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Some new Mappings in Bitopological Spaces

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〈Abstract〉

The authors introduce some new mappings in bitopological spaces and obtain some of their properties and study their relationship with some known mappings. A diagram of implications is given.

쌍위상공간의 새로운 사상들에 관하여

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〈요 약〉

쌍위상공간상에서 몇개의 새로운 사상들을 도입하고 그들의 성질들을 알아보며 도입된 새로운 사상들과 기존 사상들과의 관계를 연구한다. 끝으로 그들 상호간의 포함관계를 알아본다.

I. Introduction

A set in a bitopological space (X, P_X, Q_X) is quasi open [1] iff it is a union of a P_X-open set and a Q_X-open set. A set in a bitopological space (X, P_X, Q_X) is quasi semi open [5] iff it is a union of a P_X-semi open set and a Q_X-semiopen set (For, the concept semiopen and its properties refer to Levine [4]). Every P_X-semiopen (resp. Q_X-semiopen) set is quasi semiopen, but the converse may be false [5]. Any union of quasi semiopen sets is quasi semiopen [5]. The complement of a quasi semiopen set is termed quasi semiclosed [5]. It is obvious that every quasi open set is quasi semiopen. However, the converse need not be true. For, if $X=\{a,b,c,d\}$, $P_X=\{\phi, \{a\}, X\}$, $Q_X=\{\phi, \{b\}, X\}$ then the set $\{a,b,c\}$ is quasi semiopen but is not quasi open.

The purpose of this note is to present a study of some new mappings. Throuhout this note we mean by X a bitopolological space (X, P_X, Q_X), by Y a bitopological space (Y, P_Y, Q_Y) and so on and $f: X \to Y$ denotes a mapping f from a bitopological space X into a bitoplogical space Y.

II. Some known mappings

The following definitions are known.

Definition 1. A mapping $f: X \to Y$ is said to be pairwise continuous if the inverse image of each P_Y-open set is P_X-open and that of each Q_Y-open set is Q_X-open.

Definition 2. A mapping $f: X \to Y$ is said to

*Professors in Dept. of Math. University of Saugar, India be semi bicontinuous (resp. biirresolute) if the inverse image of each P_Y-open (resp. P_Y-semiopen) set is P_X-semiopen and that of each Q_Y-open(resp. Q_Y-semiopen) set is Q_X-semiopen [3].

Definition 3. A mapping $f: X\to Y$ is said to be quasi bicontinuous if the inverse image of each quasi open subset of Y is quasi open in X [2,6].

III. Some new mappings

We introduce some new mappings follows:

Definition 4. A mapping $f: X\to Y$ is termed quasi semicontinuous if the inverse image of each quasi open subset of Y is quasi semiopen in X.

Definition 5: A mapping $f: X\to Y$ is quasi irresolute if the inverse image of each quasi semiopen subset of Y is quasi semiopen in X.

Proposition 1. Composition of two quasi irresolute mappings is quasi irresolute.

Proof: Straightforward.

Remark 1. Composition of two quasi semi continuous mappings may not be quasi semi continuous. For,

Example 1: Let $X=\{a,b,c\}, P_X=\{\phi, \{a\}, \{b, c\}, X\}$ $Q_X=\{\phi, \{b,c\}, X\}$: $Y=\{x,y,z\}$, $P_Y=\{\phi,\{x\},Y\}$, $Q_Y=\{\phi, Y\}$; $Z=\{p, q\}, P_Z=\{\phi, \{p\}, Z\}$, $Q_Z=\{\phi, Z\}$. Define $f: X\to Y$ by $f(a)=x$, $f(b)=y$, $f(c)=z$, and $g: Y\to Z$ by $g(x)=g(y)=p$, $g(z)=q$. Then f and. g are quasi semi continuous mappings but $g\circ f$ is not quasi semi continuous. However, we have,

Proposition 2. If $f: X\to Y$ is quasi semi continuous and $g: Y\to Z$ is quasi continuous then $g\circ f$ is quasi semi continuous.

Proof. Straightforward.

Remark 2. An arbitrary restriction of a quasi semi continuous(resp. quasi irresolute) mapping may not be quasi semi continuous (resp. quasi irresolute). For, the mapping $g: Y\to Z$ of example 1 is quasi semi continuous (resp. quasi irresolute). But its restriction over the set$\{y, z\}$ is not quasi semi continuous (resp. quasi irresolute).

