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On Strongly Irresolute Mappings

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〈**Abstract**〉

In this note, a mapping $f: X \to Y$ is introduced to be strongly irresolute iff $f(\text{scl } A) \subset f(A)$ for all subsets A of X, where scl A means the semi-closure[1] of A in X. Some characterizations of these mappings are established and some of its basic algebraic properties are investigated. Further, some of its properties related to other known concepts, viz., s-connectedness, s-compactness, are discussed.

Strongly Irresolute 함수에 관하여

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〈요　약〉

Strongly irresolute 함수라는 새로운 함수를 도입하여, 성질 및 기본적인 대수적 성질을 연구한다. 더우기, 알려진 s-연결성, s-조밀성 등과의 관계를 갖는 성질들을 알아 본다.

I. Introduction

A necessary and sufficient condition for $f: X \to Y$ to be irresolute is that $f(\text{scl } A) \subset \text{scl } f(A)$ for all $A \subset X$, where scl A denotes the semi-closure of A in X [2]. In this note, it is attempted to investigate the strength of the more restrictive condition $f(\text{scl } A) \subset f(A)$ for all $A \subset X$, with which the mapping is termed strongly irresolute. Some characterizations of strongly irresolute mappings are obtained and its relation with some of the known concepts are discussed in section 1. It is shown that the class of strongly irresolute mappings contains all strongly continuous mappings defined in [7] and is contained in the class of all irresolute mappings defined in [2]. In Section 2, the algebra of strongly irresolute mappings is investigated. Section 3 is concerned with a study of these mappings associated with s-connectedness property. In section 4, strongly irresolute mapping in relation to s-compact mappings is studied.

Throughout this paper, a space means a topological space.

A set A of a space X is semi-open iff there exists an open set O in X such that $O \subset A \subset$ closure of O [9]. Any union of semi-open sets is semi-open [1]. Complement of a semi-open subset of a space X is a semi-closed set in X [1]. In a space X, intersection of all the semiclosed sets containing A is called the semi-closure of A and it is denoted by scl A

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[1] and union of all the semi-open sets contained in A is called the semi-interior of A and it is denoted by sint A [1]; also sint $A \subset A \subset$ scl A [1]. Since any union of semi-open sets is semiopen and any intersection of semi-closed sets is semi-closed, it follows that sint A and scl A are semi-open and semi-closed, respectively [1]; A is semi-closed iff $A=$ scl A and A is semi-open iff $A=$ sint A [1]. Union of two semi-closed sets need not be semi-closed [1]. A point $p \in X$ is a semi-limit point [3] of $A \subset X$ iff each semi-open set containing p, contains a point of A distinct from p. The set of all the semi-limit points of A is called the semi-derived set of A and is denoted by $D_s(A)$ [3]. $A \cup D_s(A) =$ scl A [3]. Also $A \subset B$ implies $D_s(A) \subset D_s(B)$, and $A \subset X$ is semi-closed iff $D_s(A) \subset A$ [3]. A semi-limit point p of a subset A of a space X is a limit point of A [3]. $f: X \to Y$ is irresolute [2] iff $f^{-1}(S)$ is semi-open in X for all semi-open S of Y, equivalently, iff f(scl $A) \subset$ scl $f(A)$ for every A of X. $f: X \to Y$ is pre-semi-open [2] iff $f(A)$ is semi-open in Y for all semi-open $A \subset X$. $f: X \to Y$ is a semi-homeomorphism iff f is bijective irresolute and pre-semi-open. If A is semi-open in a space X and B is semi-open in Y, then $A \times B$ is semi-open in $X \times Y$ [9]. A space X is semi-T_2[10] iff for each pair of distinct points x, y of X, there exists a semi-open set V containing y such that $x \notin$ scl V.

II. Definitions and Characterizations of Strongly Irresolute Mappings

Definition 1 : A mapping $f: X \to Y$ is termed strongly irresolute iff, for every subset A of X, f(scl $A) \subset f(A)$.

Obviously, every strongly irresolute mapping is irresolute, but not conversely. It is shown by the following example.

Example 1 : Let $X=\{a,b,c\}$ with the indiscrete topology. Then the identity mapping $i: X \to X$ is a irresolute mapping but not strongly irresolute.

It is quite evident that $f: X \to Y$ is strongly irresolute iff $f(D_s(A)) \subset f(A)$ for all $A \subset X$.

