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# Comparative Studies of Control Design Method

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<Abstract>

This paper presents the robustness of the Coefficient Diagram Method(CDM) which has been developed recently[1]. We are supposed to approach the goal through comparative studies of several design methods. These consist of two parts : the first part is of competitive comparisons of classical controller design methods such as ITAE, LQR, and dominant second order pole placement, the second part is to compare the CDM with an H design. These comparisons are carried out in the parameter space, root space and frequency domain and parametric uncertainties in both plant and controller are considered. We show that the CDM has some significant characteristics in robustness viewpoints. The results can be referred as a background when we want to choose to design a control method.

Keywords : CDM, ITAE, LQR, Pole-placement, $H^{\infty}$ design

# 제어기 설계 방법에 따른 성능 비교에 관한 연구

김한실
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<요 약>

이 논문은 최근에 발표된 계수도법의 강인성에 관한 것이다. 여러 설계 기법을 비교연구하여 적절한 제어기를 선택하는데 Guide Line을 줄 수 있는데 의의가 있다. 이것은 두 부분으로 나누어진다. 즉 첫 번째는 ITAE, LQR, Pole-placement와 같은 고전적인 설계기법

을 비교했고, 두 번째는 계수도법과 $H^\infty$제어기에 대해 비교했다. 이 비교는 파라메터 공간 및 주파수 영역에서 수행되었다. 여기서 연구된 계수도법이 가장 좋은 특징을 나타내었으며 앞으로 제어기 설계에 중요한 자료가 제시한다.

# 1. Introduction

The main reason why most of control industries still prefer classical controllers such as P, PI, PID, and lead/lag is that it is simple and intuitive. Despite modern control has been developed remarkably in past two decades, there exist few methods for designing optimally or/and robustly classical controller. Moreover, modern optimal and robust control theory can not effectively deal with fixed order controllers. Thus we still use the Ziegler and Nichols tuning rules. Recently, a novel design method, so called, the Coefficient Diagram Method(CDM) has been developed by S. Manabe[1]. Since the method enables classical and modern control concepts to be used for the design of fixed order controller, it will enhance the present design technologies of industry. Briefly speaking, CDM is an algebraic approach over polynomial ring in the parameter space, in which all equations are deal with numerator and denominator polynomials of both plant and controller. In other words, as like loop shaping in bode plot, the key concept is to shape coefficients of characteristic polynomial so that it satisfies stability, transient response, and robustness. It is well demonstrated that the CDM can be applied to the fixed order controller design problem successfully and the design approach based on the coefficient diagram is also very intuitive. However, so far there are few results for robustness characteristics of CDM.

In this paper, the objective is to investigate the issue by means of comparative studies of several design methods. Such a method can be classified two categories ; classical and modern approaches. Thus, the investigations consist of two cases. The first is to compare the CDM with classical design methods such as ITAE, LQR, and pole placement to a prototype of interval plant where I $\pm$10 percent perturbations of each coefficient will be considered. In the second case, we compare the CDM with an H design in several examples. Comparisons of robustness issue are carried out in parameter space, root space and frequency domain respectively and also parametric uncertainties in plant or controller are considered. By using parametric robust control theory[3], we will show that CDM has some significant advantages in robustness viewpoint.

# 2. Coefficient Diagram Method

In this section, the CDM is summarized briefly. A two-parameter feedback configuration shown in Fig.1 is adopted in CDM. The characteristic equation of the closed loop system is assumed as

$$\triangle(s) = a_n s^n + \cdots + a_1 s + a_0.$$

The CDM parameters, i.e., the stability index $\gamma$, the equivalent time constant $\tau$, and the stability limit $\gamma_i^*$ are defined as

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}},\ i = 1 \sim n-1\ ,\ \ \tau = \frac{a_1}{a_0},\ \ \ \gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}},\ i = 1 \sim n-1,\ \gamma_0 = \gamma_\infty = 0.$$

The stability index specifies the stability and the shape of the time response. The variation of stability index due to the plant perturbation relates to the robustness. The equivalent time constant indicates the response speed. That is, the settling time is about $2.5 \sim 3\tau$.

