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Excitation energy of a ^{3}He quasiparticle in the bulk mixture at constant pressure

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〈**Abstract**〉

A ^{3}He quasiparticle excitation energy in bulk mixture at zero pressure and 6% solution is calculated to $O(x)$ using the bulk effective interaction of Yim and Massey. The present ^{3}He quasiparticle excitation energy is in agreement with the experimental result of Hilton, Scherm and Stirling.

일정한 압력하에 체 혼합물에서 한 헤리윰-3 유사 입자의 여기 에너지

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〈요 약〉

0기압과 6퍼센트 용액하에 체 혼합물에서 한 헤리윰-3 유사입자 여기 에너지가 임과 매시의 체유효 작용을 사용하여 $O(x)$까지 계산되어 진다. 계산된 헤리윰-3 유사입자 여기 에너지는 힐튼, 셤과 스털링의 실험 결과와 부합된다.

^{3}He quasiparticle excitation energies in the bulk mixture at pressure, P, and concentration, x, have been interest over the past decade.[1,2] We also gave some qualitative explanations[3] for these excitations. Recently, Hilton et al.[1] measured the bulk quasiparticle excitation energy at $P=0$ and $x=0.06$ by the neutron experiment. These results for lower concentrations may also be used in determining the k-dependences of the effective interaction between two ^{3}He quasiparticles in the bulk mixture, $V(k)$, where $\vec{k}$ is the wave vector. (Refer to Appendix A for the electronic binding energy.) In this paper we present the quasiparticle excitation energy in the bulk mixture, $e(k)$, at $P=0$ and $x=0.06$ using the bulk effective interaction of Yim and Massey[3] (YM) and the parabolic excitation for a single ^{3}He quasiparticle at $x=0$, $e_0(k)$, i.e.,

$$e_0(k)=\frac{\hbar^2k^2}{2m}, \tag{1}$$

where the chemical potential of a ^{3}He atom at $x=0$ is neglected, m is the effective mass of a ^{3}He quasiparticle at $x=0$ and $m=2.34m_3$ at $P=0$ where m_3 is the bare mass of a ^{3}He atom. As the concentration increases, the ^{3}He quasiparticle excitation energy in the bulk mixture,[3] $e(k)$, above the ground state can be expressed as

$$e(k)=e_0(k)-\frac{1}{\Omega}\sum_{k'}V(|\vec{k}-\vec{k}'|)n_{k'}-(e_0(k_f)-\frac{1}{\Omega}\sum_{k'}V(|\vec{k}_f-\vec{k}'|)n_{k'}), \tag{2}$$

and from the above $e(k)$ to $O(x)$ is

$$e(k)=e_0(k)-\frac{x}{2}n_{40}V(k)-(e_0(k_f)-\frac{x}{2}n_{40}V(k_f)), \tag{3}$$

where k_f, Ω, n_k and n_{40} are the Fermi wave vector, the volume of the system, occupation number of ^{3}He atoms at the momentum, $\hbar\vec{k}$, and the pure ^{4}He density at the same pressure as in the bulk mixture. In Figure shown is $e(k)$ at $P=$

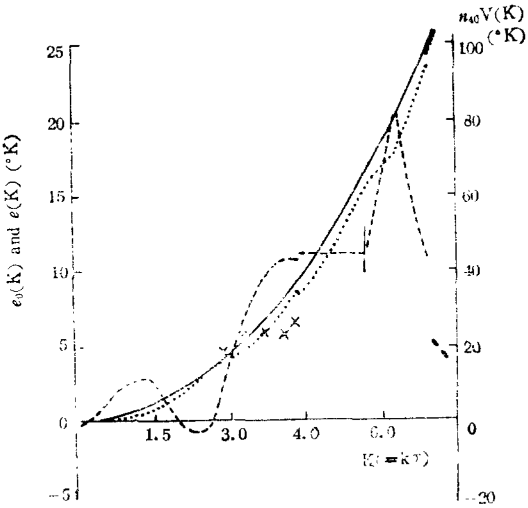

Fig. ———, ······, and ××× are $e_0(K)$. the present $e(K)$, a $e(K)$ of the neutron experiment by Hilton et al. (Ref. 1), respectively. and - - - - is the bulk effective interaction, $V(K)$, at $P=0$ of Yim and Massey (Ref. 3). $e(K)$'s are at $P=0$ and $x=0.06$. σ is 2.556 Å.

0 and $x=0.06$ using $V(k)$ calculated in YM theory[3] and compared with the neutron experiment,[1] where for the calculation of $V(k)$ we use the pure ^{4}He liquid structure function by Schiff and Verlet[1] using the cutoff $k_c=0.1565$Å^{-1},[5] and the renormalized phonon by the convolution approximation[6] for $k>2.23$Å^{-1}.

The present $e(k)$ at $P=0$ and $x=0.06$ is in agreement with the experimental result by Hilton et al.[1] at $P=0$ and $x=0.06$. $e(k)$ in the present calculation has the gaps at $k=1.68$ Å^{-1}, and 2.71 Å^{-1} resulting from $V(k)$ and the specific heat by the present $e(k)$ involves the terms like exp $\left(-\frac{xn_{40}D}{T}\right)$, where D is 1.768°K/n_{40} and the gap in $V(k)$ at $P=0$ and $k=1.68$Å^{-1}, and T is the temperature in the bulk mixture.

