연구논문집 제8권 제1호 p.55~60 1977

A Logic of Evaluating Velocity Field Around a Floating Body in a Perfect Fluid

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〈**Abstract**〉

A logic for programming to predict velocity field around a moving body with free surface is described. The nonlinearity of the free surface effect is reasonably well coped with the logic. The mathematical properties of the problem to economize the computer time are discussed at the relevant places. A detailed propramming technique is not presented.

完全 流體에서 浮遊體 주위의 速度場의 計算에 關하여

이 동 기
조 선 공 학 과

〈요 약〉

自由表面이 있는 流體에서 物體가 움직일 때 物體 周圍의 速度場을 評價하기 위한 Program 構成의 論理를 記述했다. Bernoulli 定理에 의한 自由表面의 非線型性은 比較的 合理的인 수학적인 性質과 연관하여 論議했고 구체적인 Programming은 記述되지 않았다.

Ⅰ. Introduction

The prediction of velocity field around a moving body is practically a very important question. The design as well as optimisation of a ship hull or an aircraft fuselage, or even a lifting surface such as aerofoil, are direct applications of the theory related with the question. The essence of the question may be stated as the determination of the singularity strength to represent the given body satisfying all the necessary field equations and boundary conditions.

More specifically, the problem of a non-lifting body, which is characterised by the non-existence of circulation, can be divied into two groups, namely a floating body and an immersed body. The floating body theory which is seemingly not very different from the other one demands its own distinction because of the tremendous difficulty to deal with non-linear characteristics of the free surface effect.

Since Michell published his thin ship theory in 1898, the theory has been focused a glorious light by a number of illustrious investigators. Among them, Havelock demonstrated, for the first time, how a body could be replaced by a few number of singlarities in 1943. However the development of this promising method had remained in a rather implicit form on account of the enormous amount of calculation involved until quite recent time — that is until the introduction of the computer.

With the development of the modern high-speed computer, a much more wide open possibility has been offered in the related field of interest. The first remarkable achievement was made by Hess & Smith in 1963. One regret of their work is that there is no free surface or anything corresponding to the free surface effect in their problem. This excellent formulation is still not really superseded in its own category of the problem although a few investigators here and there claim an improvement.

In 1976, Gadd showed a method to introduce the free surface into the problem. His contribution is notable especially in connection with tackling nonlinear governing equations of free surface effect since traditionally, in most cases, a linearised free surface boundary condition has been employed. This may be particularly said so from the point of view that it is generally agreed that the linearisation of the free surface boundary condition creates a considerable discrepancy between the prediction by theory and the experimental observation.

The logic of evaluating velocity field given in this note is to show how one can tackle the problem by the help of a computer. The definition of the body is to be a number of input points which constitute accordingly so many number of panels on the body surface. So eventually the body is approximated by a facetted one. Source panels are also distributed on the undisturbed water surface. Consequently the wave profile created by the body is imaginary: that is cutted crest and filled trough by the source panels. The detailed programming information is largely omitted since that is not the purpose of the description of the logic.

Ⅱ. Mathematical Properties of the Problem

In a perfect fluid a moving body does not create a circulation and hence the velocity potential can be defined. If the velocity potential is denoted by ϕ, ϕ satisfies Laplace's equation. If the possible cavitation is excluded, the continuity equation should also be applicable throughout the fluid. These two field equations are automatically fulfilled by any singularity. Inmathematical term, the above two equations are:

$$\nabla^2\phi=0 \quad (2.1)$$

$$\nabla\cdot v=0 \quad (2.2)$$

The boundary conditions to be satisfied are stated below.

(1) Hull boundary condition

$\phi_n=0$ on the hull surface (2.3)

where n denotes the normal derivative to the hull surface.

(2) Kinematic free surface boundary condition

$\phi_n=0$ on the free surface (2.4)

where n denotes the normal derivative to the free surface.

(3) Dynamic free surface boundary condition

$$\frac{1}{2}U^2=\frac{1}{2}(u^2+v^2-w^2)+g\zeta \quad (2.5)$$

This condition states that Bernoulli's equation is to be applied along the free surface where the atmospheric pressure is taken zero.

(4) The radiation condition

This condition specifies that there is no wave propagation ahead of the body.

(5) Bottom condition

When there is solid boundary at the bottom, no velocity across it should occur and if the fluid is infinitely deep the disturbed velocity approaches zero with the distance from the surface.

Since source panels are to be distributed on the undisturbed water surface, it is necessary to modify the dynamic free surface condition so that it can be directly applicable on the undisturbed flat water surface. Although it may not be absolutely true, a linear variation of velocity component along the vertical direction seems to be a reasonable approximation.

