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복수 협력 여유 관절 로보트 매니퓰레이터 시스템의 관절력 최적화

강희준
제어계측공학과

〈요 약〉

본 논문에서는, 복수의 여유 관절 매니퓰레이터들이 협력하여 하나의 공통 물체를 정해진 경로에 따라 움직이고자 할 때, 이 시스템 내부에 존재하는 여유도(redundancy)를 이용한 관절력 최적화 문제가 고려된다. 관절력 최적화 문제에서 흔히 발생하는 관절력의 불안정성을 해결하기 위하여 널 스페이스 댐핑 방법이 제안, 적용된다. 고려된 방법의 효율성은 3개의 복수 협력 로보트의 시뮬레이션을 통하여 입증된다.

Joint Torque Optimization of Multiple Cooperating Redundant Manipulators

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<Abstract>

In this article, joint torque optimization is considered for multiple cooperating redundant manipulators rigidly handling a common object. This work focuses on finding the optimal distribution of the operational forces of a multiple redundant manipulator system to the individual manipulators. Two joint torque optimization schemes(local joint torque minimization and natural joint motion) are formulated and compared. From the simulation results of a system of three cooperating redundant manipulators, the natural joint motion scheme is shown to be better than the local joint torque minimization scheme with regard to global torque minimization capability and the resulting stability of motion. However, in order to guarantee the stability, the null space damping method is required for the both schemes. The effectiveness of the null space damping method is demonstrated by simulation. Additionally, the condition for the distribution of the operational forces required to drive the given system along a natural joint motion trajectory is addressed.

1. Introduction

Cooperative use of multiple manipulators (or fingers) will allow the performance of more industrial applications than can currently be undertaken using single manipulators. Such systems can provide greater load capacity, better manipulation capability, and higher flexibility in automated manufacturing. Typical example applications include transport of heavy material, fine manipulation of objects and part assembly. A number of works dealing with cooperative execution of tasks performed by multiple cooperating manipulators have appeared recently[1-14]. Most of works, except Hu and Goldenburg[4], are not concerned with the utilization of the redundnacy of each manipulator with respect to the given task motion. Hu and Goldenburg[4] suggested the dynamic control law of multiple cooperating redundant manipulators via local and global torque optimization, and reported that the global optimal controller is better than local optimal controller in light of joint torque profile. However, the global technique is known to be not practically suitable for on-line motion control, because it requires heavy amount of computations. Therefore, we need find a local torque optimization algorithm of such case with stable joint torque profile.

In this article, joint torque optimization for multiple cooperating redundant manipulators rigidly handling a common object is considered in terms of the third load distribuion category above. From the operational space dynamic formulation for multiple cooperating redundant manipulators, the unique matrix which maps the joint torque set of the individual manipulators onto its corresponding task-dependent operational forces is obtained. Based on this functional relationship, two joint torque optimization schemes(local joint torque minimization and natural joint motion) are formulated. The joint torque instability, commonly encountered in local joint torque optimization of single redundant manipulators and seen in [4], was also observed in the multiple cooperating redundant manipulator case through simulation of three 3 DOF cooperating manipulators for planar translational operational trajectory. In order to eliminate the stability problem, the null space damping method[6] is also considered for the multiple cooperating redundant manipulator case. The effectiveness of the null space damping method is demonstrated by simulation.

2. Operational Space Dynamic Modeling of Multiple Cooperating Redundant Manipulators Rigidly Handling a Common Object

The configuration of L cooperating redundant manipulators rigidly handling a object is shown in Figure 1. The kinematic and dynamic model of each individual manipulator is assumed known. First, the object dynamics is expressed as follows, using the Newton-Euler equations

$$f = m_o \dot{r} - m_o g ;$$
$$n = [I_o]\dot{\omega} + \omega \times ([I_o]\omega) \quad (1)$$

where m_o and $[I_o]$ are the mass and inertia matrix of the object and r and w are the position of the operational point and the absolute angular velocity of the object, respectively. These equations are combined to yield the operational forces required to move the object as

$$F_o = [{}_oI^*_{uu}]\ddot{u} + {}_oC_u \quad (2)$$

where

$$[{}_oI^*_{uu}] = \begin{bmatrix} m_o[I] & [0] \\ [0] & I_o \end{bmatrix} = \begin{bmatrix} m_o[I] & [0] \\ [0] & I_o \end{bmatrix}$$

with the 3x3 identity matrix and null matrix being [I] and [0], respectively, and