Theorem 1 : $f: X \to Y$ is strongly irresolute iff $f^{-1}(B)$ is semi-closed for all $B \subset Y$.

Proof: Only if: Let $p \in D_s(f^{-1}(B))$. Then $f(p) \in f(D_s(f^{-1}(B))) \subset f(f^{-1}(B))$ (since f is strongly irresolute) $\subset B$ and hence $p \in f^{-1}(B)$. Consequently $f^{-1}(B)$ is semi-closed for all $B \subset Y$.

If: Let $A \subset X$. Then $A \subset f^{-1}(f(A))$ which implies $D_s(A) \subset D_s(f^{-}(f(A))) \subset f^{-1}(f(A))$ since $f^{-1}(B)$ is semi-closed for all $B \subset Y$. Therefore, $f(D_s(A)) \subset f(f^{-1}(f(A))) \subset f(A)$ for all $A \subset X$. Consequently f is strongly irresolute.

Corollary 1 : $f: X \to Y$ is strongly irresolute iff $f^{-1}(B)$ is semi-open for all $B \subset Y$.

Corollary 2 : $f: X \to Y$ is strongly irresolute iff $f^{-1}(B)$ is both semi-open and semi-closed for all $B \subset Y$.

Let $f: X \to Y$ be a mapping. Then a set B of X is termed an inverse set of f if $f^{-1}(f(B))=B$.

Corollary 2-A : A surjective mapping $f: X \to Y$ is strongly irresoluse iff each inverse set is semi-open as well as semi-closed.

Now, a mapping $f: X \to Y$ is set-s-connected [4] iff $f^{-1}(V)$ is both semi-open and semi-closed for every semi-open and semi-closed V of $f(X)$. Hence, we have

Corollary 3 : Every surjective strongly irresolute mapping is set-s-connected.

The converse to Corollary 3 need not be true as is evident from the following example.

Example 2 : Let $X=\{a,b,c\}$ with topologies $\mathscr{T}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathscr{U}=\{\emptyset, X, \{a\}\}$. Then the identity mapping $i: (X, \mathscr{T}) \to (X, \mathscr{U})$ is set-s-connected but is not strongly irresolute.

Obviously, if $f: (X, \mathscr{T}) \to (Y, \mathscr{U})$ is strongly irresolute, then, for any other topology $\mathscr{U}'$ for Y, the mapping $f: (X, \mathscr{T}) \to (Y, \mathscr{U}')$ is also strongly irresolute. If follows, therefore, that the knowledge of the domain of a strongly irresolute mapping does not help in knowing the topology of its range.

Evidently, every strongly irresolute mapping $f:(X,\mathscr{T})\rightarrow(Y,\mathscr{U})$ is irresolute for each and every topology on Y.

Theorem 2 : A mapping $f:X\rightarrow Y$ is strongly irresolute iff $f:X\rightarrow Y$ is irresolute with a discrete topology on Y.

Proof : Obvious.

Continuity is independent of a strongly irresolute mapping. Also, a continuous mapping and a irresolute mapping are independent notions. However, we have that a continuous mapping $f:X\rightarrow Y$, where Y is a discrete space, is irresolute. Hence, we have

Corollary 4 : If $f:X\rightarrow Y$ is continuous with a discrete topology on Y, then $f:X\rightarrow Y$ is strongly irresolute.

A weakly continuous mapping [6] into a discrete space is continuous. Hence, we have

Corollary 5 : If $f:X\rightarrow Y$ is weakly continuous with a discrete topology on Y, then $f:X\rightarrow Y$ is strongly irresolute.

$f:X\rightarrow Y$ is weakly irresolute[6] iff, for each $x\in X$ and each semi-open set H containing $f(x)$, there exists a semi-open set G containing x such that $f(G)\subset \text{scl } H$. A weakly irresolute mapping into a discrete space is, obviously irresolute. Hence

Corollary 6 : If $f:X\rightarrow Y$ is weakly irresolute with a discrete topology on Y, then $f:X\rightarrow Y$ is strongly irresolute.

$f:X\rightarrow Y$ is strongly continuous[7] iff $f^{-1}(B)$ is open as well as closed for all $B\subset Y$.

Obviously, every strongly continuous mapping is strongly irresolute but the converse may not be true as is shown by the following example.

