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Lot Streaming in a Flow Shop with Batch Setup Times

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<Abstract>

This paper considers a multi-stage flow shop scheduling problem with lot streaming allowed where makespan is the performance measure. Lot streaming is the process of splitting a job into sublots so as to make the job process accelerated. Each sublot of the job is initiated with an individual setup at each stage where the batch setup time is independent of sublot size and non-separable from processing times. This study characterizes the optimal solution single job scheduling problem in a two-stage flow shop problem. Also, a heuristic procedure is introduced in a multi-stage flow shop problem.

뱃치별 준비시간을 갖는 흐름생산시스템에서의 롯 분할에 관한 연구

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<요 약>

본 논문에서는 뱃치별 준비시간을 갖는 흐름생산시스템에서 롯 분할에 관한 문제를 다루고 있다. 뱃치별 준비시간은 공정소요시간과 분리될 수 없는 경우를 다루었다. 롯 분할에 따른 개별장비의 유휴시간의 감소로 생산 가속화의 효과가 발생한다. 반면, 배치별 준비시간의 추가 소요로 인한 개별 장비에서의 소요시간이 늘어나는 부담이 있게된다. 이러

한 상황에서 총생산소요시간을 최소화하는 롯의 갯수와 롯의 크기를 결정하는 문제를 다룬다. 본 연구에서는 두 단계 흐름생산시스템에 대한 문제분석과 아울러 일반적인 문제에 대한 발견적인 해법을 제안한다.

1. Introduction

This paper consider a multi-stage flow shop scheduling problem with lot streaming allowed where makespan is the performance measure. Lot streaming is the process of splitting a job into sublots so that the sublots are processed sequentially but treated as individual jobs so as to make the job process accelerated. Each sublot of the job is initiated with an individual setup at each stage where the batch setup time is independent of sublot size and non-separable from processing times. It is assumed in the problem that the whole job is composed of a large number of items so as to be treated as to be infinitely devisable. The analysis of the infinitely divisible case is expected to provide some of the general insights of the solution properties for many application variations of the problem.

In many practical situations, job processing times are greatly dependent on the way of job batching or job splitting. For example, consider a manufacturing facility with a queue of several items that are waiting for processing. The waiting jobs may need be grouped into appropriately sized batches due to a work configuration at the facility. On the other hand, a lot of items may be required to split into smaller batches by a work flow management policy. Such batching decisions can incur a significant influence on time related performance measures, such as flow times, makespan, and due date performance.

In recent years, lot streaming has received greater attention with the growing practical concern about manufacturing lead times. Nevertheless, there have been few formal studies of lot streaming in the research literature. Szendrovits[7] has analyzed a makespan problem in a flow shop where one job composed of equally-sized sublots, and no machine idle times were permitted once processing began. Potts and Baker[5] have considered the makespan measure for a flow shop schedule where lot streaming was allowed. They have shown that it is optimal for one-job model to use the same sublot size all machines, and proposed a heuristic solution procedure. Baker and Pyke[1] have presented an algorithm for solving a two-sublot problem with respect to the makespan measure, and examined several heuristic approaches to a problem with more than two sublots involved. Kropp and Smunt[3] have considered the lot splitting policies in a multi-process flow shop environment with the objective of minimizing either mean flow time or makespan.

It is assumed in our study that items in batch are available individually for processing at a machine only after the completion of the whole batch production run

on its preceding machine. This situation is referred to Santos and Magazine[6] as the case of *batch availability* (rather than *item availability*) because no items in sublot are available until the entire sublot is completed.

The objective of this paper is to find a schedule which minimizes the makespan of all the sublots with respect to the batch availability. This objective can contribute to minimize work-in-process inventory, which is significant in a work flow management where demand and due dates can be manipulated, and can also contribute to item delivery lead time shortening so as to reduce the level of safety stocks required by downstream customers.

2. Problem Description

This paper considers the lot streaming scheduling for one job in a multi-stage flow shop with the objective of minimizing makespan under measuring scheme of batch availability. The makespan of a lot is the period from starting the first operation on the first item of the lot until the whole lot is processed in the production system.