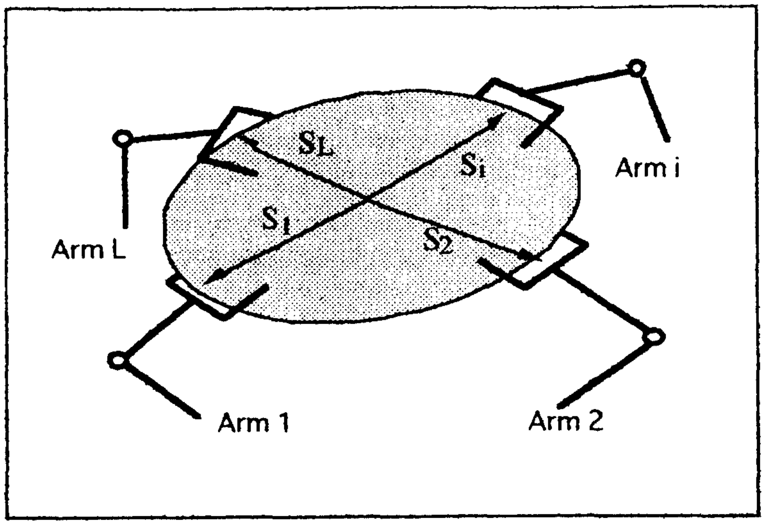

Figure 1. Multiple Cooperating Redundant Manipulators Handling a Common Object

$$\ddot{u} = \begin{bmatrix} \ddot{r} \\ \dot{\omega} \end{bmatrix}; \ {}_0C_u = \begin{vmatrix} -m_0 g \\ \omega \times (I_0 \omega) \end{vmatrix}; \ F_o = \begin{bmatrix} f^T, n^T \end{bmatrix}$$

The operational forces due to the object dynamics are obtained from the contact forces and moments exerted by the L cooperating manipulators as

$$f = \sum_{i=1}^{L} f_i \ ; \ n = \sum_{i=1}^{L} n_i + \sum_{i=1}^{L} s_i \times f_i \qquad (3)$$

where f_i and n_i are the contact forces and moments applied to the object by the i^{th} end-effector, with respect to the global coordinates Eqs. (3) may be combined to yield the constraint relationship between the operational forces(F) required to move the object and the contact forces and moments(F_c) applied by the L manipulators at the contact points as

$$F = [W] F_c \qquad (4)$$

where

$$F_c = [f_1^T, n_1^T, .., .., .., f_L^T, n_L^T]^T;$$
$$[W] = [[W_1], [W_2], .., [W_i], .., [W_L]]$$

with

$$[W_i] = \begin{bmatrix} [I] & [0] \\ [S_i] & [I] \end{bmatrix} \quad \text{and}$$

$$[S_i] = \begin{bmatrix} 0 & {}_iS_z & -{}_1S_y \\ -{}_1S_z & 0 & -{}_1S_x \\ {}_iS_y & {}_iS_x & 0 \end{bmatrix}$$

The force relationship between the operational forces due to the object dynamics and the contact forces gives the dual kinematic velocity relationship between the operational point and the contacting points(the end effector tip positions of each manipulator) as

$$\dot{u} = [W_i]^{-T} \dot{e}_i \qquad i = 1, 2, ..., L \qquad (5)$$

where $\dot{u}$ and $\dot{e}_i$ are the velocity of center of mass of the object and the velocity of end-effector tip of the i^{th} manipulator, respectively.

Combined with the known Jacobians of the manipulators $[G_i]$, Eq. (5) relates the Cartesian velocity of the center of mass of the object to the joint coordinate velocities of the i^{th} manipulator, with the resulting Jacobian J_i, as

$$\dot{u} = [W_i]^{-T} [G_i] \dot{\phi}_i = [J_i] \dot{\phi}_i \qquad i = 1, 2, ..., L \qquad (6)$$

where $[G_i]$ is Jacobian relating the velocities of the end-effector tip and the joint velocities ($\dot{\phi}_i$) of the ith manipulator.

Now, the operational space dynamic model of each individual manipulator(i = 1, 2, ...,L) is considered based on its joint space dynamic model and the previously computed Jacobian, J_i. The joint space dynamic model of the i^{th} manipulator is expressed as

$${}_i\tau_\phi = [{}_iI^*_{\phi\phi}] \ddot{\phi}_i + C(\phi_i, \dot{\phi}_i) \qquad i = 1, 2, ..., L \qquad (7)$$

where ${}_i\tau_\phi$ is joint load vector of the i^{th} manipulator with $[{}_iI^*_{\phi\phi}]$ and $C(\phi_i, \dot{\phi}_i)$ being its nxn inertia matrix and nx1 vector of Coriolis, centripetal and gravity components, respectively.

The joint space dynamic model can be transferred to the operational space dynamic model according to the formulation given in [15]. The effective operational forces due to the i^{th} manipulator can be written as

$$F_i = [{}_iI^*_{uu}]\ \ddot{u} + {}_iC_u \tag{8}$$

where the mxm effective inertia matrix and the mx1 vector of Coriolis, centripetal and gravity components of the i^{th} manipulator are, respectively,

$$[{}_iI^*_{uu}] = (J_i\,[{}_iI^*_{\phi\phi}]^{-1} J_i^T)^{-1} \tag{9}$$

and

$${}_iC_u = -[{}_iI^*_{uu}]\,H(\phi_i,\dot{\phi}_i) + ({}_iJ_I^+)^T C(\phi_i,\dot{\phi}_i)$$
$$\text{with } ({}_iJ_I^+)^T = [{}_iI^*_{uu}]\,J_i\,[{}_iI^*_{\phi\phi}]^{-1} \tag{10}$$