Example 3 : Let $X=\{a,b,c\}$ with topology $\mathscr{T}=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$ and $Y=\{p,q,r\}$ with any topology. Then the mapping $f:X\rightarrow Y$, defined by $f(a)=f(c)=p, f(b)=r$, is evidently strongly irresolute but is not strongly continuous.

However, a strongly irresolute mapping is strongly continuous if it is defined on a discrete space.

Thus, we have the following implications diagram:

Strongly continuous mapping $\Longrightarrow$ Strongly irresolute mapping $\Longrightarrow$ Irresolute mapping $\Longrightarrow$ Weakly Irresolute mapping

Theorem 3 : If X is a discrete space, then $f:X\rightarrow Y$ is strongly irresolute.

Proof : Since every subset of X is semi-open, it follows that the inverse image of every subset of Y is semi-open in X.

The converse to Theorem 3 does not hold, in general. It may be seen by the following example.

Example 4 : Let $X=\{a,b,c\}$ with topology $\mathscr{T}=\{\emptyset,X,\{a,b\},\{c\}\}$ and $Y=\{p,q,r\}$ with any topology. Then, the mapping $f:X\rightarrow Y$, defined by $f(a)=f(b)=q, f(c)=r$, is strongly irresolute but X is not a discrete space.

However, we have

Theorem 4 : An injective mapping $f:X\rightarrow Y$ is strongly irresolute iff X is a discrete space.

Proof : The if part follows from Theorem 3.

Only if : If f is injective strongly irresolute, then every point subset $\{x\}=f^{-1}(f(x))$ is semi-open in X. But every non-empty semi-open set has a nonvoid open set[1]. Hence every singleton $\{x\}$ is open in X. Consequently, X is a discrete space.

Example 1 proves also that even a semi-homeomorphism may fail to be strongly irresolute. However, we have

Corollary 7 : A semi-homeomorphism $f:X\rightarrow Y$ is strongly irresolute iff X and Y both are discrete spaces.

Proof : If : Obvious from Theorem 3.

Only if : By Theorem 4, X is a discrete space and so every subset of X is semi-open. Since f is a semi-homeomorphism, semi-open sets have semi-open images, and hence every subset of Y is semi-open. Consequently, Y is a discrete space.

Theorem 5 : $f: X \to Y$ is strongly irresolute iff $f^{-1}(y)$ is semi-open for each $y \in Y$.

Theorem 6 : If $f: X \to Y$ is strongly irresolute, then $f^{-1}(y)$ is semi-closed for each $y \subset Y$.

The converse to Theorem 6 may fail to be true as the following example shows.

Example 5 : Let $X = \{a, b, c, d\}$ with topology $\mathscr{T} = \{\emptyset, X, \{a, b, c\}, \{c\}, \{a, b\}\}$ and $Y = \{p, q, r\}$ with any topology. Let the mapping $f: X \to Y$ be defined by $f(a) = f(b) = p$, $f(c) = q$, $f(d) = r$ Then, obviously, $f^{-1}(y)$ is semiclosed for each $y \subset Y$ but f is not strongly irresolute.

Ⅲ. Algebra of Slrongly Irresolute Mappings

Theorem 7 : If $f: X \to Y$ is a strongly irresolute mapping and $g: Y \to Z$ is any mapping, then $g \circ f: X \to Z$ is strongly irresolute.

Corollary 8 : The composition of two strongly irresolute mappings is strongly irresolute.

Theorem 7 is not necessarily true for irresolute mappings as is shown by the following example.

Example 6 : Let $X = \{a, b, c\}$ with an indiscrete topology $\mathscr{T}$ and a discrete topology $\mathscr{D}$. Then, obviously, the identity mappings $f: (X, \mathscr{T}) \to (X, \mathscr{D})$ is irresolute and $g: (X, \mathscr{D}) \to (X, \mathscr{T})$ is any mapping but $g \circ f$ is not irresolute.

Lemma 1 : $f: X \to Y$ is weakly irresolute [6, Theorem 1], iff for each semi-open subset H of Y, $f^{-1}(H) \subset \text{sint } f^{-1}(\text{scl } H)$.