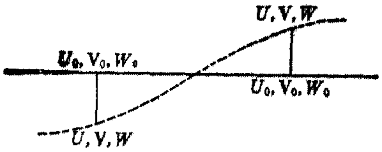

Fig. 1

Hence, $u=u_0+\zeta\left(\frac{\partial u}{\partial z}\right)_0$, $v=v_0+\zeta\left(\frac{\partial v}{\partial z}\right)_0$,

$$w=w_0+\zeta\left(\frac{\partial w}{\partial z}\right)_0 \qquad (2.6)$$

where z direction is chosen vertically upward and the small circular subscript means that the values are evaluated at the undisturbed flat surface.

Substituting the equations (2.6) into (2.5), we have

$$\frac{1}{2}U^2=\frac{1}{2}\left[\left\{u_0+\zeta\left(\frac{\partial u}{\partial z}\right)_0\right]^2+\left[v_0+\zeta\left(\frac{\partial v}{\partial z}\right)_0\right]^2+\left[w_0+\zeta\left(\frac{\partial w}{\partial z}\right)_0\right]^2\right\}+g\zeta$$

or by rearrangement

$$\zeta=\frac{1}{2g}\left\{U^2-\left[u_0+\zeta\left(\frac{\partial u}{\partial z}\right)_0\right]^2-\left[v_0+\zeta\left(\frac{\partial v}{\partial z}\right)_0\right]^2-\left[w_0+\zeta\left(\frac{\partial w}{\partial z}\right)_0\right]^2\right\} \qquad (2.7)$$

On the other hand, if we apply the continuity condition to a small piece of wave profile, we get

$$w_0=\frac{\partial}{\partial x}\left\{\left[u_0+\frac{\zeta}{2}\left(\frac{\partial u}{\partial z}\right)_0\right]\zeta\right\}+\frac{\partial}{\partial y}\left\{\left[v_0-\frac{\zeta}{2}\left(\frac{\partial v}{\partial z}\right)_0\right]\zeta\right\}=WO. \qquad (2.8)$$

The last equation (2.8) expresses the upward velocity magnitude required by the Bernoulli equation and the continuity equation is denoted specifically by WO

Ⅲ. The Formation of Matrices

1. Formation of Source Panels

The body is defined by a number of points which constitute the input data to the computer and which create a series of source panels eventually. The undisturbed water surface is also divided into a number of panels by the similar procedure to that for the hull.

The number of source panels on the hull: N_h
The number of source panels on the surface: N_s

We assume that on each source panel a uniform source strength is distributed. In fact this source strengths are the unknowns and we try to decide them satifying all the conditions described previously. Once the source strengths are decided it is straightforward to calculate velocity components at an arbitrary point outside the body.

At the same time as the formation of source panels, we specify a control point on each panel which will represent the panel in the subsequent calculation.

2. Formation of Influence Coefficient

Assuming a unit source strength on each panel in turn, the velocity components u, v, w and the velocity gradient components $\frac{\partial u}{\partial z}$, $\frac{\partial v}{\partial z}$, $\frac{\partial w}{\partial z}$ are calculated at each control point. For convenience, these whole sets of numbers are grouped as the below and denoted U, V, W matrix and z^U, z^V, z' matrix.

velocity and velocity gradient reduced at		due to a unit source strength on		U, V, W matrices			U, V, W matrices		
Hull panel	i	Hull panel	j	U^1_{ij}	V^1_{ij}	W^1_{ij}	${}_zU^1_{ij}$	${}_zV^1_{ij}$	${}_zW^1_{ij}$
Hull panel	i	Surface panel	m	U^2_{im}	V^2_{im}	W^2_{im}	${}_zU^2_{im}$	${}_zV^2_{im}$	${}_zW^2_{im}$
Surface panel	m	Hull panel	i	U^3_{mi}	V^3_{mi}	W^3_{mi}	${}_zU^3_{mi}$	${}_zV^3_{mi}$	${}_zW^3_{mi}$
Surface panel	m	Surface panel	n	U^4_{mn}	V^4_{mn}	W^4_{mn}	${}_zU^4_{mn}$	${}_zV^4_{mn}$	${}_zW^4_{mn}$

These matrices are calculated only once and stored for the later use. This is possible owing to the linearity of the velocity potential due to a source.