The operational forces of the considered robotic system can be expressed as a sum of effective operational force contribution of each individual manipulator and the common object as

$$F = F_o + \sum_{i=1}^{L} F_i = [I^*_{uu}]\ \ddot{u} + C_u \tag{11}$$

where the corresponding mxm inertia matrix of the given robotic system is

$$[I^*_{uu}] = [{}_oI^*_{uu}] + \sum_{i=1}^{L} [{}_iI^*_{uu}] \tag{12}$$

with the mx1 vector of Coriolis, centripetal and gravity components being

$$C_u = {}_oC_u + \sum_{i=1}^{L} {}_iC_u \tag{13}$$

3. Functional Relationship between Joint Torques and Operational Forces

Using Eqs. (11)-(13), the operational forces required to drive the given robotic system along the desired operational space trajectory are obtained from the control structure of the given robotic system. Here, the required joint torque is not unique. The surjective function mapping the joint torque set onto its effective operational forces can be represented by augmenting the matrices, $[{}_iJ_I^+]^T$, seen in Eq. (10).

Proposition 1) Any set of joint torques of multiple redundant manipulators rigidly handling a common object can be mapped onto its effective operational force vector by a unique transformation matrix, $[J_I^+]^T$.

$$F = [J_I^+]^T \tau_\phi$$
$$\text{with } [J_I^+]^T = [({}_1J_I^+)^T : \cdots : ({}_iJ_I^+)^T : \cdots ({}_LJ_I^+)^T] \tag{14}$$

where the joint torque set

$$\tau_\phi = [{}_1\tau_\phi^T \cdots {}_i\tau_\phi^T \cdots {}_L\tau_\phi^T]^T \text{ and}$$
$$({}_iJ_I^+)^T = [{}_iI^*_{uu}]\ J_i\,[{}_iI^*_{\phi\phi}]^{-1}$$

Proof) Refer to [15].

The main difference between single and multiple redundant manipulators is the level of redundancy: multiple cooperating redundant manipulators generally have more force redundancy than kinematic redundancy, while kinematic redundancy implies force redundancy for single redundant manipulators. For the case of a multiple cooperating redundant manipulator system, the joint torque set required to drive the system along a given joint trajectory is not unique, whereas it is unique for a single redundant manipulator. The procedure to determine minimum torque set for a given joint trajectory is explained via the natural joint motion of the multiple cooperating redundant manipulators. The next two propositions are given to illustrate the condition for the distribution of operational forces to a joint torque set of the multiple cooperating redundant manipulator system for inducing natural joint motion. This condition might help one to understand the natural behavior of the total system and will be used for a joint torque optimization scheme to follow.

Proposition 2) For a given operational force vector(F), a system independent joint torque set(τ_a) which is mapped by a transformation matrix, $(J_a^u)T$, drives the multiple cooperating redundant manipulator system along the natural joint motion, where the Jacobian matrix (J_a^u) relates the system's operational space velocity vector to the system independent joint velocity vector. (For clarity, see Eq. (21).)

Proof) When the multiple manipulators hold a common object, a closed chain mechanism is formed. Due to the closed chain constraint equations, the Lagrangian coordinates employed to initially describe the system are divided into a system independent coordinate set and a system dependent coordinate set. Then, the equations of motion of the given system can be completely described in terms of system independent coordinate set. Once the minimal kinematic and dynamic models are obtained, the argument for the natural joint motion of the multiple cooperating manipulator system can be treated in the same fashion as that for a single redundant manipulator. From the torque solution in Eq.(12) of Kang and Freeman[6], the proposition is evident. (Q.E.D)

Proposition 3)

$$ {}_i\tau_\phi = J_I^T F_i \qquad i = 1, 2, \ldots, L \tag{15}$$

and

$$ F = F_1 + F_2 + \ldots + F_L \tag{16}$$

Any joint torque set obtained from the operational force distribution according to Eqs. (15) and (16) drives the multiple cooperating redundant manipulators rigidly handling a common object along the natural joint motion.

Proof) The system independent actuating torque set, τ_a, is effectively equivalent to the corresponding Lagrangian torque set, τ_ϕ, in that both joint torque sets drive the system along the same joint trajectories. The relationship is written as

$$ \tau_a = (J_a^\phi)^T \tau_\phi \tag{17}$$

where (J_a^ϕ) is Jacobian matrix relating the system Lagrange coordinate set and the system independent coordinate set. The detailed expression of Eq.(17) can be written as

$$\tau_a = [({}_1J_a^{\phi})^T \cdots ({}_iJ_a^{\phi})^T \cdots ({}_LJ_a^{\phi})^T] \begin{bmatrix} {}_1\tau_{\phi} \\ \vdots \\ {}_i\tau_{\phi} \\ \vdots \\ {}_L\tau_{\phi} \end{bmatrix} \quad (18)$$