Proof : Let $x \in f^{-1}(H)$. Then $f(x) \in H$. Therefore, by definition of a weakly irresolute, there exists a semi-open set G containing x such that $f(G) \subset \text{scl } H$. This implies $x \in f^{-1}(\text{scl } H)$, i.e., $x \in \text{sint } f^{-1}(\text{scl } H)$. Conversely, let $x \in X$ and $f(x) \in H$(semi-open in Y). Then $x \in f^{-1}(H) \subset \text{sint } f^{-1}(\text{scl } H) = G$(say). Therefore, $f(G) = f(\text{sint } f^{-1}(\text{scl } H)) \subset f(f^{-1}(\text{scl } H)) \subset \text{scl } H$. Hence, f is weakly irresolute.

Theorem 8 : If $f: X \to Y$ is weakly irresolute and $g: Y \to Z$ is strongly irresolute, then $g \circ f: X \to Z$ is strongly irresolute.

Proof : Let A be any subset of Z. Then $g^{-1}(A)$ is a semi-open as well as semi-closed subset of Y. Since f is weakly irresolute, by the above Lemma 1, $f^{-1}(g^{-1}(A)) \subset \text{sint } f^{-1}(\text{scl } g^{-1}(A)) = \text{sint } f^{-1}(g^{-1}(A))$, i.e., $(g \circ f)^{-1}(A) \subset \text{sint } ((g \circ f)^{-1}(A))$. It follows, therefore, that $(g \circ f)^{-1}(A)$ is semi-open in X. Consequently $g \circ f$ is strongly irresolute.

Corollary 9 : If $f: X \to Y$ is irresolute and $g: Y \to Z$ is strongly irresolute, then $g \circ f: X \to Z$ is strongly irresolute.

Lemma 2 : [2] If A is semi-open and Y is open in a space X, then $A \cap Y$ is semi-open in Y.

Restriction of a strongly irresolute mapping to any subset of the domain need not be strongly irresolute. As, in Example 3, f is strongly irresolute, but $f|A: A \to Y$, where $A = \{b, c\} \subset X$, is not strongly irresolute. However, we have

Theorem 9 : If $f: X \to Y$ is a strongly irresolute mapping and A is an open set in X, the $f|A: A \to Y$ is strongly irresolute.

Proof : Let B be any set in Y. Then $(f|A)^{-1}(B) = f^{-1}(B) \cap A$. f being strongly irresolute, $f^{-1}(B)$ is semi-open in X. It follows, then, by Lemma 2, that $(f|A)^{-1}(B)$ is semi-open in A. Hence $f|A$ is strongly irresolute.

In Theorem 9, if A is semi-open in X, then $f|A$ is not always strongly irresolute, as shown by just before Theorem 9 where $A = \{b, c\}$ is semi-open in X.

If the restriction of a mapping to an open subset of the domain is strongly irresolute, then it is not necessary that the mapping is strongly irresolute. As in Example 5, $f|A: A \to Y$, where $A = \{a, b, c\}$ is open in X, is strongly irresolute but f is not strongly irresolute.

Theorem 10 : Let $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$ be strongly irresolute mappings and let $f: X \to Y$, where $X = X_1 \times X_2$ and $Y = Y_1 \times Y_2$, be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f is strongly irresolute.

Proof : Let $y \in Y$. Then $y=(y_1, y_2)$ where $y_1 \in Y_1$ and $y_2 \in Y_2$ and $f^{-1}(y)=f_1^{-1}(y_1)\times f_2^{-1}(y_2)$. Since f_1 and f_2 are strongly irresolute, $f_1^{-1}(y_1)$ and $f_2^{-1}(y_2)$ are semi-open in X_1 and X_2, respectively, for each $y_1 \in Y_1$ and $y_2 \in Y_2$. It follows then, due to Levine [9], that $f^{-1}(y)$ is semi-open in X. Hence f is strongly irresolute.

Theorem 11 : Let $f: X \to \prod_{\alpha \in \Lambda} X_\alpha$ be a strongly irresolute mapping and let $f_\alpha : X \to X_\alpha$ for each $\alpha \in \Lambda$ be defined as $f_\alpha(x)=x_\alpha$ where $f(x)=(\{x_\alpha\})$. Then f_α is strongly irresolute for each $\alpha \in \Lambda$.