The standard values $\gamma_i$, proposed by Manabe[1] is $\gamma_1 = 2.5$, $\gamma_2 = \gamma_3 = \cdots = \gamma_{n-1} = 2$. These value has the favorable characteristics which are almost no overshoot for system type I, almost equal response forms irrespective to the system order etc. The design process in CDM is summarized as follows ; For a given plant transfer function and specification, first reformulate the specification into the CDM parameters, $\gamma_i$ and $\tau$. For most ordinary problems, it is recommended that the standard form be good. Subsequently, select a proper order of controller. Then drive the Diophantine equation form characteristic polynomial of which the unknowns are controller parameters and solve it. Alternatively, a graphical shaping method on the coefficient diagram may be better for some cases[6].

## 3. Robustness characteristics of some classical control design methods

We attempt to investigate the robustness characteristics of the CDM controller by comparing with those of ITAE, LQR, and dominant pole-placement methods. It is assumed, for our robustness analysis, that all the coefficients of plant are perturbed by $\pm 10$ percent from the nominal values. To make a fair comparison between different control methods, we must impose some constraints on the settling time.

Consider the general robust feedback system shown in Figure 1 and consider an interval plant

$$P(s) = \frac{B_p(s)}{A_p(s)} = \frac{b_0}{s(a_3 s^2 + a_2 s + a_1)}$$

where $b_0 \in [1.8 \quad 2.2]$, $a_3 \in [0.9 \quad 1.1]$, $a_2 \in [0.225 \quad 0.275]$, $a_1 \in [5.625 \quad 6.875]$.

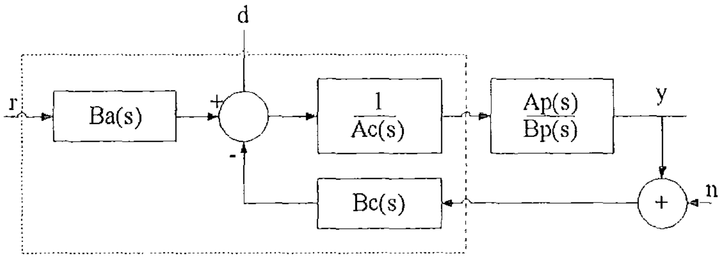



Figure 1. Two degree of freedom feedback configuration

The nominal values of the plant are chosen as $b_0 = 2$, $a_3^0 = 1$, $a_2^0 = 0.25$, $a_1^0 = 6.25$. We find a feedback controller such that will meet the following specifications:
(i) the order of controller is fixed as two.
(ii) the settling time of 1% shall be 2 sec.

### 3.1 Design of 4 classical controllers

We begin with designing on ITAE optimal system. Using a 3rd order optimal ITAE system with $w_0 = 4.2$, the overall closed loop system $G_0(s)$ and its corresponding controller are

$$G_o(s) = \frac{1306.9123}{s^2 + 11.76s^4 + 407.484s^2 + 1057.9766s + 1306.9123}$$

$$B_a(s) = 653.4562$$

$$B_c(s) = 157.8892s^2 + 281.8868s + 653.4562$$

$$A_c(s) = s^2 + 11.51s + 79.0725$$

We next design the quadratic optimal system. The main objective is to minimize the quadratic performance index

$$J = \int_0^{\infty} [q(y(t) - r(t))^2 + u^2(t)]dt$$

The given constraint is achieved by selecting q=492. And the overall transfer function $G_0(s)$ and the controller are

$$G_0(s) = \frac{44.3621}{s^3 + 5.9233s^2 + 23.7613s + 44.3621}$$

$$B_a(s) = 22.1811/(s+20)^2$$

$$B_c(s) = 1458.9597s^2 + 3630.049s + 8872.42$$
$$A_c(s) = s^2 + 45.6733s + 643.0249$$

The overall system poles in the dominant pole-placement method are chosen at -10, -2, ±1.9j. These canceling pole-zero pairs represent the observer poles that are selected at s = -15 and s = -25. Then the overall system $G_0(s)$ and the controller are

$$G_0(s) = \frac{76.1}{(s+10)(s^2+4s+7.61)}$$
$$B_a(s) = 38.06/(1+s/15)(1+s/25)$$
$$B_c(s) = 8.8718s^2 + 19.8394s + 38.05$$
$$A_c(s) = 0.0027s^2 + 0.1433s + 235678$$