However, we have,

Proposition 3. Let $f: X\to Y$ be a quasi semi continuous (resp. quasi irresolute) mapping. If A is a biopen subset of X then f_A is quasi irresolute.

We shall need the following lemma:

Lemma. If B is quasi semi open in X and A is biopen in X then $A\cap B$ is quasi semi open in A[5].

Proof of proposition 3. Let V be any quasi open (resp. quasi semi open) set in Y. Since f is quasi semi continuous (resp. quasi irresolute) it follows that $f^{-1}(V)$ is quasi semi open in X. Therefore by the above lemma, $f^{-1}(V)\cap A$ is quasi semi open in A because A is biopen in X. But, $f_A^{-1}(V)=f^{-1}(V)\cap A$. It follows that $f_A^{-1}(V)$ is quasi semi open in A. Consequently, f_A is quasi semi continuous(resp. quasi irresolute).

Proposition 4. Let $f: X\to Y$. Then the following statements are equivalent:

(a) f is quasi semi continuous (resp. quasi-irresolute).

(b) The inverse image by f of every quasi semi open(resp. quasi semi open) subset of Y is quasi semi open in X.

(c) For each $x\in X$ and each quasi open(resp. quasi semi open) set O in Y such that $f(x)\in O$, there is a quasi semi open set A in X such that $x\in A$ and $f(A)\subset O$.

(d) The inverse image by f of each quasi semi closed (resp. quasi semi closed) subset of Y is quasi semi closed in X.

Proof (a)$\Longleftrightarrow$(b). Definition.

(b)$\Longrightarrow$(c). Let O be quasi open(resp. quasi semi open) set in Y, and $f(x)\in O$. Then $x\in f^{-1}(O)$ which is quasi semi open in by (b). Put $A=f^{-1}(O)$. Then, $x\in A\subset O$.

(c)$\Longrightarrow$(b). Let O be quasi open (resp. quasi semi open) set in Y and $x\in f^{-1}(O)$. Then, $f(x)\in O$. Therefore by(c) there exists a quassi semi open set A_x in X such that $x \in A_x$ and $f(A_x) \subset O$ Then, $x \in A_x \subset f^{-1}(O)$. Consequently, $f^{-1}(O)$ is a union of quasi semi open sets in X and hence is quasi semi open sets in X. Thus (b) holds.

(b)$\Longleftrightarrow$(d). For if $B \subset Y$, then $f^{-1}(Y-B) = X - f^{-1}(B)$.

Remark 3. The intersection of two quasi semi open sets may not be quasi semi open [5].

Definition 6. A bitopological space X is termed strongly bitopological if the intersection of any two quasi semiopen sets is quasi semi open.

Definition 7. A bitopological space (X, P_X, Q_X) is said to be quasi semi T_2 if for each pair of distinct points x, y of X there exist disjoint quasi semi open sets U, V in X such that $x \in U$, $y \in V$ [5].

Proposition 5. If f and g are quasi semi continuous (resp. quasi irresolute) mappings from a strongly bitopological space X into a quasi Hausdorff [1] (resp. quasi semi T_2) space Y then the set $A = \{x \mid f(x) = g(x)\}$ is quasi semi closed.

Proof: Let $y \in X - A$. Then $f(y) \neq g(y)$. Since Y is quasi Hausdorff(resp. quasi semi T_2) there exist quasi open (resp. quasi semi open) sets U and V such that $f(y) \in U$, $g(y) \in V$ and $U \cap V = \phi$. Since f and g are quasi semi continuous (resp. quasi irresolute), $f^{-1}(U)$ and $g^{-1}(V)$ are quasi semi open and $y \in f^{-1}(U) \cap g^{-1}(V) = B$ say. Since X is strongly bicopological and so B is a quasi semi open set in X. Moreover, $A \cap B = \phi$. For, if it is not so then we reach to a contradiction that $U \cap V \neq \phi$. Thus, $y \in B \subset X - A$. Andso, $X - A$ is a union of quasi semi open sets in X. Consequently, $X - A$ is quasi semi open in X. Hence, A is quasi semi closed in X.

Ⅳ. Interrelations

In this section we study interrelations among the mappings discussed in the previous sections.