Proof : Let p_α be the projection of $\prod_{\alpha \in \Lambda} X_\alpha$ onto X_α. Then, obviously, $f_\alpha = P_\alpha$ of for each $\alpha \in \Lambda$. Since f is strongly irresolute, each f_α is strongly irresolute in view of Theorem 7.

IV. s-Connectedness and Strongly Irresolute Mappings

Lemma 3 [4] : If A is semi-closed and Y is open in X, then $A \cap Y$ is semi-closed in Y.

In a space X, a set is s-connected iff it is s-connected as a subspace of X [11]. A space X is s-connected iff nonempty proper subset of X is both semi-open and semi-closed [11]. A space X is locally s-connected [11] iff, for each $x \in X$ and each open set 0 containing x, there exists an open s-connected set G such that $x \in G \subset 0$.

Theorem 12 : If $f: X \to Y$ is strongly irresolute, then, for every non-empty open s-connected subset A of X, $f(A)$ is a single point.

Proof : Suppose $f(A)$ contains more than one point. Let $p \in f(A)$. Then, f being strongly irresolute, $f^{-1}(p)$ is a nonempty semi-closed as well as semi-open subset of X. Therefore, in view of Lemma 2 and Lemma 3, $f^{-1}(p) \cap A$ is a nonempty proper semi-closed and semi-open subset of A. Thus A is not s-connected, a contradiction.

Corollary 10 : Image under a strongly irresolute mapping $f: X \to Y$ of a nonempty open s-connected set of X is s-connected.

Proof : Obvious, since every singleton is s-connected [11].

The converse to Theorem 12 does not hold, in general, as in Example 5, image under f of every nonempty open s-connected subset of X is a single point, but f is not strongly irresolute. However, we have

Theorem 13 : If the image under $f: X \to Y$, where X is locally s-connected, of every nonempty open s-connected subset of X is a single point, then f is strongly irresolute.

Proof : Let A be any subset of X. We show that $f(D_s(A)) \subset f(A)$. Let $x \in D_s(A)$. Then $x \in D(A)$ ($D(A)$ denotes the set of all limit points of A). Since X is locally s-connected, there exists an open s-connected set G containing x and so $G \cap A \neq \emptyset$. Now, obviously, $f(x) \in f(G)$ and since $f(G)$ is a single point, $f(x)=f(G)$. Also, $\emptyset \neq f(G \cap A) \subset f(G)=f(x)$. Hence $f(x)=f(G \cap A) \subset f(A)$ and thus $f(x) \in f(A)$. This proves the theorem.

Every s-connected space is connected [11]. A space X is totally disconnected iff the singletons are the only connected subsets of X.

The converse to Corollary 10 may fail to be true, as, in Example 5, image under f of every non-empty open s-connected subset of X is s-connected, but f is not strongly irresolute. However, we have

Theorem 14 : If the image under $f: X \to Y$, where X is locally s-connected and Y is totally disconnected, of every nonempty open s-connected subset A of X is s-connected, then f is strongly irresolute.

Proof : $f(A)$ is s-connected and hence connected for every A. Since Y is totally disconnected, $f(A)$ is a singleton for every A. Consequently, by Theorem 13, f is strongly irresolute.

Theorem 15 : A space X is s-connected iff every strongly irresolute mapping on X is constant.

Proof : Only if : It is obvious in view of Theorem 12.

If : Let X be not s-connected. Then there exists a nonempty proper subset A of X which is both semi-open and semi-closed. Let $Y=\{p, q\}$, where $p\neq q$, with any topology. Now define a mapping $f: X\to Y$ such that $f(A)=\{p\}$ and $f(X-A)=\{q\}$. Obviously, f is a strongly irresolute but not a constant mapping on X. This, being a contradiction, proves the theorem.

In a locally s-connected space X, the s-component [11] of $p\in X$ is the union of all open s-connecsed sets which contain the point p. Each s-component is open, s-connected and closed [11].

Theorem 16 : Let X be a locally s-connected space and let m be the cardinality of the family C, of all s-components of X. Let Y be any space. Then the following statements are equivalent:

(a) $f(X)=Y$ for some strongly irresolute mapping $f: X\to Y$.

(b) Cardinality of $Y\leqq m$.