3. Coefficient Matrices

Once influence coefficient matrices are formed, it is a straightforward manipulation to form the coefficient matrices which comprises the induced normal velocity at each control point by each panel. We denote these matrices as a matrices. More explicitly, for instance, a component of A matrix is calculated as the following. Suppose i-th control point at which the normal vector to the panel is n_i. Surppose the compponts of induced velocity at this point by j-th panel is v_{ij}. Then the induced normal velocity is

$$n_i \cdot v_{ij} = A_{ij}. \quad (3.3.1)$$

Again for convenience, these matrices are grouped as the following table:

Normal velocity induced at	due to a unit source strength on	Matrices
Hull panel i	Hull panel j	A^1_{ij}
Hull panel i	Surface panel m	A^2_{im}
Surface panel m	Hull panel i	A^3_{mi}
Surface panel m	Surface panel n	A^4_{mn}

4. Reflected Upward Velocity Coefficients

To minimize the number of equations we are dealing with, it is necessary to reduce the number of unknows as much as possible. Fortunately the variation of source strength on the hull panels can be replaced by that of surface panels so that they satisfy the hull boundary condition at every stage of change in the source strengths on the surface panels. Here we fully appreciate the linearity of the velocity field created by the distributed sources. In mathematical sense, the source strengths on the surface panels may vary independently but the source strengths on the hull panels are not free to vary, in other words, dependent by the condition that they should satisfy the hull boundary condition. However, this argument is not absolutely true. We may take the hull source strengths as independent variables and the surface source strengths as dependent variables satisfying the dynamic free surface boundary condition at every stage of the alteration of source strengths and not satisfying the tangential velocity condition on the hull surface. But this procedure would be much more complicated to handle compared to the former one. Since it is true that one set of the source strengths is independent and the other dependent, we choose the surface source as the independent variable for the sake of simplicity.

On the basis of this argument, the induced upward velocity at the control point of m-th surface panel due to the unit source strength on n-th surface panel and its reflected source strengths (image sources) at every hull panel creates the reflected upward velocity coefficient matrix. This matrix is called ν-matrix and may be formed by the following procedure.

a. Set zero source strength for every surface panel except n-th surface panel.
b. Set zero free stream velocity.
c. Calculate source strength on the hull panels to satisfy the hull boundary condition for the velocity field created by the single n-th surface panel. This set of source strengths on the hull composes the image sources whose strength on i-th hull panel is denoted λ_{in}.
d. Calculate the induced upward velocity at m-th surface panel due to the unit source strength on n-th surface panel and its image sources. If we denote this induced upward velocity by ν_{mn} then

$$\nu_{mn} = W^4_{mn} + \sum_{i=1}^{N_h} \lambda_{in} W^3_{mi} \quad (3.4.1)$$

Where $W^4_{mn} = 2\pi$ when $m = n$

$W^4_{mn} = 0$ when $m \neq n$.

Ⅳ. Iteration

The physical property of the boundary conditions, especially the dynamic free surface boundary condition, suggest that it is very unlikely to get the solution of the whole variable by just one go. Hence the iteration procedure may be inevitable and this logic is based on that scheme. Since we try to decide source strengths on the surface panel, it is natural to choose some physical quantity there on which the whole iteration process will be based. Two possibilities are open — one is the variation of the surface source strength itself and the other is the discrepancy between the required upward velocity (eq. 2.8) and the calculated upward velocity at the same control point. Although both are almost identical, the latter appears to be of the more current interest.

1. Initial Value

To satisfy the hull boundary condition in the fluid field created by the free stream there should be a set of source strengths on the hull panels. This set of source strengths on the hull panels with zero source strengths on the surface panels can be used as the initial values for the iteration. In this case there is an initial wave profile created by the sources on the hull panel. The other alternative is a set of source strengths to satisfy the hull boundary condition and flat water surface condition. In this case, the source strength on the surface would not be zero.

2. k-th Stage of Iteration

At this stage, the state of the variables and the necessary procedure are described below.

a. Source strength on the i-th hull panel: ${}_H\sigma_i{}^k$
Source strength on the n-th surface panel: ${}_s\sigma_n{}^k$

b. Calculation of the induced upward velocity WS at the m-th surface panel due to these sets of source strengths.

$$WS_m^k=\sum_{n=1}^{N_s}W^4{}_{mn\,s}\sigma_n^k+\sum_{i=1}^{N_h}W^3{}_{mi\,H}\sigma_i^k \qquad (4.2.1)$$

c. Calculation of u_0, v_0, w_0, $\left(\frac{\partial u}{\partial z}\right)_0$, $\left(\frac{\partial v}{\partial z}\right)_0$, $\left(\frac{\partial w}{\partial z}\right)_0$ and then ζ from equation(2.7) for each surface panel.

d. Calculation of WO^k from equation (2.8) using the above result for each surface panel. The value at m-th surface panel is denoted by $WO_m{}^k$

e. Imposing a small increase of source strength on n-th surface panel only (with its image sources), calculate the change of WO^k at m-th surface panel and hence $(\partial WO_m/\partial_s\sigma_n)^k$ which comprises μ-matrix.