Remark 4. Every pairwise continuous mapping is quasi continuous. But the converse may be false. For,

Example 2. Let $X = \{a, b, c\}$, $P_X = \{\phi, \{a\}, X\}$, $Q_X = \{\phi, \{b\}, X\}$; $Y = \{x, y, z\}$, $P_Y = \{\phi, \{x\}, \{x, y\}, Y\}$, $Q_Y = \{\phi, \{y\}, Y\}$. Define $f: X \to Y$ by $f(a) = x$, $f(b) = y$, $f(c) = z$. Then f is quasi continuous but it is not pairwise continuous.

Remark 5. Every quasi continuous map is quasi semi continuous. But the converse may be false. For,

Example 3. Let X, Y be the spaces of example 2. Define $f: X \to Y$ by $f(a) = f(c) = x$, $f(b) = y$. Then f is quasi semi continuous but it is not quasi continuous.

Remark 6. Every pairwise continuous map is semi bicontinuous. The converse may be false. For, the mapping $f: X \to Y$ in example 3 is semi bicontinuous but it is not pairwise continuous.

Remark 7. Every semi bicontinuous map is quasi semicontinuous. But the converse need not be true. For,

Example 4. Let X, Y be the spaces of example 2. Define $f: X \to Y$ by $f(b) = f(c) = x$, $f(a) = y$. Then f is quasi semi continuous but it is not semi bicontinuous.

Remark 8. The mapping $f: X \to Y$ of example 3 is semi bicontinuous but it is not quasi continuous.

Example 5. Let X, Y be the spaces of example 2. Define $f: X \to Y$ by $f(a) = y$, $f(b) = x$, $f(c) = z$. Then f is quasi continuous but it is not semi bicontinuous.

Remark 9. Every biirresolute mapping is semi bicontinuous. The converse may be false. For,

Example 6. Let $X = \{a, b, c\}$, $P_X = \{\phi, \{a\}, \{b, c\}, X\}$, $Q_X = \{\phi, \{b\}, \{a, c\}, X\}$: $Y = \{x, y, z\}$, $P_Y = \{\phi, \{x\}, Y\}$ $Q_Y = \{\phi, \{y\}, Y\}$. Define $f: X \to Y$ by $f(a) = x$, $f(b) = y$, $f(c) = z$. Then f is semi bicontinuous (in fact, pairwise continuous) but it is not biirresolute.

Remark 10: Every biirresolute mapping is quasi irresolute. The converse may be false. For, the map of example 5 is quasi irresolute but it is not biirresolute.

Remark 11. Every quasi irresolute mapping is quasi semi continuous. The converse may be false. For,

Example 7. Let $X=\{a,b,c\}$, $P_X=\{\phi,\{a\},\{b,c\},X\}$, $Q_X=\{\phi,\{b\},X\}$, and (Y,P_Y,Q_Y) and $f:X\longrightarrow Y$ be as in example 6. Then f is quasi continuous(and hence quasi semicontinuous) but it is not quasi irresolute.

Remark 12. The mapping $f:X\longrightarrow Y$ defined in example 4 is quasi irresolute but it is neither quasi continous nor semi bicontinuous.

Remark 13. The mapping $f:X\longrightarrow Y$ in example 7 is semi bicontinuous (in fact, pairwise continuous) but it is not quasi irresolute.

Remark 14. A biirresolute mapping may fail to be pairwise continuous. For, if we let X, P_X, Q_X, Y, P_Y as in example 2 and let $Q_Y=\{\phi,Y\}$. Define $f:X\longrightarrow Y$ by $f(a)=f(c)=x$, $f(b)=y$. Then $f:X\longrightarrow Y$ is biirresolute but it is not quasi continuous(and hence it is not pairwise continuous).

V. Diagram

The study in section IV leads us to the following complete diagram of implications.

$$\begin{array}{ccccc} PC & \Longrightarrow & SBC & \Longleftarrow & BI \\ \Downarrow & & \Downarrow & & \Downarrow \\ QC & \Longrightarrow & QSC & \Longleftarrow & QI \end{array}$$

where,

PC: Pairwise continuous
QC: Quasi continuous
BI: Biirresolute
QI: Quasi irresolute
SBC: Semibicontinuous
QSC: Quasi semi continuous.

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