where $({}_iJ_a^{\phi})$ is the Jacobian matrix relating the system Lagrange coordinate set of the ith manipulator and the system independent set (Refer to [5] for generation of the Jacobians). Substituting Eq.(15) into Eq.(18) yields

$$\tau_a = [({}_1J_a^{\phi})^T \cdots ({}_iJ_a^{\phi})^T \cdots ({}_LJ_a^{\phi})^T] \begin{bmatrix} J_1^T F_1 \\ \vdots \\ J_i^T F_i \\ \vdots \\ J_L^T F_L \end{bmatrix}$$

$$= ({}_1J_a^{\phi})^T J_1^T F_1 + \cdots + ({}_iJ_a^{\phi})^T J_i^T F_i + \cdots + ({}_LJ_a^{\phi})^T J_L^T F_L \quad (19)$$

The Jacobian matrix relating the operational space velocity vector to the system independent coordinate velocity vector is obtained by substituting the internal joint kinematic relationship(i.e., J_a^{ϕ}) into the first order kinematics of a selected manipulator. Regardless of which manipulator is taken to describe the operational space motion, the same Jacobian matrix is obtained. Therefore,

$$(J_a^u) = J_1({}_1J_a^{\phi}) = \cdots = J_i({}_iJ_a^{\phi}) = \cdots = J_L({}_LJ_a^{\phi}) \quad (20)$$

Combining Eqs.(15), (19) and (20) yields

$$\tau_a = (J_a^u)^T F \quad (21)$$

From Proposition 2), the joint torque set obtained from this distribution scheme drives the system along the natural joint motion.

(Q.E.D)

4. Joint Torque Optimization

Eq. (14) is a relationship between the manipulator joint torques and the states of the system which are specified by the trajectory of the common object. What remains is to specify the manipulator joint torques required to achieve the specified operational trajectory of the object grasped by a group of redundant manipulators. The underdetermined problem of solving the joint torques from Eq.(14) allows one to optimize a given performance criterion as an additional control constraint, which then uniquely determines the joint torques. In this section, two local joint torque optimization schemes are treated, based on the functional relationship given in Eq.(14). They are the joint torque minimization scheme and the natural joint motion scheme. The resulting torques of both schemes given here include fictitious damping forces acting in the null space of $[J_I^+]^T$, since stability problems are expected here as they were in the joint torque optimization schemes[6] for single redundant manipulators. In that paper, the condition for null space damping matrix to achieve the positive damping is addressed.

4.1 Joint Torque Minimization(JTM)

$$\text{Min } \left(\frac{1}{2}\tau_{\phi}^T \tau_{\phi}\right) \quad \text{subject to } F = [J_I^+]^T \tau_{\phi}$$

Incorporating null space dissipative forces, the command torques become

$$\tau_{\phi} = P_2 F - K_d (I - P_2 [J_I^+]^T) \dot{\phi} \quad (22)$$

where

$$P_2 = [J_I^+]([J_I^+]^T[J_I^+])^{-1} \quad (23)$$

The particular solution seen in Eq. (22) provides the local absolute minimum joint torque norm required to drive the total system along a specified operational trajectory.

4.2 Natural Joint Motion(NJM)

As seen in [10], the joint torque optimization scheme may be obtained using the inverse inertia weighted joint torque norm as an objective function:

$$\text{Min}\left(\frac{1}{2}\tau_\phi^T[I_{\phi\phi}^*]^{-1}\tau_\phi\right) \quad \text{subject to} \quad F = [J_I^+]^T\tau_\phi$$

where the inverse inertia matrix is constructed such that its ith diagonal submatrix is the inverse of the joint inertia matrix of the ith manipulator. The command torques, in terms of the weighted Moore-Penrose generalized inverse, is obtained as

$$\tau_\phi = P_1 F \quad (24)$$

where

$$P_1 = [I_{\phi\phi}^*][J_I^+]([J_I^+]^T[I_{\phi\phi}^*][J_I^+])^{-1} \quad (25)$$

and the inertia matrix, $[I_{\phi\phi}^*]$, is constructed such that its ith diagonal submatrix is the joint inertia matrix of the ith manipulator. Decoupling Eq.(24), the joint torques of the ith manipulator can be expressed as

$$_i\tau_\phi = J_i^T[_iI_{uu}^*][I_{uu}^*]^{-1}F \qquad i = 1, 2, \ldots, L \quad (26)$$

It is recognized from proposition 3) that the joint torque solution in Eq.(26) drives the given system along natural joint motion(here, $F_i = [_iI_{uu}^*][_iI_{uu}^*]^{-1}F$ in Eq. (15)). Eq. (26) also shows that the operational forces are distributed to the individual manipulators according to their effective operational inertia contribution. Although the resulting torques drive the system along natural joint motion, the fact that the joint torque set corresponding to a given joint motion is not unique due to the additional force redundancy of multiple cooperating redundant manipulators, give rises to a question: Does the operational inertia distribution scheme generates the true minimum norm among all other possible sets of joint torques which can drive the given system along the natural joint motion? This question leads to the consideration of the following joint torque optimization scheme.