Finally, let us design a controller for the CDM. The standard form of $\gamma_i = [\gamma_1\gamma_2\gamma_3\gamma_4]$=[2.5 2 2 2] with the equivalent time constant $\tau = 1.08255$ meets our constraint. Imposing the zero steady state error condition, we have

$$G_0(s) = \frac{1681.9012}{s^5 + 18.4758s^4 + 170.6767s^3 + 788.3449s^2 + 1820.6581s + 1681.9012}$$
$$B_a(s) = 840.9506$$
$$B_c(s) = 317.233s^2 + 410.7345s + 840.9506$$
$$A_c(s) = s^2 + 18.2258s + 159.8703$$

## 3.2 Robustness comparisons of 4 classical controller

We will compare the actuating signal of different controllers. Figure 2 and Table 1 shows that CDM controller has minimum control input energy under the constraint that they have the same settling time.

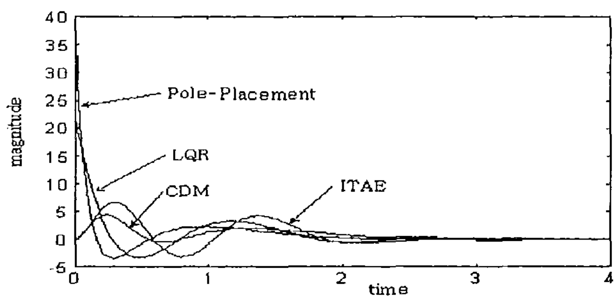



Figure 2 The actuating signal of different controller

Table 1 The comparison of control input energy

| | Maximum | Minimum | Energy |
|---|---|---|---|
| ITAE | 6.5638 | -3.2115 | 28.1171 |
| LQR | 22.1811 | -3.3555 | 42.2141 |
| Pole placement | 38.0025 | -3.4096 | 51.9248 |
| CDM | 4.2703 | -0.4358 | 6.7057 |

Next, the parametric stability margin of four controllers is supposed to compared with. To begin with, define the $l_2$-stability margin in parameter space is defined as the radius, $\rho(x^o)$, of the largest ball centered at $x^0$ for which the characteristic polynomial $\triangle(s, x)$ remains stable. That is,

$$\triangle_{\rho}(s,x) := \triangle(s, x^0 + \triangle x) : \| \triangle x \| \leq \rho$$

where $x$, $x^0$, $\triangle x$ are uncertain parameter vector, nominal vector, and perturbation vector respectively. The computation algorithm of $\rho$ is referred to [3]. In case of controller parameter perturbation, the parametric $l_2$-stability margin of each controller is listed in Table 2. Now the contrary case is considered, namely, a fixed controller and plant parameters entering perturbation. The results in Table 3 are obtained by the same procedure except replacing the perturbation variables by plant parameters. As shown in Table 2, Table 3, the CDM design gives the maximum tolerance to controller coefficient and the second robustness to plant variation.

Table 2 $l_2$-stability margins of controller parameter variation

| Methodology | $\rho$ | $\rho / \| x^0 \|_2$ |
|---|---|---|
| ITAE | 9.488028476869520e-001 | 1.293819255720045e-003 |
| Quadratic Optimal | 7.142205753258777e+000 | 7.349394397574021e-004 |
| Dominant Pole-Placement | 3.288950332884828e-003 | 7.492888956583186e-005 |
| CDM | 2.569208104375958e+000 | 2.566093205241946e-003 |

Table 3 $l_2$-stability margins of plant perturbation

| Methodology | $\rho$ | $\rho / \| x^0 \|_2$ |
|---|---|---|
| ITAE | 4.758965755071945e-001 | 7.164241555938469e-002 |
| Quadratic Optimal | 8.863466296528300e-001 | 1.334323818228192e-001 |
| Dominant Pole-Placement | 5.381273924304336e-001 | 8.101076632312598e-002 |
| CDM | 7.264996602914076e-001 | 1.093687016152147e-001 |

To analyze the root sensitivity with respect to plant perturbation, the Edge Theorem [3] is applied to four control systems above. Figure 3 and Figure4 show the whole root region and their damping characteristics. Figure 4 represents the step response of four control systems. The best transient behavior is shown by the CDM.

