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Analysis of the Effect of Continuous Deviation of Symmetric Axis on the Extrusion Pressure

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〈**Abstract**〉

An analytical expression for the effect of deviation of die exit from symmetric axis in extrusion is found by the upper bound method for the straight and the curved die profile. The upper bound extrusion pressures are compared with various variables such as reduction of area, friction factor and deviation magnitude. The die lengths are obtained by minimizing the extrusion power with respect to chosen variables.

전방압출시 대칭축의 일정한 편심이 압출하중에 미치는 영향에 대한 해석

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(1981. 12. 30 접수)

〈**요　약**〉

전방압출시 금형이 대칭축으로부터 일정하게 편심되게 제작되는 경우에 압출하중은 축대칭 압출시보다 증가하게 된다. 이에 대한 해석을 상계해 이론을 적용하여 Curved Die와 Straight Die Profile에 대해 비교하여 보았다. 이경우 Straight Die Profile이 동일한 편심도에 있어서 좀더 적은 압출압력이 계산되었으며, 상계해 값을 최소로 하는 최적금형의 길이도 작게 계산되었다. 출구에서 편심되는 크기를 크게 할수록 마찰계수가 압출압력에 미치는 영향은 급격히 크게 되었으며 단면 감소율이 증가함에 따라 이 편심도의 영향은 점차 적어짐을 알 수 있었다.

Ⅰ. Introduction

In the number of studies on the axially symmetric extrusion, the upper bound method has been used extensively to calculate a maximum value of the ram pressure. In the case, the axis of symmetry is deviated continuously from its original position, it will become a three dimensional shape extrusion and the ram pressure will be changed in proportion to the magnitude of eccentricity. In this paper the effect of eccentricity is investigated with lubricated round billet through the curved and the straight boundary.

Ⅱ. Analysis

Considering the eccentric extrusion from a

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round billet with a unit radius as shown in Fig. 1, the function $R(z)$ is the die profile function in axisymmetric extrusion and the function $G(z)$ represents the deviation of axis of symmetry. The equation of the die profile is given by

$$\left.\begin{array}{l} y=R(z)+G(z) \\ R(0)=1,\ R(L)=R_f \\ G(0)=0,\ G(L)=G_f \end{array}\right\} \quad (1)$$

For the straight die

$$R_1(z)=\frac{R_f-1}{L}z+1 \quad (2\text{-a})$$

and for the curved die

$$R_2(z)=\left(\frac{R_f^2-1}{L}z+1\right)^{1/2} \quad (2\text{-b})$$

$$G(z)=\frac{G_f}{L}z \quad (3)$$

The billet is devided into three zones. The zone I consists of billet and the zone II is the deformation zone which is separated from the outgoing zone III.

Assuming entrance and exit boundaries of plastic flows in the transform velocity field are flat planes and are perpendicular to the axial direction, the velocity field in the zone II is found out by the condition of velocity continuity across the surface Γ_1 and Γ_2. And the velocity fields are described in a Cartesian coordinate system.

The components of velocity are

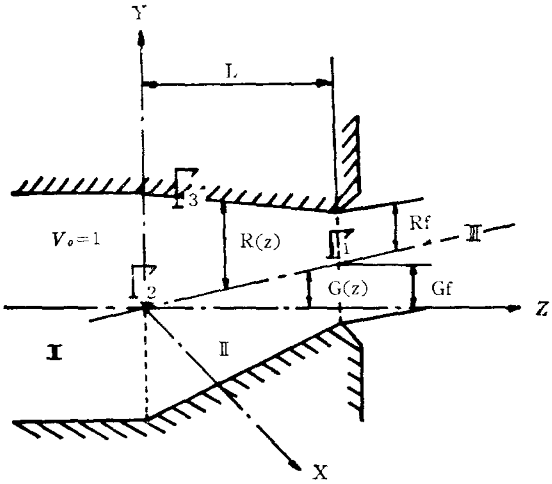

Fig. 1 Geometry of extrusion process

$$V_x=\frac{a_1V_0}{R_1^3(Z)}x \quad (4\text{-a})$$

$$V_y=\frac{V_0}{R_1^3(z)}\left\{a_1(y-G(z))+\frac{G_f}{L}R_1(z)\right\} \quad (4\text{-b})$$

$$V_z=\frac{V_0}{R_1^2(z)} \quad (4\text{-c})$$

where $a_1=\frac{R_f-1}{L}$

along the straight die profile

$$V_x=\frac{a_2V_0}{2R_2(z)}x \quad (5\text{-a})$$

$$V_y=\frac{V_0}{R_2(z)}\left\{\frac{a_2}{2}(y-G(z))+\frac{G_f}{L}(a_2z+1)\right\} \quad (5\text{-b})$$

$$V_z=\frac{V_0}{R_2^2(z)} \quad (5\text{-c})$$

where

$$a_2=\frac{R_f^2-1}{L}$$

along the curved die profile.