For a single job with lot streaming model, let t_i denote the processing time of the job at machine i (i = 1, 2, ... , m). And let s_i denote the batch setup time for a sublot at machine i, which is independent of the sublot sizes and non-separable from the processing times. To accelerate the progress of the job, its work can be split into sublots, where x_j (j = 1, 2, ... , n) represents the proportions the work assigned to the jth sublot and $n \geq 2$. Moreover, these proportions are assumed to be identical for all machines.

The basic lot streaming model involves a single job and sequence of machines at which the single-job operations are performed. Figure 1 depicts the model with two machines having the processing times of 10 and 8, and having the batch preparation times of 2 and 3, respectively. If the job is produced without its lot splitting and so x_1=1, its makespan will be 23 time units. However, if the lot is split into two equal sublots such as x_1=1/2 and x_2=1/2, the makespan of the lot is reduced to 21 time units.

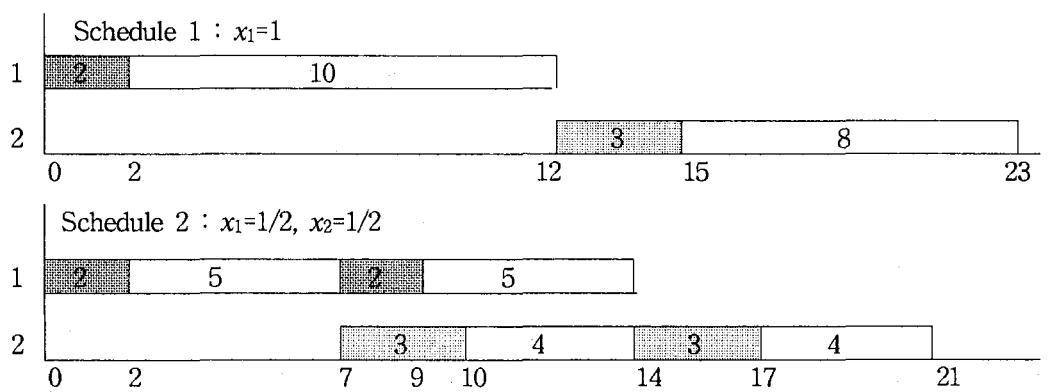

Figure 1. Two schedules for a two-machine flow shop.

This paper considers the identically proportional sublot size x_j as a continuous variable, despite of dealing with discrete items, so that the total item quantity is treated as being homogeneous and divisible in any proportion with the (given) number of sublots n. This is because similar analysis results can be derived even if x_j's are restricted to integer. The solution in the integer case may look quite different, especially in items of the number of sublots produced.

In the aforementioned references, n was implicitly assumed given. In practice, however, the size of n is dependent on the work process control system for tracing sublots in the shop. It may also be constrained by the number of item carries on the shop floor, the design of processing equipment, the packaging requirements of vendors or the need to trace individual sublots for subsequent field service.

A network representation of the problem is given in Figure 2. The figure shows an activity-on-node diagram with node (i, j) representing the processing of sublot j on machine i, which takes time $s_i + t_i x_j$. The makespan corresponding to the longest path in the network, but in contrast to the usual critical path model, this one has variable-length activity times because sublot sizes are decision variables. Thus, the problem of minimizing the makespan involves allocating work to sublots to minimize the length of the critical path in the network.

$$
\begin{array}{ccccccc}
(1, 1) & \rightarrow & (1, 2) & \rightarrow & \cdots & \rightarrow & (1, n) \\
\downarrow & & \downarrow & & & & \downarrow \\
(2, 1) & \rightarrow & (2, 2) & \rightarrow & \cdots & \rightarrow & (2, n) \\
\downarrow & & \downarrow & & & & \downarrow \\
\vdots & & \vdots & & & & \vdots \\
\downarrow & & \downarrow & & & & \downarrow \\
(m, 1) & \rightarrow & (m, 2) & \rightarrow & \cdots & \rightarrow & (m, n)
\end{array}
$$

Figure 2. Network representation of a lot streaming problem.

The timing of sublot j on machine i is constructed by two events: the completion of sublot j on the previous machine (machine i-1) and the completion of the previous sublot (sublot j-1) on machine i. The later of the these two times determines when sublot j starts and when it is completed on machine i. Let $C_{(i, j)}$ denote the completion time of sublot j on machine i.

