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Effort Level, Layoffs and Procyclical Productivity

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<Abstract>

This paper shows that productivity can move procyclically over business cycle while the previous implicit contract and efficiency wage models incur countercyclical movement which is opposite to the empirical results. It also shows layoffs are possible even when there is work sharing, and layoffs occur in order of effort level or efficiency unit of labor. Moreover, this layoff pattern strengthens our arguments about procyclical productivity movement. If effort level increases by seniority this results implies that layoffs occur by seniority rule. This result also can be applied to explain high unemployment rate in young, woman, or minor ethnical worker groups. finally, the basic model was extended to derive self-enforceable contract when the contract is not legally enforceable.

노력의 수준이 해고의 순서 및 생산성에 미치는 영향

이재기
경제학 전공

<요 약>

본 논문은 노동시장에서 관찰되는 비자발적 실업과 해고의 존재를 노동계약모형을 통하여 설명하는 동시에, 생산성이 경기변동에 따라 순경기적으로 변화하는 현상을 설명하려고 하는 시도이다. 대부분 동실적 노동을 가정하고 있는 노동계약모형의 경우 노동시간공유(work-sharing)가 가능하다면 해고 및 자발적 실업의 존재를 설명할 수 없다. 본 연구에서는 노동이 관찰가능한 비동질성을 보유하고 있는 경우 노동시간공유의 가능성 여부에 관계

없이 해고 및 비자발적 실업이 존재할 수 있다는 것을 보인다. 한편 암묵적 계약모형과 효율임금모형 모두 생산성이 순경기적(procyclical)으로 변화한다는 경험적인 사실을 설명하지 못하고 오히려 경험적 사실과 반대되는 예측을 하고 있다. 본 논문은 노동계약모형에 효율임금모형의 요소인 노력수준을 명시적으로 도입할 경우 순경기적인 생산성의 변화를 설명할 수 있음을 밝히고 있다.

I. Introduction

The early implicit contract model attempted to explain real wage rigidity as the outcome of a risk shifting agreement between risk neutral firms and risk averse workers.[1] In these literature that there is homogeneous labor, however, effort is exogenously given; thus the only reason for inefficiencies in the allocation of labor stem from imperfect attempts at risk sharing between firms and workers.

On the other hand, recently there has developed a new class of models - efficiency wage models - which can explain several stylized facts characterizing business cycles.[2] In this paper, by using a simple contractual structure with explicit consideration of worker's effort we explain several economic phenomena which can not be explained by contract and efficiency wage models.

In most contract literature assuming homogeneous labor, if we assume work sharing is feasible the existence of layoffs or involuntary unemployment can not be explained (Cooper, 1987). This paper shows that there can exist layoffs regardless of the feasibility of work sharing when labor is composed of various observationally different groups. Neither efficiency wage models like Shapiro and Stiglitz(1984) nor implicit contract models can explain the empirical results that productivity changes procyclically. Predictions in those models are the opposite to the empirical results. This paper shows that productivity can move procyclically in a contract model with explicit consideration of effort level. Finally this paper tries to construct a self-enforceable contract in which non-binding agreement is incentive compatible, when the third party (courts) can not make the agreement legally enforceable. The feature of the model presented here that distinguishes it from earlier work on implicit contract theory is that it explicitly considers worker's effort level which comes from the spirits of efficiency wage models.

The following section presents the basic model and its implications for the behaviors of effort level, productivity, real wage and employment level. Section III discusses the basic model with heterogeneous labor which is distinguishable in effort levels, and shows that during economic downturn layoffs occur from the group with the lowest

1 See Azariadis(1975) and Baily(1974) For surveys of the more recent work in implicit contract theory, see Rosen(1985) and Cooper(1987).

2 Efficiency wage models are all based on a convincing explanation as to why firms may find it unprofitable to cut wages in the presence of involuntary unemployment. For the details, see Weiss(1980), Shapiro and Stiglitz(1984) and Katz(1986).

effort level. This result implies that work sharing is feasible within a group but not inter groups; therefore, layoffs are possible even if there exists work sharing. Section IV constructs a self-enforcing contract by extending the basic model. Final section presents a summary and conclusion.

II. The Basic Model

Consider a contracting process between a risk neutral firm and a group of n homogeneous workers. The firm uses only labor as inputs to produce a single output according to production technology

(1) $y(s) = f(e(s)q(s)n)$,

where

s = state of nature
e(s) = effort level of employed worker in state s
q(s) = probability of employment in state s.

Production function $f(\cdot)$ is twice differentiable, concave, non-decreasing such that $f(0)=0$, $\lim n \to 0\ f'(n)=\infty$ and $\lim n \to \infty\ f'(.)=0$. The output market is competitive and output price p(s) depends on the prevailing state of nature. Firm is assumed to maximize expected profit.