Proof : Let Y be any space with cardinality $n\leqq m$. Let C_s' be the subfamily of C, of cardinality n. Then there is a one-to-one mapping g from C_s' to Y. Define a mapping $f: X\to Y$ such that $f(x)=g(D_x)$ when $x\in D_x\in C_s'$ and $f(x)=g(D_0)$ when $x\in D\in C$, but $D\notin C_s'$, D_0 being some fixed member of C_s'. Since each $D\in C$, is semi-open, $f^{-1}(y)$ is semi-open for each $y\in Y$. Hence f is strongly irresolute. Further, if $f: X\to Y$ is a strongly irresolute mapping of X onto y, then f can take at most m different values. Therefore, the cardinality of $Y\leqq m$.

V. s-Compact Mappings and Strongly Irresolute Mappings

A cover of X is termed semi-open iff the union of its members, being semiopen, is X. A space X is s-compact iff every semi-open cover of X has a finite subcover. A subset Y of a space X is s-compact iff Y is s-compact as a subspace of X.

Lemma 4[10] **:** Let X be a topological space and B a semi-open set(semi-closed) in X containing a subset A of X. Then, A is semi-open (semi-closed) in X iff A is semi-open(semi-closed) in the subspace B.

Lemma 5 : Every semi-closed and semi-open subspace Y of an s-compact space X is s-compact.

Proof : Let $\{S_\alpha : \alpha\in \Lambda\}$ be a cover of Y by semi-open subsets of Y. Then $Y-S_\alpha$ is semi-closed in Y for each $\alpha\in\Lambda$. By Lemma 4, $Y-S_\alpha$ is semi-closed in X. Hence $X-(Y-S_\alpha)=(X-Y)\cup S_\alpha$ is semi-open in X for each $\alpha\in\Lambda$. Therefore, $\{(X-Y)\cup S_\alpha: \alpha\in\Lambda\}$ is a semi-open cover of X. Since X is s-compact, there is a finite number of the S_α's, say, $S_{\alpha 1}, S_{\alpha 2},\dots, S_{\alpha n}$, such that $\{(X-Y)\cup S_{\alpha_i}: i=1,2,\dots,n\}$ is also a semi-open cover of X. Thus, obviously, $\{S_{\alpha_i}: i=1,2,\dots,n\}$ is a finite subcover of Y. Hence Y is s-compact.

Defintion 2 : A mapping $f: X\to Y$ is said to be s-compact iff the inverse image of every s-compact subset of Y is an s-compact subse of X.

Theorem 17 : Every strongly irresolute mapping on an s-compact space is s-compact.

Proof : Let A be any s-compact subset of Y. Then $f^{-1}(A)$ is a semi-closed as well as semi-open subset of X. Since X is s-compact, it follows by Lemma 5 that $f^{-1}(A)$ is s-compact. Hence the theorem is proved.

Theorem 18 : Image under a strongly irresolute mapping $f: X\to Y$ of every open s-compact subset A of X is a finite set.

Proof : Since f is strongly irresolute, $f^{-1}(y)$ is semi-open in X for each $y\in Y$. Thus $\{f^{-1}(y) : y\in Y\}$ is a semi-open cover of X and so a cover of A by semi-open subsets of X. Since A is open in X, by Lemma 2 $\{f^{-1}(y)\cap A : y\in Y\}$ is a semi-open cover of X and so a cover of A by semi-open subsets of X. Since A is open in X, by Lemma 2 $\{f^{-1}(y)\cap A : y\in Y\}$ is a cover of A by semi-open subsets of A itself. Since A is s-compact, there exits finitely many points $y_1, y_2, \dots, y_n$ in Y such that $A=\bigcup_{i=1}^{n}\{f^{-1}(y_i)\cap A\}$. Therefore, $f(A)=f(\bigcup_{i=1}^{n}\{f^{-1}(y_i)\cap A\})\subset\bigcup_{i=1}^{n} y_i$($a$ finite subset of Y). Hence $f(A)$ is a finite set.

Corollary 11 : Image under a strongly irresolute mapping of an open s-compact(compact).

Proof : It is obvious since every finite space is s-compact.

Evidently, image under a strongly irresolute mapping of an s-compact space is s-compact (compact).

Definition 3 : A space X is a s-s space iff the semi-open sets in X coincide with the s-compact sets in X.

Every singleton in a space is s-compact.

Theorem 19 : Let $f: X\rightarrow Y$ be a mapping with X a s-s space. Then f is strongly irresolute iff it is s-compact.