Then completion times can be determined as small as possible, subject to the following constraints ;

a) machine capacity constraints, $C_{(i,\ j)} \geq C_{(i,\ j-1)} + (s_i + t_i\ x_j)$

b) production constraints, $C_{(i,\ j)} \geq C_{(i-1,\ j)} + (s_i + t_i\ x_j)$

c) the initialization constraints, $C_{(1,\ 1)} \geq s_1 + t_1\ x_1$

That is,

$$C_{(i,\ j)} = max\ \{\ C_{(i,\ j-1)},\ C_{(i-1,\ j)}\ \} + (s_i + t_i\ x_j),\quad 1 \leq i \leq m;\ 1 \leq j \leq n$$

where $C_{(i,\ 0)} = 0$ and $C_{(0,\ j)} = 0$. The objective is to schedule the sublots in such way that the entire job is completed as early as possible. Thus, the objective is to minimize $C_{(m,\ n)}$, the makespan of the schedule.

3. A Two-Machine Problem

This section wants to characterize the optimal solution of single job scheduling problem in a two-machine flow shop where the batch setup times are additionally incorporated for each sublot and the makespan is to be minimized under the measuring scheme of batch availability.

In the case m=2, the makespan of the schedule can be viewed as the solution to the critical path problem in the network representation. To develop an expression for the network's longest path, the following notation is introduced as :

$$X_{(j,\ k)} = x_j + x_{j+1} + \ldots + x_{j+k},$$

where $X_{(j,\ k)} = 0$ for $j > k$. In other words, $X_{(j,\ k)}$ represents the sum of cumulative proportional sizes from sublot j to sublot k.

Let M denote the makespan of the schedule. Then M is the longest path in the network. For any sublot j ($1 \leq j \leq n$), the makespan must be at least as large as the sum of

(a) the processing time of sublots 1 through j on machine 1 and
(b) the processing time of sublot j through n on machine 2.

It can be expressed mathematically as

$$M \geq j\ s_1 + t_1\ X_{(1,\ j)} + (n-j+1)\ s_2 + t_2\ X_{(j,\ n)}\ ,\quad 1 \leq j \leq n$$

Therefore, the makespan can be determined as

$$M = max_j \ \{ j\, s_1 + t_1 X_{(1,\, j)} + (n-j+1)\, s_2 + t_2 X_{(j,\, n)} \}$$

Let h denote an index j with which the maximum is attained. Then sublot h is called critical sublot in this case.

Lemma 1.
If $(n-1) \geq \min \{ t_2/s_1,\ t_1/s_2 \}$, then the makespan of the case with n sublots is larger than that of the no-splitting case.

Proof. Let M_1 denote the makespan of no-splitting case. Then

$$M_1 = s_1 + t_1 + s_2 + t_2$$

For the case with n sublots, the makespan M_n must satisfy the following two relations:

$$M_n \geq n\, s_1 + t_1 + s_2 + t_2\, x_n > s_1 + t_1 + s_2 + (n-1)\, s_1 \geq M_1$$

and

$$M_n \geq s_1 + t_1\, x_1 + n\, s_2 + t_2 > s_1 + t_1 + s_2 + (n-1)\, s_2 \geq M_1$$

Therefore, $M_n > M_1$ under the above conditions.
Thus, the proof is completed.

This implies that the upper bound of n can be calculated when the number of sublot is also a decision variable for the makespan problem where the batch setup times are considered.

Lemma 2.
In the optimal solution for the case $m = 2$, all sublots are critical.

The proof can be easily done by a similar way to that in Potts and Baker[7]. As the results of Lemma 2, the following relation is obtained as

$$s_1 + t_1\, x_{j+1} = s_2 + t_2\, x_j \quad \text{for } j = 2, 3, \ldots, n$$

And the additional conditions, $\sum_{j=1}^{n} x_j = 1$, are introduced. We can determine the optimal sublot sizes.

Note that the optimal makespan is a convex function according to the number of

sublots, which is bounded below in Lemma 2. Thus, we can also determine the optimal number of sublots and each sublot size.

Numerical Example 1.

Consider a two-stage flow shop problem where processing times have 5 and 10 time units, respectively. And setup times have 2 and 1 time units. For the results of Lemma 1, the upper bound of n can be calculated as

$$(n - 1) < min \{ t_2/s_1, t_1/s_2 \} = 5.$$

Then, the results of the example are given by Table 1.