Table 4 Comparison of Controller performance

| Performance factor | Comparison |
|---|---|
| Min | ITAE < LQR < CDM < Pole-place |
| $M^{\omega}{}_{n}ax$ | ITAE < CDM < LQR < Pole-place |
| Stability degree | LQR ITAE < Pole-place CDM |

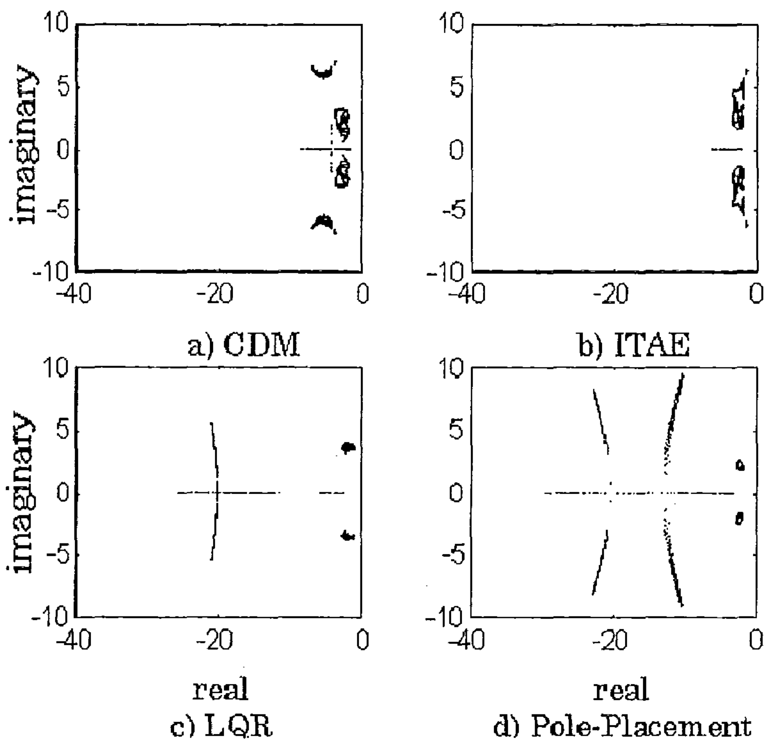



Figure 3. The root set of different control system

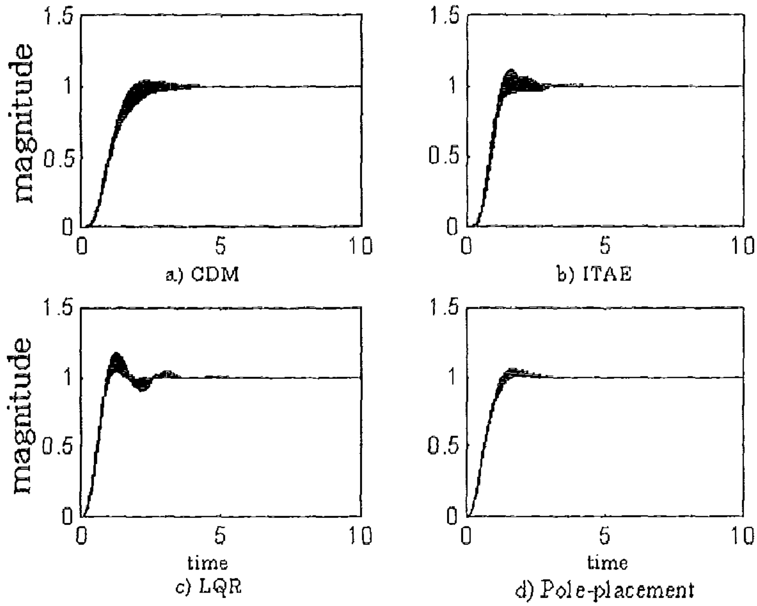



Figure 4. Step responses of 4 control system

We will examine stability margin in the frequency domain relative to parameter uncertainty. Figure 5 shows Nyquist envelopes of 4 design methods. It is shown that the dominant pole placement gives us the largest gain/phase margins and the CDM has the second phase margin and third gain margin.