In conclusion the present calculation is also indirect proof of the physical interpretation such as a ^{3}He quasiparticle excitation at $x=0$ in the mixture is approximately a parabolic form from the phase equilibrium argument between the mixture and pure ^{3}He at the same pressure. The gap in $V(k)$ at $k=2.71$Å^{-1} is 22.738°K/n_{40}. One may detect these gaps using one particle-excitation experiment.

Appendix *A*: Electronic binding energy of He atom

The well-known solutions for He atom are due to the ordinary perturbation theory and variational method. Recently, 1/N expansion theory (it is somewhat similar to dimensional analysis) becomes popular particularly for quantum chromodynamics. Here a simple method of this solution for He atom different from the above is given and the result is compared with the result of 1/N expansion theory.

The Hamiltonian of He atom consisted of two electrons, H, is given by

$$H=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}-\frac{2e^2}{r_1}-\frac{2e^2}{r_2}+\frac{e^2}{r_{12}}, \quad \text{(A1)}$$

where m, e, $\vec{p}_i$ and $\vec{r}_i$ are the mass, the charge, the momentum operator and the positional coordinate of ith electron. In this Hamiltonian kinetic energies of protons and neutrons and potential energies of them contributed by two electrons are neglected because of the large mass of nucleus compared with the strength of interaction.

Since the momentum of an isolated atom conserves and also, the momentum of the nucleus can be neglected, we have

$$\vec{p}_1+\vec{p}_2=\vec{0}\quad, \qquad \text{(A2)}$$

and from *Eq.* (A2) we have

$$\vec{r}_1+\vec{r}_2=\vec{c}\quad, \qquad \text{(A3)}$$

where $\vec{c}$ is a constant vector, but we can choose it zero within 0.0003 of Bohr radius for He atom in considering nucleus. Therefore, $\vec{c}$ can be chosen zero here. It is also physically reasonable assumption. Using Eqs. (A2) and (A3) we have

$$H=2\left(\frac{p_1^2}{2m}-\frac{7e^2}{4r_1}\right)=2\left(\frac{p_1^2}{2m}-\left(\frac{\sqrt{7}}{2}e\right)^2/r_1\right)$$
$$\equiv 2H_0 \quad . \qquad \text{(A4)}$$

H_0 in *Eq.* (A4) is the Hamiltonian of hydrogen atom with the mass, m, and the charge, $(\sqrt{7}/2)e$. Therefore, we have electronic binding energy of the ground state of He atom to be $-3.0625(me^4)$ compared with the experimental result, $-2.91(me^4)$. This result is better one than one of 1/N expansion theory, however, not better one than those of the ordinary perturbation theory and variational method. It also gives lower value than the experimental result while others give upper values. However, it is a straightforward method for also excitations of He atom. Let me call it rather quite modest.

Appendix *B*: Errata of "Ground state of a mass-3 boson system and Superfluid ^{3}He" (Moo Bin Yim, UIT Report Vol. 11, No. 2, pp. 271 (1980))

Please notes the following corrections:

Page 272: In Table -2.59 at $n/n_0=0.814$ in fifth column has to be corrected: -2.59 at $n/n_0=0.7608$.

Page 273: Column 2, the second line from the top: $n=0.01597 \text{Å}^{-3}$ has to be corrected: $n=0.01493 \text{A}^{-3}$.

Eq. (14) to *Eq.* (15) have to be corrected as follow:

$$e_0(n)=A(P)(n/n_0)+B(P)(n/n_0)^2, \quad (14)$$

where P is the pressure of the mass-3 boson system, n_0 is the equilibrium density of the liquid ^{4}He and particularly, $e_0(n_{eq})$, $A(0)$ and $B(0)$ are given by

$$e_0(n_{eq})=-2.59^\circ\text{K at } n_{eq}=0.7608n_0,$$

$$A(0)=-2.59^\circ\text{K},$$

and

$$B(0)=-1.07^\circ\text{K},$$

and

$$\epsilon(n)=e_0(n)+P/n. \quad (15)$$

($\epsilon(n)$ is the chemical potential of a mass-3 boson in the mass-3 boson system and also it is equal to that of a ^{3}He (or a mass-3 boson) in the mixture at zero concentration)

Column 2, line 24 from the top: -0.5807 $^\circ\text{K}/n_0$ has to be corrected as follows: -0.8140 $^\circ\text{K}/n_0$.

Page 274: $(2/3)(n_{eq}/n_0)^2V_m(k)$ in ref. 9 has to be corrected as follows: $(n_{eq}/n_0)V_m(\text{k})$.

Adds in ref. 9 the following: For other pressures the ratios of the effective interactions between two ^{3}He atoms in pure liquid ^{3}He, $V(k)$, to the effective interactions between two ^{3}He quasiparticles in the mixture, $V_m(k)$, at the same pressures are also n/n_4, where n and n_4 are the densities of pure liquid ^{3}He and pure liquid ^{4}He at the same pressures, respectively.

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