Table 1. The results of example 1.

sublots (n)	size of sublots (x_j)	makespan	otimal schedule
1	(1)	18	
2	(0.4, 0.6)	16	*
3	(0.26, 0.31, 0.43)	16.27	
4	(0.21, 0.23, 0.25, 0.31)	17.07	
5	(0.2, 0.2, 0.2, 0.2, 0.2)	18	

4. Heuristic Procedures

This section considers an m-stage n-sublot problem with lot streaming allowed. The upper bound of n can be calculated when the number of sublot is also a decision variale foe the makespan problem where the batch setup times are considered. In a simlar way to that of the two-machine case the following property is obtained.

Lemma 3.
If $(n - 1) \geq min_i \{ (T_{(1, m)} - t_i) / s_i \}$, then the makespan of the case of n sublots is larger than that of the no-splitting case.

Proof. Let M_1 denote the makespan of the no-splitting case. Then

$$M_1 = S_{(1, m)} + T_{(1, m)}$$

For the n sublots case, the makespan M_n must be satisfy the following relations for all i (i = 1, 2, ... , m) ;

$$M_n \geq S_{(1,\ i-1)} + T_{(1,\ i-1)}\, x_1 + n\, s_i + t_i + S_{(i+1,\ m)} + S_{(i+1,\ m)}\, x_n$$
$$> S_{(1,\ m)} + t_i + (n-1)\, s_i \geq M_1$$

Therefore, the relation $M_n > M_1$ holds under the above conditions.
Thus, the proof is completed.

In an approach of solving a linear programming, there is no known method of finding optimal solution to the m-machine n-sublot version of the problem. rather, in an attempt to devise effective heuristic procedures for the problem, it makes sense to build on the concepts of solving the two-machine problems. This approach reflects the lessons of the traditional flow shop literature. In particular, the makespan problem can be solved efficiently for the case with two machines by using the rsult of Johnson [2], but no efficient optimization procedure exists for cases with m machines.

Equal-Sublot Heuristic

In order to get some perspective on how well a heuristic procedure might be expected to perform, a very simple procedure is given in which the work is allocated equally among the n sublots.

Two-Machine Heuristic

Step 0. Calculate the uppper bound of n.
Step 1. For each machine i, calculate the total processing time ($n\ s_i + t_i$).
Step 2. Determine two machines; one has the largest processing time and the other has the second largest.
Step 3. For selected two machines, solve the two-machine problem and calculate the makespan.

Numerical Example 2.

Consider a three-stage flow shop problem described as follows ;

machine i	1	2	3
setup time s_i	1	3	2
processing time t_i	5	6	7

As the results of Lemma 3, the upper bound of n can be calculated as

$$(n - 1) < min_{i} \{ (T_{(1, m)} - t_i) / s_i \} = 4.$$

Then, according to the heuristic algorithms for the problem, the results of the example are given by Table 2.

Table 2. The results of example 2.

n	Equal Heurstic		Two-Machine Heuristic	
	x_j	makesapn	x_j	makesapn
1	1	24	1	24
2	(1/2, 1/2)	21	(0.54, 0.34)	20.92
3	(1/3, 1/3, 1/3)	22	(0.43, 0.34, 0.23)	21.76
4	(1/4, 1/4, 1/4, 1/4)	24	(0.41, 0.32, 0.20, 0.07)	23.54

5. Conclusion

This paper has presented a solution algorithm for a multi-stage lot-streaming problem with batch setup times required. The solution algorithm provides an easy method for computing the optimal sublots for the two-machine case, and near optimal sublots for general case. The calculations required to determin the allocation of the lot to the sublots can be done readily by hand in simple steps.

The solution algorithm is exploited by use of the concept of a critical sublot based on a critical path analogy in network theory. It is demonstrated that an optimal schedule for the two-machine problem must have all critical sublots. Also, a heuristic procedure is introduced in a multi-stage flow shop problem, it makes sence to build on the concepts of solving the two-machine problems.

Further research on the lot streaming problem will consider more than on e case, along with performance measures other than makespan. Another interesting subject will be a dynamic version of the problem.

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