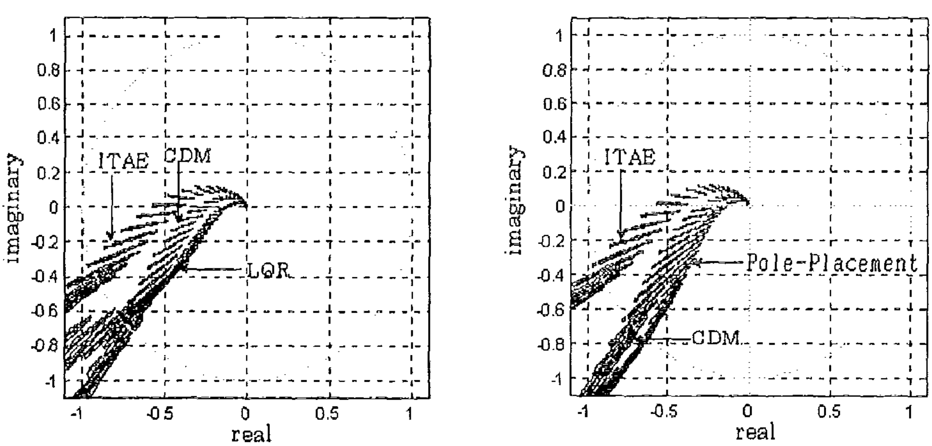



Figure 5. Relative stability from the Nyquist Plot

# 4. Comparison of CDM and $H^{\infty}$ Design.

We now attempt to compare the robustness of the CDM controller with those of $H^{\infty}$ controller. Consider the following non-minimum phase plant

$$P(s)=\frac{B_p(s)}{A_p(s)}=\frac{s^3+s62+0.5s+5}{sw^4+0.62s^3+7.351s^2+0.7269s+6.3046}$$

We first find a feedback controller such that will meet the following specification:

(i) The overshoot to the step response must be lesser than 5%

(ii) The settling time of 2% shall be with in 15 sec.

(iii) The resulting system has good balance between the sensitivity function and complementary sensitivity function

(iv) If it has lower order, it becomes more acceptable. Any control configuration can be permitted

An $H^{\infty}$ controller obtained is as follows :

$$B_a(s)=1$$

$$B_c(s)=-0.0081s^5+5.8165s^4+3.5495s^3+42.8794s^2+4.1798s+36.9138$$

$$A_c(s)=s^6+6.609s^5+29.0048s^4+59.4159s^3+87.7597s^2+84.6595s+0.0008$$

Let us design a controller for the CDM.

Stability index $\gamma_i=[\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6]=[2.5354\ 1.953\ 1.4241\ 1.7349\ 2.2404\ 2.2434]$ and the equivalent time constant $\tau=4.8723$ meets the design specification. Imposing the zero steady state error condition, we have

$$B_a(s)=0.14$$

$$B_c(s)=-0.25s^3+0.5s^2+0.14$$

$$A_c(s)=s(0.025s^2+0.55s+1)$$

Figure 6 shows that both controllers satisfy the settling time condition. However, CDM controller has less control input signal as shown in Figure 7.

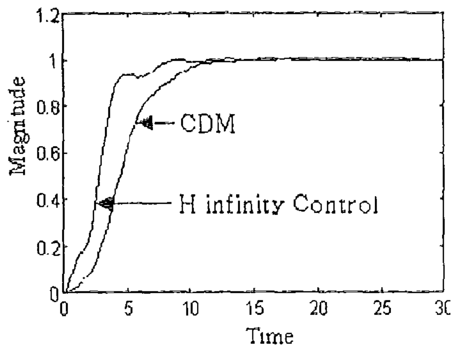


Figure 6. Step response

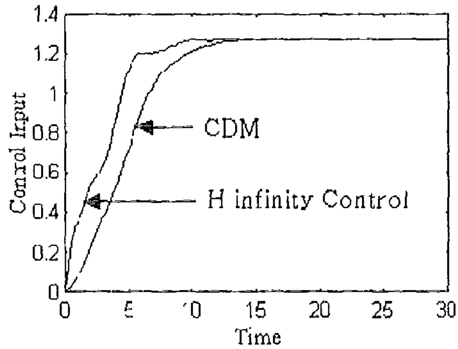


Figure 7. Control input Signal

In case of controller parameter perturbation, the parametric $l_2$-stability margin of each controller is listed in Table 5. The results in Table 6 are $l_2$-stability margin with respect to plant perturbation. As shown in Table5, Table6, the CDM design gives more tolerance to controller coefficient and plant variation.