It is assumed that the given system makes natural joint motion. With the aid of Proposition 3), the joint torque optimization scheme for natural joint motion is reformulated as

$$\text{Min}\left(\frac{1}{2}\tau_\phi^T\tau_\phi\right)$$

$$\text{subject to} \quad _i\tau_\phi = J_i^T F_i \quad i = 1, 2, \ldots, L \quad \text{and}$$

$$F = \sum_{i=1}^{L} F_i \quad (31)$$

Using the Lx1 vector of Lagrange multipliers, λ, and substituting Eq. (31) into the objective function, the augmented objective function, I, can be written as

$$I = \sum_{i=1}^{L}\frac{1}{2}F_i^T J_i J_i^T F_i + \lambda^T\left(F - \sum_{i=1}^{L} F_i\right) \quad (32)$$

The problem is now reduced to determining the minimum solution of the unconstrained objective function, I, in terms of the independent parameters, λ and F_i (i = 1, 2, ..., L). This requires that

$$\frac{\partial I}{\partial F_i} = J_1 J_i^T F_i - \lambda = 0 \quad i = 1, 2, ..., L \qquad (32)$$

and

$$\frac{\partial I}{\partial \lambda} = F - \sum_{i=1}^{L} F_i = 0 \qquad (34)$$

Assuming that there is no kinematic degeneracy in any of the individual manipulators, solving Eq.(33) for F_i yields

$$F_i = (J_1 J_i^T)^{-1} \lambda \quad i = 1, 2, ..., L \qquad (35)$$

Substituting Eq.(35) into Eq.(34) and solving for λ gives

$$\lambda = \left(\sum_{i=1}^{L} (J_1 J_i^T)^{-1}\right)^{-1} F \qquad (36)$$

Substituting Eq.(36) into Eq.(35) yields

$$F_i = (J_1 J_i^T)^{-1} \left(\sum_{i=1}^{L} (J_1 J_i^T)^{-1}\right)^{-1} F \quad i = 1, 2, ..., L \qquad (37)$$

Substituting Eq.(37) into Eq.(31), the joint torques required to satisfy the objective function are

$$_i\tau_\phi = J_i^T (J_1 J_i^T)^{-1} \left(\sum_{i=1}^{L} (J_1 J_i^T)^{-1}\right)^{-1} F \quad i = 1, 2, ..., L \qquad (38)$$

From [15], the physical interpretation of the force distribution of Eq.(37) is that the operational forces are distributed to the individual manipulators according to their effective operational stiffness contribution, for the case in which all joints of the given system are assumed to have equal motor stiffness. Based on this physical interpretation, Eq.(38) can be written as

$$_i\tau_\phi = J_i^T [_iK^*_{uu}][K^*_{uu}]^{-1} F \quad i = 1, 2, ..., L \qquad (39)$$

where the effective operational linear stiffness matrix of the ith manipulator is

$$[_iK^*_{uu}] = (J_1 J_i^T)^{-1} \quad i = 1, 2, ..., L \qquad (40)$$

with the total effective operational linear stiffness matrix of the system being by

$$[K^*_{uu}] = \sum_{i=1}^{L} (J_1 J_i^T)^{-1} \qquad (41)$$

This solution shows that the local torque minimum for natural joint motion is obtained from the distribution of the operational forces to the individual manipulators according to their operational stiffness contribution rather than their operational inertia contribution. Incorporating the null space damping forces, which satisfy the stability condition, the command torques from this scheme become

$$_i\tau_\phi = J_i^T [_iK^*_{uu}][K^*_{uu}]^{-1} F - K_d([I] - J_i^T ({}_iJ_I^{+})^T [_iI^*_{\phi\phi}]) \ddot{\phi}_1 \quad i = 1, 2, ..., L \qquad (42)$$

In this section, the joint torque minimization and natural joint motion schemes are presented for the desired performance: the resulting torques are globally stable and have good torque minimization capability. The performances of the joint torque minimization scheme(Eq. (26)) and the natural joint motion scheme(Eq. (42)) will be compared in light of the following simulation results.