In the case of axi-symmetrical extrusion, when $G_f=0$ along the curved die profile, the velocity field is same as Chang and Choi's result.[1]

The velocity discontinuities across the surface Γ_1, Γ_2 and Γ_3 along the straight die are

$$\Delta V_1=V_0\left\{(a_1x)^2+\left(a_1y+\frac{G_f}{L}\right)^2\right\}^{1/2} \quad (6\text{-a})$$

$$\Delta V_2=\frac{V_0}{R_f^3}\left\{(a_1x)^2+\left(a_1y+\frac{G_f}{L}\right)^2\right\}^{1/2} \quad (6\text{-b})$$

$$\Delta V_3=\frac{V_0}{R_1^3(z)}\left\{R_1^2(z)(a_1^2+1)+2a_1\frac{G_f}{L}(y-G(z))\cdot R_1(z)+\left(\frac{G_f}{L}\right)^2R_1^2(z)\right\}^{1/2} \quad (6\text{-c})$$

And along the curved die are

$$\Delta V_1=V_0\left\{(a_2x)^2+\left(\frac{a_2}{2}y+\frac{G_f}{L}\right)^2\right\}^{1/2} \quad (7\text{-a})$$

$$\Delta V_2=\frac{V_0}{R_f^4}\left[\left(\frac{a_2}{2}x\right)^2+\left\{\frac{a_2}{2}(y-G_f)+\frac{G_fR_f^2}{L}\right\}^2\right]^{1/2} \quad (7\text{-b})$$

$$\Delta V_3=\frac{V_0}{R_2^3(z)}\left[\left(\frac{a_2}{2}\right)^2+\left(\left(\frac{G_f}{L}\right)^2+1\right)(a_2z+1)+a_2^2\frac{G_f}{L}\cdot\left(y-\frac{G_f}{L}z\right)\right]^{1/2} \quad (7\text{-c})$$

Deformation occurs only the in the zone II. From the velocity field given by equation (6) and (7), strain rates are derived as follows

$$\dot{\varepsilon}_{xx}=\frac{a_1V_0}{R_1^3(z)} \tag{8-a}$$

$$\dot{\varepsilon}_{yy}=\frac{a_1V_0}{R_1^3(z)} \tag{8-b}$$

$$\dot{\varepsilon}_{zz}=-\frac{2a_1V_0}{R_1^3(z)} \tag{8-c}$$

$$\dot{\varepsilon}_{xy}=0 \tag{8-d}$$

$$\dot{\varepsilon}_{xz}=-\frac{3}{2}\frac{a_1V_0}{R_1^4(z)}x \tag{8-e}$$

$$\dot{\varepsilon}_{yz}=\frac{V_0}{2R_1^4(z)}\left\{-a_1^2(y-G(z))-3a_1\frac{G_f}{L}\right\} \tag{8-f}$$

for the straight die

$$\dot{\varepsilon}_{xx}=\frac{a_2V_0}{2R_2^4(z)} \tag{9-a}$$

$$\dot{\varepsilon}_{yy}=\frac{a_2V_0}{2R_2^4(z)} \tag{9-b}$$

$$\dot{\varepsilon}_{zz}=-\frac{a_2V_0}{R_2^4(z)} \tag{9-c}$$

$$\dot{\varepsilon}_{xy}=0 \tag{9-d}$$

$$\dot{\varepsilon}_{xz}=-\frac{a_2^2V_0}{2R_2^6(z)}x \tag{9-e}$$

$$\dot{\varepsilon}_{yz}=\frac{V_0}{2R_2^4(z)}\left\{-a_2^2\frac{(y-G(z))}{R_2^2(z)}-\frac{3}{2}a_2\frac{G_f}{L}\right\} \tag{9-f}$$

for the curved die.