Proof : If f is strongly irresolute and A is any s-compact subset of Y, then $f^{-1}(A)$ is a semi-open subset of X. Since X is a s-s space, $f^{-1}(A)$ is s-compact and hence f is s-compact. Conversely, let $y\in Y$. Then $\{y\}$ is s-compact and hence semi-open. Consequently, f is strongly irresolute.

Lemma 6 [5] : A mapping $f: X\rightarrow Y$ has a strongly semi-closed graph $G(f)$ iff, for each $x\in X$, $y\in Y$ such that $f(x)\neq y$, there exist semi-open sets U in X and V in Y containing x and y, respectively, such that $f(U)\cap \text{scl } V=\emptyset$.

A strongly irresolute mapping fails to have a strongly semiclosed graph as is shown by the following example.

Example 6 : Let $X=\{a, b, c\}$ with topology $\mathcal{T}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $Y=\{p, q, r\}$ with topology $\mathcal{T}'=\{\emptyset, Y, \{p\}\}$. Then, obviously, the mapping $f: X\rightarrow Y$, defined by $f(a)=f(c)=p$, $f(b)=r$, is strongly irresolute but $G(f)$ is not strongly semi-closed.

However, we have

Theorem 20 : Let $f: X\rightarrow Y$ be a strongly irresolute mapping and Y be a semi-T_2 space. Then $G(f)$ is strongly semi-closed in $X\times Y$.

Proof : Let $x\in X$, $y\in Y$, such that $y\neq f(x)$. Since Y is semi-T_2, there is a semi-open set V containing y such that $f(x)\notin \text{scl } V$. Therefore, $f^{-1}(\text{scl } V)$ is a semi-open as well as a semi-closed set in X and $x\notin f^{-1}(\text{scl } V)$. Taking $U=X-f^{-1}(\text{scl } V)$, U is a semi-open set containing x, and then $f(U)\cap \text{scl } V=\emptyset$. Hence, by Lemma 6, $G(f)$ is strongly semi-closed.

In view of the following example, the converse to the above Theorem 20 does not hold, in general.

Example 7 : Let $X=\{a, b, c\}$ with topologies $\mathcal{T}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{T}'=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then, obviously, the graph of the identity mapping $i: (X, \mathcal{T})\rightarrow(X, \mathcal{T}')$ is strongly semiclosed but i is not strongly irresolute.

A space X is said to be almost compact (or quasi H-closed) iff every open cover of X has a finite subfamily whose closures cover X.

Theorem 21 : Every strongly continuous image of an almost compact space is s-compact.

Proof : Let $f: X\rightarrow Y$ be a strongly continuous mapping of an almost compact space X onto a space Y. If $\{U_\alpha : \alpha\in\Lambda\}$ be any semi-open cover of Y, then $\{f^{-1}(U_\alpha) : \alpha\in\Lambda\}$ is a cover of X by clopen sets of X. Since X is almost compact, there exists a finite subfamily $\{f^{-1}(U_{\alpha_i}) : 1\leq i\leq n\}$ of $\{f^{-1}(U_\alpha) : \alpha\in\Lambda\}$ which covers X. It follows then that $\{U_{\alpha_i} : 1\leq i\leq n\}$ is a finite subfamily of $\{U_\alpha : \alpha\in\Lambda\}$ which covers Y and hence Y is s-compact.

Lemma 7 [2] : If A is semi-open in U and U is open in X, then A is semi-open in X.

Theorem 22 : Let $f: X\rightarrow Y$ be a mapping and $\{A_\alpha : \alpha\in\Lambda\}$ an open (semi-open) cover of X. If $f|A_\alpha : A_\alpha\rightarrow Y$ is strongly irresolute.

Proof : Let V be any arbitrary set in Y. Thus for each $\alpha\in\Lambda$, $(f|A_\alpha)^{-1}(V)=f^{-1}(V)\cap A_\alpha$ is semi-open in A_α because $f|A_\alpha$ is strongly irresolute. Hence, by Lemma 7(Lemma 4), $f^{-1}(V)\cap A_\alpha$ is semi-open in X for each $\alpha\in\Lambda$. Therefore, $\bigcup_{\alpha\in\Lambda}\{f^{-1}(V)\cap A_\alpha\}=f^{-1}(V)$ is semi-open in X. This implies that f is strongly irresolute.

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