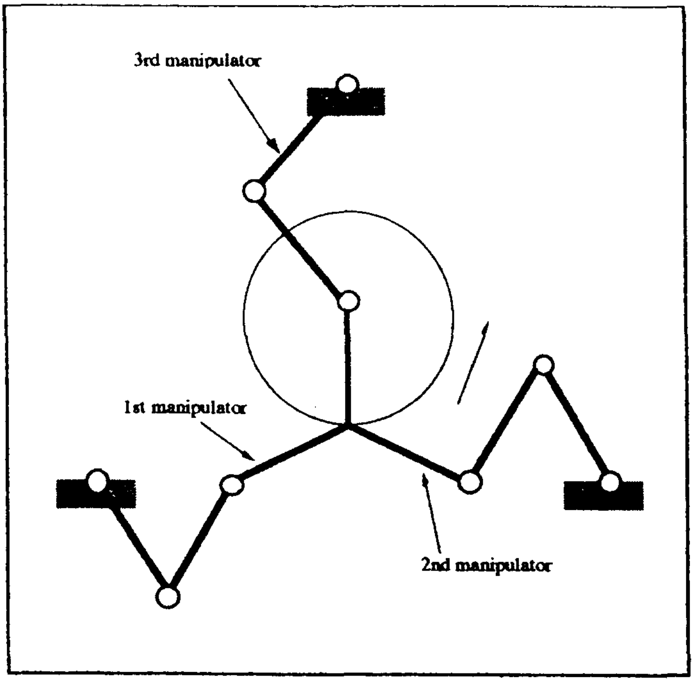

Fig. 2 Three Cooperating Redundant Manipulators to the X-Y Task Space

5. Numerical Simulation and Discussion

The joint torque minimization and natural joint motion schemes, for both the undamped and damped cases, are simulated for a long time circular trajectory. In this example, the simulated manipulator system, seen in Fig 2, consists of three cooperating 3R planar manipulators operating in X-Y task space without gravity. Each link is modeled as a thin uniform rod with a length of 1.0 m and a mass of 10 kg. The simulated desired hand trajectory is a circular path with high acceleration as followings: radius = 0.5 m, $\ddot{x}$ = - radius $\pi^2 \cos(\pi t)$ m/s^2, and $\ddot{y}$ = - radius $\pi^2 \sin(\pi t)$ m/s^2. The total simulation interval is 4 sec, completing 2 periodic cycles. The initial state of the manipulator is $\phi = [\phi_1^T, \phi_2^T, \phi_3^T]^T$ = [-64.34, 128.68, -34.34, 115.66, 128.68, -94.34, -131.41, 82.82, -41.41]^T (deg) and $\dot{\phi} = [\dot{\phi}_1^T, \dot{\phi}_2^T, \dot{\phi}_3^T]^T$ = [0.812, -1.789, 1.060, -0.977, 1.788, -0.729, 0.992, 0, 0.909]^T (rad/sec). The manipulator dynamics are integrated at an interval of 0.01 sec with the fourth order Runge Kutta integration routine. For a forward dynamic simulation of the three cooperating manipulators, the dynamic models in terms of system generalized coordinate set, are required. First, the optimal joint torque sets, tϕ, are obtained from either Eq. (26) or Eq. (42) according to the employed joint torque optimization scheme. Recalling Eq. (21), the effectively equivalent joint torques, ta, are obtained according to the selected minimum set of coordinates(fa) (selected as the three base joint coordinates of each manipulator in this example). Then, the joint accelerations are evaluated by solving the dynamic equation based on the selected minimum set of coordinates. For details of this procedure for forward dynamic simulation, refer to Kang and Freeman(1990-b). The simulation results of the joint velocity norm, the acceleration norm, and the torque norm trajectories are given for each algorithm, and for both the undamped and damped cases. The units of the plotted velocity, acceleration and torque norms are (rad/sec)2, (rad/sec2)2 and ((N m)2, respectively.

The simulation results, seen in Figs. (3)-(14), show generally similar characteristics to the results of the single redundant manipulator case in Kang and Freeman (1993) with respect to the global torque minimization capability, the resulting stability, and the convergence to a certain null space damped joint trajectory. From comparison of Figs. (3)-(6) and Figs. (9)-(11), however, it is noted that the undamped NJM scheme generally leads to stable motion and torque trajectories during the simulation interval(4 sec), while the undamped JTM scheme easily deviates from the desired system performance. For the single redundant manipulator case, both undamped schemes show the same level of stability problems. The global characteristics(i.e., the global minimization of the constrained Lagrangian) of the undamped NJM scheme are prominently illustrated in the multiple cooperating redundant manipulator case, compared to the results of the undamped JTM scheme. However, the motion and torque trajectories of the undamped NJM (Figs. (9)-(11)) reveal, via the increases in the velocity, acceleration, and torque norms, that they are deteriorating the desired system performance, even though at a slow rates.

The elimination of the stability problem is accomplished by adding fictitious dissipating forces, without affecting the operational forces, to the undamped solutions. Figs.(6)-(8) and Figs. (12)-(14) demonstrate the effectiveness of this approach. From the comparison of those results, it is concluded that the null space damped natural joint motion(NDNJM) scheme is superior to the null space damped joint torque minimization (NDJTM) scheme with respect to the global torque minimization capability and the resulting stability. In addition, the NDJTM scheme had a much smaller range of damping gains yielding a stable response than did the NDNJM scheme, and also, led to a torque peak and high speed joint motion in its initial stage. Those problems were also observed in the NDJTM scheme for single redundant manipulators. Therefore, the NDNJM scheme is deemed more appropriate for joint torque optimization of the multiple cooperating redundant manipulator system.