Each one satisfies the condition of incompressibility

$$\dot{\varepsilon}_{xx}+\dot{\varepsilon}_{yy}+\dot{\varepsilon}_{zz}=0 \tag{10}$$

Avitzur[2] formulated the upper bound theorem. The expression (11) is minium power for the actual velocity distribution.

$$J^*=\frac{2}{\sqrt{3}}\sigma_0\int_V\left(\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{1/2}dV+\int_{Sr}\tau|\Delta V_t|ds-\int_{St}T_tV_tdS_t \tag{11}$$

The last term means power due to predetermined body traction and, in this paper, this term is not taken into consideration. The internal power of deformation is computed over the zone Ⅱ.

$$\dot{E}_i=\frac{4\sigma_0V_0}{\sqrt{3}}\int_0^L\int_{-R(z)+G(z)}^{R(z)+G(z)}\int_0^{[R^2(z)-\{y-G(z)\}^2]^{1/2}} R^{-3}(z)\cdot\left[3a^2+\frac{9a^4x^2}{8R^2(z)}+\frac{1}{8R^2(z)}\cdot\left\{-3a(y-G(z))-3a\frac{G_f}{L}R(z)\right\}^2\right]dxdydz \tag{12}$$

$R_1(z)$ and a_1, $R_2(z)$ and a_2 are substituted into $R(z)$ and a for the straight die profile and the curved die profile respectively. Shear losses due to velocity discontinuity over the surface Γ_1 and Γ_2 are given by

$$E_{S_1}=\frac{2\sigma_0V_0}{\sqrt{3}}\int_{-1}^{1}\int_0^{\sqrt{1-y^2}}\left[(a_1x)^2+(a_1y+\frac{G_f}{L})^2\right]^{1/2}dxdy \tag{13-a}$$

$$\dot{E}_{S_2}=\frac{2\sigma_0V_0}{\sqrt{3}}\int_{G_f-R_f}^{G_f+R_f}\int_0^{\sqrt{R_f^2-(y-G_f)^2}}\frac{1}{R_f^3}\cdot\left\{(a_1x)^2+a_1y-R_f\frac{G_f}{L}-G_fa_1)^2\right\}^{1/2}dxdy \tag{13-b}$$

Power consumption over die profile for the straight die profile is given by

$$\dot{E}_{S_3}=\frac{2\sigma_0V_0m}{\sqrt{3}}\int_0^L\int_{-R_1(z)+G(z)}^{R_1(z)+G(z)}\left[\frac{R_1^2(z)+\left\{a_1R_1(z)-\frac{G_f}{L}(y-G(z))\right\}^2}{R_1^2(z)-(y-G(z))^2}\right]^{1/2}\cdot R_1^{-2}(z)\left\{1+a_1^2+\left(\frac{G_f}{L}\right)^2+\frac{2a_1G_f(y-G(z))}{R_1(z)\cdot L}\right\}^{1/2}dydz \tag{13-c}$$

For the curved die profile, shear losses and power consumption are given by

$$\dot{E}_{S_1}=\frac{2\sigma_0V_0}{\sqrt{3}}\int_{-1}^{1}\int_0^{\sqrt{1-y^2}}\left\{\left(\frac{a_2x}{2}\right)^2+\left(\frac{a_2y}{2}+\frac{G_f}{L}\right)^2\right\}^{1/2}dxdy \tag{14-a}$$

$$\dot{E}_{S_2}=\frac{2\sigma_0V_0}{\sqrt{3}}\int_{G_f-R_f}^{G_f+R_f}\int_0^{\sqrt{R_f^2-(y-G_f)^2}}\frac{1}{R_f^4}\left[\left(\frac{a_2x}{2}\right)^2+\left\{\frac{a_2}{2}(y-G_f)+\frac{R_f^2G_f}{L}\right\}^2\right]^{1/2}dxdy \tag{14-b}$$

$$\dot{E}_{S_3}=\frac{2m\sigma_0V_0}{\sqrt{3}}\int_0^L\int_{-R_2(z)+G(z)}^{R_2(z)+G(z)}\left[\frac{R_2^2(z)+\left\{\frac{G_f}{L}(y-G(z))\right\}^2}{R_2^2(z)-(y-G(z))^2}\right]^{1/2}\cdot R_2^{-4}(z)\left[R_2^2(z)\left\{\left(\frac{a_2}{2}\right)^2+\left(\frac{G_f^2}{L^2}+1\right)R_2^2(z)+\frac{a_2G_f}{L}\cdot(y-G(z))\right\}\right]^{1/2}dydz \tag{14-c}$$

Substituting equation (12), (13), (14) into (11) and the integration was done by the numerical method, and the value L is used to minimize the value of equation (11) in the calculation.

Ⅲ. Results and Discussion

As proposed in this study, the extrusion pressure was calculated as changing the value G_f. Fig.2 shows the variations of extrusion pressure with curved and straight die profiles. As expected, the extrusion pressure increases as the value G_f becomes large and the straight die profile requires little lower pressure than the curved die profile.

