P_{test} [MPa]	P_{test}/P_E	P_{test}/P_{pred} API	BS5500[4]	FEM
1	1.37	16.3	15.20	0.32	1.53	1.08	1.36
2	1.73	16.0	18.60	0.33	1.72	0.90	1.16
3	1.48	24.0	14.10	0.41	1.51	1.14	1.50
4	1.51	32.0	12.40	0.56	1.59	1.16	1.38
5	1.14	34.9	9.32	0.59	1.45	1.20	1.19
6	0.93	13.9	12.80	0.39	1.76	1.12	1.36
7	1.00	37.2	7.45	0.68	1.44	1.39	1.24
8	0.71	40.1	6.00	0.77	1.41	1.53	0.97
9	0.99	11.2	7.87	0.43	1.75	1.13	1.33
10	0.96	12.0	8.31	0.44	1.70	0.97	1.12
11	1.34	16.6	8.28	0.62	1.87	1.15	1.18
12	2.44	45.3	3.11	0.78	1.06	1.16	0.68
13	0.74	16.0	2.93	1.00	1.92	1.08	1.18
14	0.78	31.4	1.44	0.97	1.21	1.87	0.62
Mean					1.57	1.21	1.16
COV					0.15	0.20	0.21

These ratios can be approximated as the ratio between two collapse loads calculated by FEM, respectively for both initial imperfections. Table 3 shows the shape imperfection sensitivities and the modification of test data.

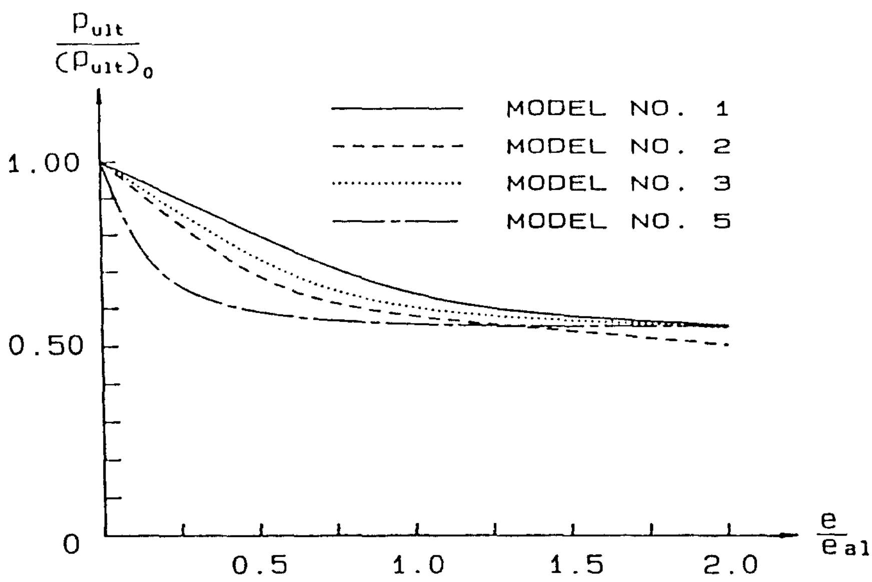

FIGURE 2. Ultimate strength varying with the magnitude of shape imperfection[6]

Table 3. Imperfection sensitivities within the tolerance limit and the modification of the actul test data

Model No.	$(P_{ult})_0$	$(P_{ult})_{al}$	$\frac{(P_{ult})_{al}}{(P_{ult})_0}$	$(P_{ult})_{act}$	Correction Ratio	Actual test data	Modified test data
1	19.00	12.20	0.64	11.20	1.09	15.20	16.57
2	31.00	18.00	0.58	16.00	1.13	18.60	21.02
3	16.80	10.00	0.59	9.40	1.06	14.10	14.95
4	16.80	9.40	0.55	9.00	1.04	12.40	12.90
5	14.20	7.80	0.54	7.80	1.00	9.32	9.32
6	14.90	9.20	0.62	9.40	0.98	12.80	12.54
7	10.60	6.00	0.56	6.00	1.00	7.45	7.45
8	8.50	6.20	0.72	6.20	1.00	6.00	6.00
9	10.20	5.80	0.57	5.90	0.98	7.87	7.71
10	14.00	7.40	0.52	7.40	1.00	8.31	8.31
11	13.10	7.00	0.53	7.00	1.00	8.28	8.28
12	5.00	4.50	0.90	4.60	0.98	3.11	3.05
13	4.48	2.46	0.54	2.48	0.99	2.93	2.90
14	2.20	2.34	1.06	2.32	1.01	1.44	1.45

CONCLUSIONS

In this paper the validity and accuracy of the semi-empirical method were investigated and the recently developed one-dimensional FEM[6] was applied to seek the improvement measures in the semi-empirical method and following conclusions were drawn.