6. Summary

In this paper, joint torque optimization for multiple cooperating redundant manipulators rigidly handling a common object was considered. This work focused on finding the optimal distribution scheme of the operational forces of a multiple redundant manipulator system to the individual manipulators. Two undamped/damped joint torque optimization schemes(local joint torque minimization and natural joint motion) were formulated and compared with respect to the global torque minimization capability and the resulting stability. From the simulation results of a system of three cooperating redundant manipulators, the null space damped natural joint motion(NDNJM) scheme is deemed best among the considered schemes for the desired system performance. Additionally, the condition for the distribution of the operational forces required to drive the given system along natural joint motion trajectory was addressed.

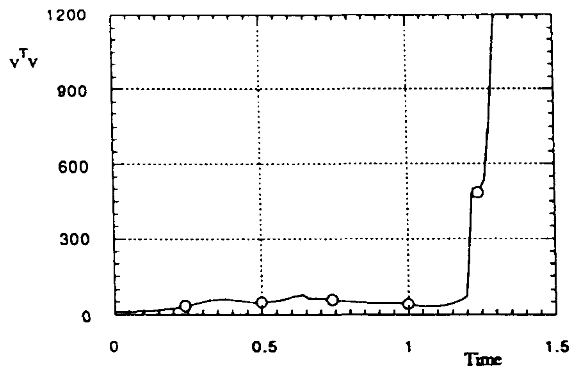

Fig. 3 Velocity Norm Trajectory of Undamped JTM(K_d = 0)

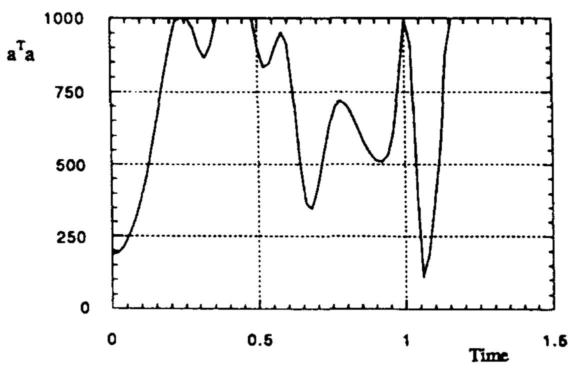

Fig. 4 Acceleration Norm Trajectory of Undamped JTM(K_d = 0)

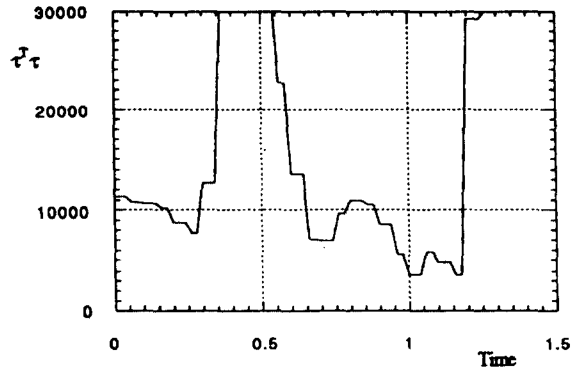

Fig. 5 Torque Norm Trajectory of Undamped JTM($K_a = 0$)

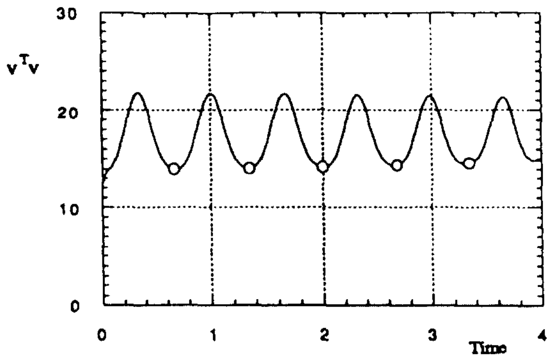

Fig. 6 Velocity Norm Trajectory of Null Space Damped JTM($K_a = 200$)

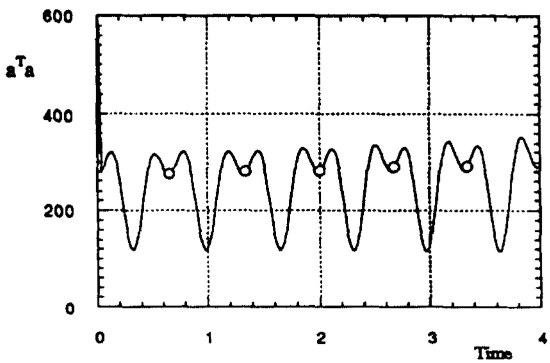

Fig. 7 Acceleration Norm Trajectory of Null Space Damped JTM($K_a = 200$)

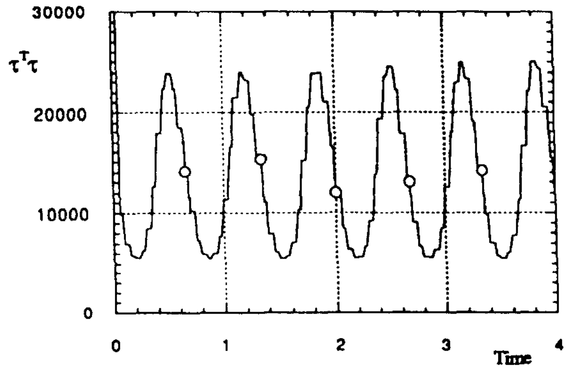

Fig. 8 Torque Norm Trajectory of Null Space Damped JTM(K_d = 200)