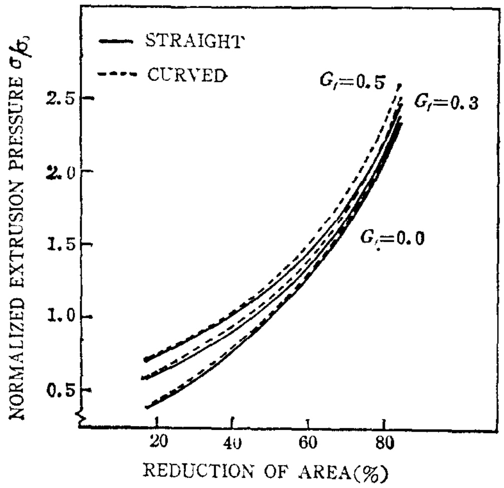

Fig.2 Effect of eccentricty on extrusion
(m=0.1)

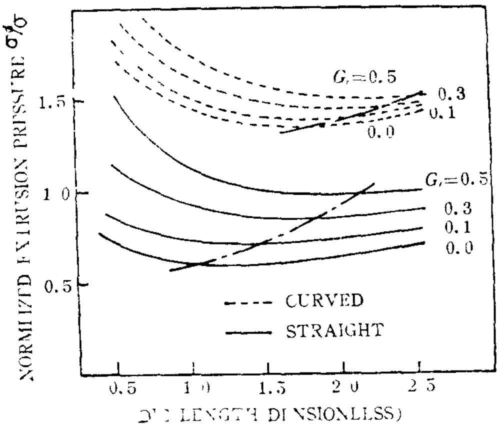

Fig.3 Effect of die length on extrusion pressure
(reduction of area; 64%)
(friction factor; m=0.1)

The optimal die length becomes considerably longer as increases the magnitude of deviation G_f and the straight die profile has little shorter die length as shown in Fig.3.

Fig.4 shows the extrusion pressure for the straight die with different friction factor. It can be seen here that, as the deviation magnitude G_f increases, the friction factor is more significant for extrusion pressure. The extrusion pressures with the friction factor m=0.1, 0.3, 0.5 increase more defintely as the deviation magnitude G_f increases.

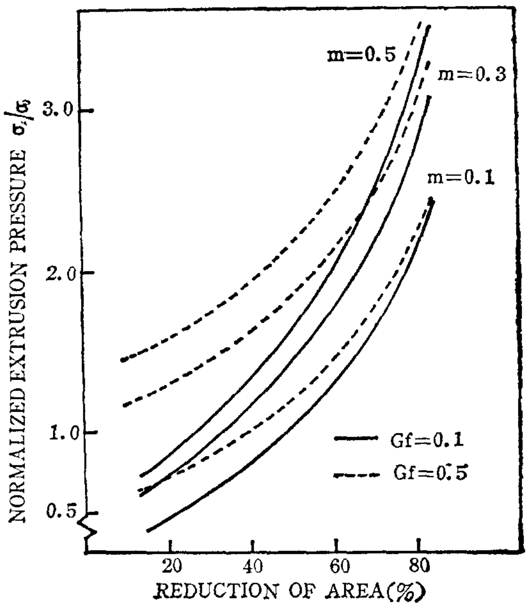

Fig.4 Effect of friction factor in the straight die profile

Ⅳ. Conclusions

1. The upper bound extrusion pressure, when the center of exit is deviated from its symmetric axis, are compared with various variables such as reduction of area, die length, friction factor and deviation magnitude G_f for the straight and the curved die profile.
2. The straight die profile requires less extru-

sion pressure and optimal die length than the curved die profile. And the optimal die length becomes longer as the deviation magnitude increases.

3. The extrusion pressure increases with increasing the friction factor and, in the extrusion which has larger deviation magnitude, the increase of extrusion pressure as the friction factor increases is more definite than that of small deviation magnitude.

Nomenclature

E_i=internal power of deformation

E_{sk}=power due to shear or friction along the surface of velocity discontinuity where $k=1,2,3$

G_f=magnitude of deviation at die exit (dimensionless)

$G(z)$=magnitude of deviation at generic section

J^*=applied power

L=die length (dimesionless)

m=friction factor

R_f=radius of round bar at die exit (dimensionless)

$R(z)$=radius of round bar at generic section

x, y, z=coordinates of a Cartesian coordinate system

$S\Gamma$=surface of integration where the velocity discontinuity is involved

V_x, V_y, V_z=components of velocity field

V_0=entrance velocity (=1)

$\Delta V_1, \Delta V_2, \Delta V_3$=velocity difference

$\Gamma_1, \Gamma_2, \Gamma_3$=surface of velocity discontinuities

σ_0=yield stress for the rigid-perfectly plastic material

$\dot{\varepsilon}_{ij}$=components of strain rate

τ=shear stress

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