1. The poor agreement between the predictions of API code and BS5500 says that there is much room for improvement in the present semi-empirical formulae.

2. Analysis results of the applied FEM show enough accuracy for the FEM to be used in constructing strength formulae together with test results[refer to Table 2].

In addition, it was proposed as a way of considering shape imperfection effects to modify the test data by the finite element analysis. This modification of the test data is deemed to make it possible to apply test data more reasonably and save the test results which should otherwise have been discarded because of large initial imperfection of the test model.

Finally, followings are recommended as future works.

- Further investigation of semiempirical formulae with more test models.
- Improvement of the present semi-empirical strength formulae with modified test data and finite element analysis data for the range for which no test data is available.

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NOTATION LIST

L length of the cylinder
L_r distance between stiffening rings
R shell radius
t shell thickness
h_w web height of ring-stiffener
t_w web thickness of ring-stiffener
E Young's modulus
σ_y yield stress of material
e amplitude of shape imperfection
e_{act} actually measured value of e
e_{al} allowed value of e by rules or codes
p_{test} test value of collapse pressure
P_E elastic buckling pressure
P_{pred} theoretically predicted collapse pressure
$(P_{ult})_0$ ultimate (or collapse) pressure when $e = 0$
$(P_{ult})_{al}$ ultimate (or collapse) pressure when $e = e_{al}$
$(P_{ult})_{act}$ ultimate (or collapse) pressure when $e = e_{act}$

APPENDIX:API code for the ring stiffened cylinders under hydrostatic pressure

The value of M_x appearing in the following equations is defined as;

$$M_x = \frac{L_r}{\sqrt{Rt}}$$

where
L_r= distance between stiffening rings
R= shell radius
t= shell thickness

1) Local Buckling of Unstiffened or Ring stiffened Cylinders

a. Theoretical elastic buckling pressure, P_{eL}.

$$P_{eL} = \begin{cases} \dfrac{1.27}{A^{1.18}+0.5} E(t/R)^2 & \text{if } M_x > 1.5 \text{ and } A < 2.5 \\ \dfrac{0.92}{A} E(t/R)^2 & \text{if } 2.5 < A < 0.208\ R/t \\ 0.836 C_p^{1.061} E(t/R)^3 & \text{if } 0.208 < C_p < 2.85 \\ 0.275 E(t/R)^3 & \text{if } C_p > 2.85 \end{cases}$$

where

$$A = M_x - 0.636$$

$$C_p = \frac{A}{R/t}$$

b. Failure Pressure, P_{cL}

$P_{cL} = \eta \alpha_L P_{eL}$

where

α_L: imperfection factor, 0.8 is recommended for fabricated cylinders which meet the fabrication tolerances

η: plasticity reduction factor given as

$$\begin{cases} 1.0 & \text{if } \Delta \le 0.55 \\ (0.45/\Delta)+0.18 & \text{if } 0.55 < \Delta \le 1.6 \\ 1.31/(1+1.15\Delta) & \text{if } 1.6 < \Delta < 6.25 \\ 1/\Delta & \text{if } \Delta \ge 6.25 \end{cases}$$

Where

$$\Delta = \frac{\sigma_{eL}}{\sigma_y}$$

where

$\sigma_{eL} = \dfrac{P_{eL} P_o}{t}$: elastic buckling stress

σ_y : yield stress of material

2) General Instability of Ring Stiffened Cylinders

a. Theoretical elastic buckling pressure, P_{eG}

$$P_{eG} = \frac{E(t/R)\lambda_G^4}{(n^2+0.5\lambda_G^4-1)(n^2+\lambda_G^2)^2} + \frac{EI_{er}(n^2-1)}{L_r R_c^2 R_o}$$

where $\lambda_G = \pi R/L$. R_c is the radius to the centroid of the effective section, R_o is the radius to the outside of the shell and I_{er} is the moment of inertia of the effective section given by the following equation;

$$I_{er} = I_r + A_r Z_r^2 \frac{L_e t}{A_r + L_e t} + \frac{L_e t^3}{12}$$

where Z_r is the distance from the centerline of the shell to the centroid of the stiffener ring (positive outward) and

$$L_e = \begin{cases} 1.1\sqrt{Dt}+t_w & \text{if } M_x > 1.56 \\ L_r & \text{if } M_x \le 1.56 \end{cases}$$

and n is the non-integer value which gives the minimum value of P_{eG}.

b. Failure Pressure

$P_{cG} = \eta \alpha_G P_{eG}$

Where imperfection factor α_G and plasticity reduction factor η are the same respectively as those values in local buckling.