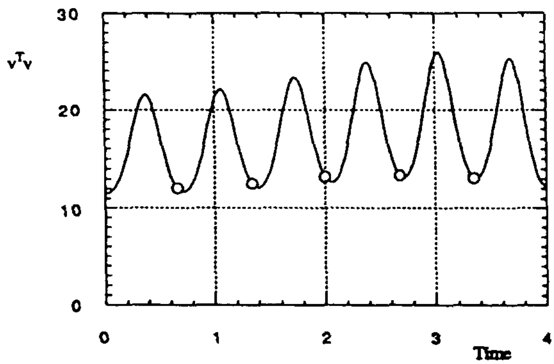

Fig. 9 Velocity Norm Trajectory of Undamped NJM(K_d = 0)

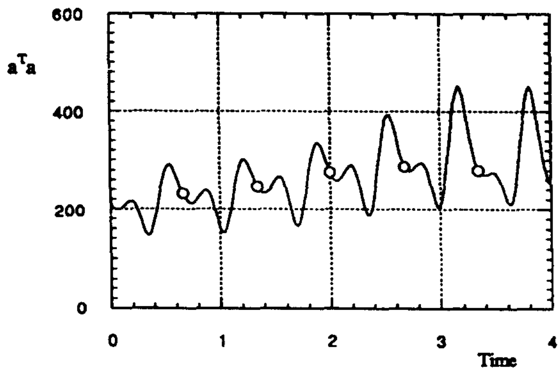

Fig. 10 Acceleration Norm Trajectory of Undamped NJM(K_d = 0)

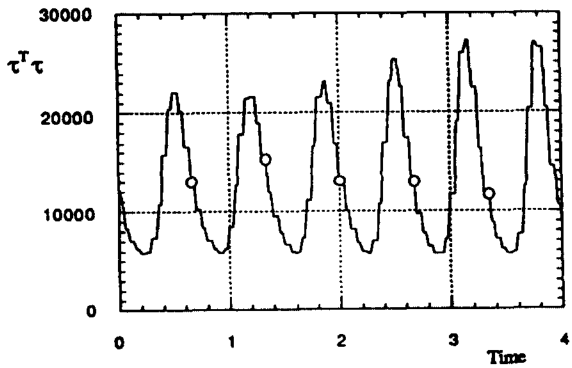

Fig. 11 Torque Norm Trajectory of Undamped NJM($K_d = 0$)

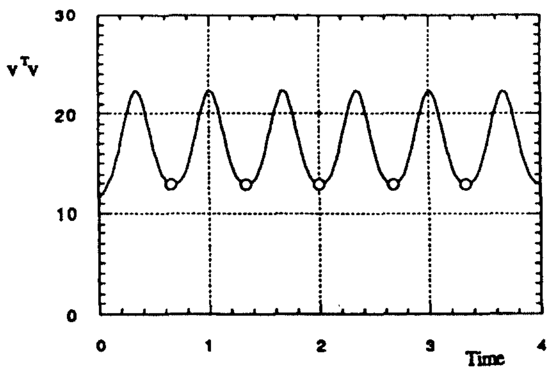

Fig. 12 Velocity Norm Trajectory of Null Space Damped NJM($K_d = 5$)

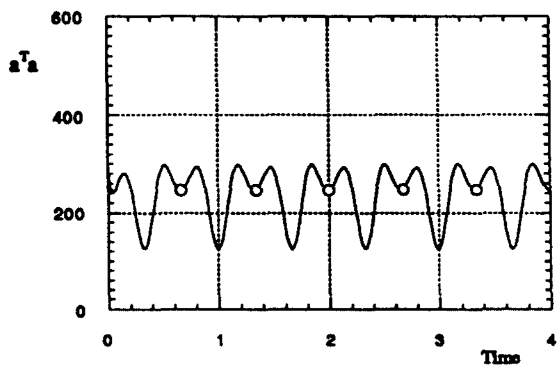

Fig. 13 Acceleralion Norm Trajectory of Null Space Damped NJM($K_d = 5$)

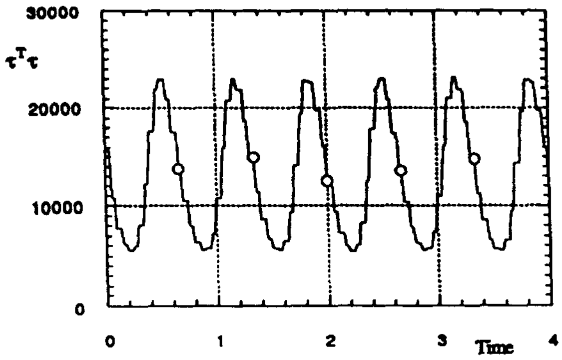

Fig. 14 Torque Norm Trajectory of Null Space Damped NJM(K_d = 5)

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