For the homogeneous workers, when employed, utility level is measured by

(2) $U = u(w(s))-v(e(s))$

where w(s) is wage level in state s.[3] It is also assumed that $u'>0$, $u''<0$, $v'>0$, $v''>0$ and $v(0)=0$. If a worker is laid off, his utility is $u(k)-v(0)$ where k is wage level in a competitive labor market or his value of leisure. Assume that state s are ex post observable to both agents and effort level e(s) is also observable to all parties of contract including the third party (courts). At this stage, there is no problem of enforceability or incentive compatibility.[4]

The optimal contract $d^* = \{w^*(s), q^*(s), e^*(s)\}$ can be characterized by the solution of the following problem:

3 Separable utility function is assumed. Even if it is very restrictive this type of function makes our analysis accessible and does not change the basic properties of our argument.

4 If we admit private information, we need to analyze a model with two sided private informations. Firm can have a private informations about state of nature and workers can have those about effort level.

Maxd Es $q(s)[u(w(s))-v(e(s))]+(1-q(s))u(k)$

subject to

Es $p(s)f(e(s)q(s)n)-q(s)nw(s) > 0$,

$0 < q(s) < 1$.

The objective function is a representative worker's valuation of state dependent utility and the first constraint ensures that firm is willing to participate in the contract. The first-order conditions for all s are:

(3) $u'(w(s)) = m_1 n$

(4) $u(w(s))-v(e(s))-u(k) = m_1 n[w(s)-p(s)e(s)f'(e(s)q(s)n)]+m_2$

(5) $v'(e(s)) = m_1 np(s)f'(e(s)q(s)n)$

where m_1 and m_2 are Lagrange multipliers. The equation (3) implies optimal risk sharing between agents. If a worker is employed his income is independent of state and effort level.[5]

By combining (3) and (4) we can obtain

(6) $p(s)e(s)f'(e(s)q(s)n) = w(s)-[u(w(s))-v(e(s))-u(k)]/u'(w(s))+m_2/m_1 n$.

By using (6), we can show the well-known result in contract model that employment level is higher than that in a competitive labor market.[6] Effort level which is related with productivity is determined by condition (4) and (5), and productivity can be procyclical.

Proposition 1: Effort is independent of p(s) when $q(s)<1$, but it is increasing when $q(s)=1$.

proof: If $q(s)<1$, from (3) and (4)

$m_1 np(s)e(s)f'(e(s)q(s)n) = u'(w*)w*-[u(w*)-v(e(s))-u(k)]$.

By combining (5) and the above,

$e(s)v'(e(s)) = u'(w*)w*-[u(w*)-v(e(s))-u(k)]$.

Effort level is, therefore, independent of p(s) when $q(s)<1$.

On the other hand, if $q(s)=1$, from totally differentiating (5),

$de(s)/dp(s) = u'(w*)f'(\cdot)/[v''(\cdot)-p(s)q(s)nu'(w*)f''(\cdot)] > 0$. Q.E.D.

The above proposition says that effort increases, but not strictly, with p(s). The

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5 This result comes from the assumption of separable utility function.

6 To prove overemployment result, assume $q(s)<1$. Then $m_2=0$. Add $u'(w(s))k$ to both sides of equation (6); then, $u(w(s))\ v(e(s))-u(k)+u'(w(s))(k-w(s)) = u'(w(s))(k\ p(s)e(s)f'(\cdot))$. The strict concavity of $u(\cdot)$ proves the result. This overemployment result is discouraging if our major concern is to understand underemployment. We can make it less serious if we assume that firm maximizes the discounted value of future profit stream but workers maximize period by period utility because of capital market imperfection.

following figure shows the relationship between effort and p(s) when p* is the minimum price which guarantees full employment.

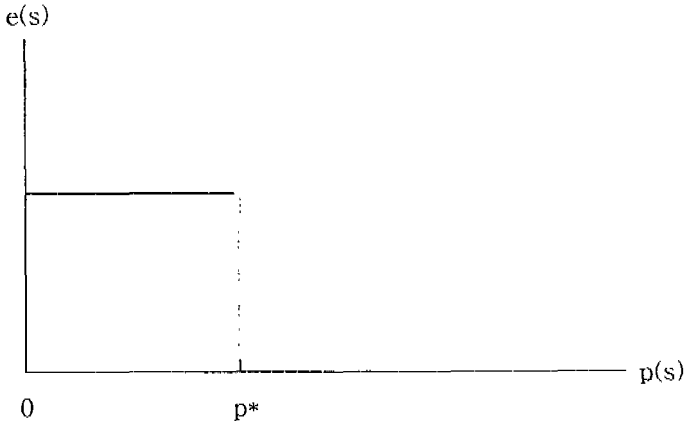

Figure 1: Effort Schedule

From the above figure, we can infer labor productivity initially declines as p(s) and q(s) increase as a result of diminishing marginal productivity of labor, but both productivity and output increase with p(s) when q(s)=1. Productivity can be higher in times of full employment than average productivity in times with unemployment.[7]

Proposition 2: It is not profitable for the firm to fix the minimum effort level of workers.

proof: The addition of constraint $e(s)>e_0$ changes (5) to the following:

$$v'(e(s)) = u'(w^*)p(s)f'(e(s)q(s)n) - m_3,$$

where m_3 is a Lagrange multiplier of the new constraint. Since m_3 is non-negative, effort with minimum constraint can not be higher than that without it. Q.E.D.