Unfortunately the μ-matrix may not straightforwardly be calculated as the ν-matrix since the components in μ-matrix are not the linear function of the source strengths.

3. (k+1)-th Stage of Iteration

If the change of source strengths occurs, the calculated and the required upward velocity at m-th surface panel may be expressed as the followings respectively.

$$\begin{aligned}WS_m^{k+1}&=WS_m^k+\Delta WS_m\\&=WS_m^k+\sum_{n=1}^{N_s}W^4{}_{mn}\Delta_s\sigma_r+\sum_{i=1}^{N_h}W^3{}_{mi}\Delta_H\sigma_i\\&=WS_m^k+\sum_{n=1}^{N_s}\left(W^4{}_{mn}+\sum_{i=1}^{N_h}W^3{}_{mi}\lambda_{in}\right)\Delta_s\sigma_r\\&=WS_m^k+\sum_{n=1}^{N_s}\nu_{mn}\Delta_s\sigma_n\end{aligned} \qquad (4.3.1)$$

As mentioned earlier, since the surface source strengths are taken independent variable and the change of the hull source strength is replaced by that of the surface source strenghth as in equation(4.3.1), the following expression is valid.

$$dWO=\frac{\partial WO}{\partial_s\sigma_1}d_s\sigma_1+\frac{\partial WO}{\partial_s\sigma_2}d_s\sigma_2+\cdots+\frac{\partial WO}{\partial_s\sigma_{N_s}}d_s\sigma_{N_s}$$

The only restriction is that every $d_s\sigma_i$ should be small as obvious from the differential calculus.

$$\begin{aligned}WO_m^{k+1}&=WO_m^k+\sum_{n=1}^{N_s}\left(\frac{\partial WO_m}{\partial_s\sigma_n}\right)^k\Delta_s\sigma_n\\&=WO_m^k+\sum_{n=1}^{N_s}\mu_{mn}^k\Delta_s\sigma_n\end{aligned} \qquad (4.3.2)$$

We try make $WS_m^{k+1}=WO_m^{k+1}$ so equating eq. (4.3.1) and eq. (4.3.2), we get

$$WS_m^k+\sum_{n=1}^{N_s}\nu_{mn}\Delta_s\sigma_n=WO_m^k+\sum_{n=1}^{N_s}\mu_{mn}^k\Delta_s\sigma_n \qquad (4.3.3)$$

$$\sum_{n=1}^{N_s}(\nu_{nm}-\mu^k_{nm})\ \Delta_s\sigma_n = WO^k_m - WS^k_m \qquad (4.3.4)$$

Eq. (4.3.4) is a linear simultaneous equation with N_s unknown values of $\Delta_s\sigma$. As there are as many numbers of equations it is straightforward to solve. The new source strength are as the followings:

a. On the hull panel

$$_H\sigma_i^{k+1} = {}_H\sigma_i^k + \sum_{n=1}^{N_s}\lambda_{in}\Delta_s\sigma_n \qquad (4.3.5)$$

b. On the surface panel

$$_s\sigma_m^{k+1} = {}_s\sigma_m^k + \Delta_s\sigma_m \qquad (4.3.6)$$

With these new sets of source strengths, the iteration would be repeated until all the discrepancies between WS_m and WO_m become satisfactorily small.

Ⅴ. Conclusion

It is generally agreed that the linearised hull boundary condition and the linearised free surface boundary condition are hardly justifiable in the field of the ship hydrodynamics. Especially with the introduction of extremely full form ships such as tankers, the prediction of the linearised theory has been proved very poor. Not only the blunt geometrical shape of the bow can be hardly linearised but the velocity field created around that area is also highly nonlinear.

The logic described in this note has its aim to cope this shortcoming. By the distribution of the source panels on the body surface, the linearisation of the geometry of the body is, at least theoretically, avoided. The application of the Bernoulli equation on the water surface means that we are trying to take the nonlinearity of the free surface into account, with reservation that, of cource, how precisely it is applied.

Since the problem itself is extremely large, if perhaps not prohibitively, even for the modern high speed computer, it is essential how to make the programme as much efficient as possible. This logic is a possibility for that purpose. A similar procedure may be found from Hess and Smith's work for an immersed body and Gadd's work for a floating body with slightly different notion.

Nomenclature

ϕ : velocity potential

U : the free stream velocity

u, v, w: the local x, y, z component of velocity respectively

g : the acceleration due to gravity

ζ : the wave height

N_h : the number of panels on the body

N_s : the number of panels on the free surface

$_H\sigma$: the source strength on the body panel

$_s\sigma$: the source strength on the surface panel

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