Table 5 $l_2$-stability margins of controller parameter variation

| Methodology | $\rho$ | $\rho/\lVert x^0 \rVert_2$ |
|---|---|---|
| CDM | 1.400000000000000e-001 | 9.911333661631940e-002 |
| $H^\infty$ controller | 7.155716447697677e-001 | 4.765961094185456e-003 |

Table 6 $l_2$-stability margins of plant perturbation

| Methodology | $\rho$ | $\rho/\lVert x^0 \rVert_2$ |
|---|---|---|
| CDM | 4.458473000223930e-001 | 4.013496998436312e-002 |
| $H^\infty$ Controller | 5.213362919896760e-002 | 4.693045450698904e-003 |

The sufficient condition for Hurwitz stability of polynomial is given by $\gamma_i > 1.12\gamma_i^*$[6], In CDM, $\gamma_i$,, and $\gamma_i^*$ are as follows:

$\gamma_i$ = [2.5354 1.953 1.4241 1.7349 2.2404 2.2434]

$\gamma_i^*$ =[0.5152 1.0966 1.0884 1.1486 1.0221 0.4464

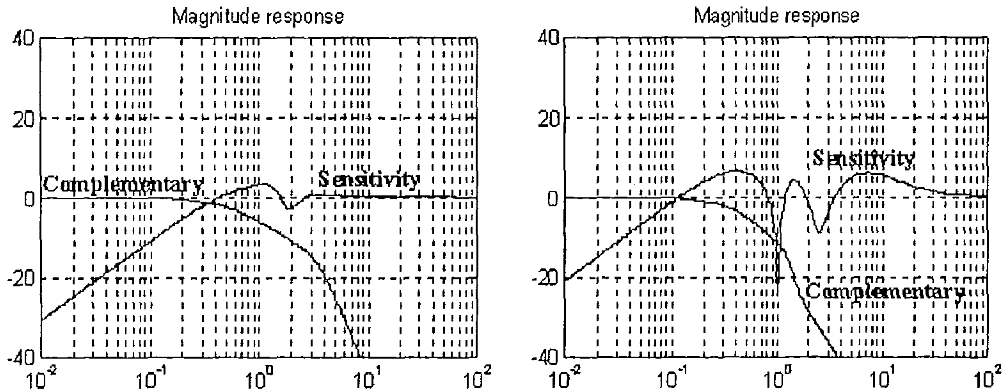
In an $H^\infty$ controller, $\gamma_i$ and $\gamma_i^*$ are as follows :

$\gamma_i$ = [2.0649 1.1518 1.4965 1.2779 1.3178 1.4082 1.2117 1.7076 1.2921]

$\gamma_i^*$ = [0.8682 1.1525 1.6507 1.4271 1.4926 1.5814 1.2958 1.5992 0.5856]

The CDM controller satisfies the sufficient condition but $H^{\infty}$ controller does not. With a viewpoint of parametric perturbation, CDM controller is more robust than $H^{\infty}$ controller.

We will investigate that the resulting system has good balance between the sensitivity function and complementary sensitivity function. As shown in Figure 8, $H^{\infty}$ controller has good balance than that of CDM.




(a) $H^{\infty}$ controller (b) CDM controller

Figure 8. Sensitivity and complementary sensitivity function.

## 5. Conclusion

Some robustness characteristics of CDM have been investigated by comparative studies. In this paper we test the 3rd order system. It can be extended to any order SISO system. We considered two cases. The first is to compute the CDM with other classical design method such as ITAE, LQR, and dominant pole-placement. It was applied to a third order interval plant. In the second case, we compare the CDM with an $H_{\infty}$ design in several examples. Robustness items for comparisons were i) parametric stability margin, ii) sizes of root region with respect to parametric uncertainty, iii) stability margins in frequency domain, and iv) total amount of control input energy as well as the maximum magnitude of input signals. As a result, we showed that the CDM is in general more robust than some other design methods

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