The intuition here is that since wage is independent of effort the imposition of minimum effort is always binding and aggravates productivity.[8]

7 The intuition behind this result is that employees usually work faster and more efficiently when there is a lot of work to do.

8 Another possible explanation of this result is that when workers have imperfect information about the firm's minimum effort requirement, the imposition of explicit criteria can lead to decreased worker effort. In this case, minimum effort level is a kind of signal to worker's uncertainty.

III. Heterogeneous Labor

In section II, we assumed that all workers had identical characteristics. We shall now treat a more general case in which differences in observable attributes among workers enable firms to partition the labor force (or union) into different types. Assume workers in different types only differ in their effort levels in each state. For simplicity, assume that there are two different types (or groups) composed of ni, i=1, 2, workers and e1(s)>e2(s) for all s where ei(s) is effort level of a worker in group i in state s.

The optimal contract, $d^* = \{w_1(s)^*, w_2(s)^*, q_1(s)^*, q_2(s)^*, e_1(s)^*, e_2(s)^*\}$, can be characterized by the solution of the following maximization problem:

$$\begin{aligned} \underset{d}{\text{Max}}\ Es\ & Bn_1\{q_1(s)(u(w_1(s))-v(e_1(s)))+(1-q_1(s))u(k)\} \\ & +(1-B)n_2\{q_2(s)(u(w_2(s))-v(e_2(s)))+(1-q_2(s))u(k)\} \end{aligned}$$

subject to

$$Es\ p(s)f(e_1(s)q_1(s)n_1+e_2(s)q_2(s)n_2)-q_1(s)n_1w_1(s)-q_2(s)n_2w_2(s) > 0,$$
$$0 \le q_1(s) < 1, \quad 0 \le q_2(s) \le 1,$$

where B>1/2 is the bargaining power of high effort group. In production process, two types of workers are assumed to be completely substitutable. The necessary first order conditions for the above problem are: for all s,

(7) $u'(w_1(s)) = m_1/B$

(8) $u'(w_2(s)) = m_1/(1-B)$

(9) $Bn_1\{u(w_1(s))-v(e_1(s))-u(k)\}+m_1\{p(s)e_1(s)n_1f'(.)-n_1w_1(s)\}-m_2 = 0$

(10) $(1-B)n_2\{u(w_2(s))-v(e_2(s))-u(k)\}+m_1\{p(s)e_2(s)n_2f'(.)-n_2w_2(s)\}-m_3 = 0$

(11) $Bv'(e_1(s)) = m_1p(s)f'(.)$

(12) $(1-B)v'(e_2(s)) = m_1p(s)f'(.)$,

where mi, i=1, 2, 3, is a Lagrange multiplier. Condition (7) and (8) imply optimal risk sharing between firm and each group of workers. Each worker in the same group has state and effort independent wage level, but high effort group will get higher wage since B>1/2, that is, high effort group usually have more bargaining power.

(13) $w_1 > w_2 \ge k$

This relationship is true because if workers of different types receive the same wage the firm would almost always make positive profits on one type of workers and those workers would be bid away by competing firms. In addition, if wages between two different groups are equal workers in high effort group can increase their utility levels

by moving to low effort group.

Proposition 3: Effort is independent of p(s) when $q_i(s)<1$, but it is increasing when $q_i(s)=1$.

proof: Apply the proof of proposition 1 to each group.

This proposition 3 intensifies our argument about procyclical productivity by being combined with the following proposition 4.

Proposition 4: The whole of group 2 will be laid off before a worker in group 1 is laid off.

proof: From condition (7) and (9), for $q1(s)>0$,

(14) $p(s)e_1(s)f'(.) \geq w_1-R_1$

where $R_1=\{u(w_1)-v(e_1(s))-u(k)\}/u'(w_1)$ and equality holds when $0<q_1(s)<1$. Similarly, from condition (8) and (10), for $q2(s)>0$,

(15) $p(s)e_2(s)f'(.) \geq w_2-R_2$,

where $R_2=\{u(w_2)-v(e_2(s))-u(k)\}/u'(w_2)$ and equality holds when $0<q_2(s)<1$. By (13) and strict concavity of u(.), (for proof, draw a diagram)

(16) $w_1-R_1 < w_2-R_2$.

We now need to show that if $0<q_1(s)<1$, $q_2(s)$ must be 0. Suppose that $0<q_1(s)<1$ and $q_2(s)=1$. Then (15) itself holds and equality holds in (14). By (16) and the assumption that $e_1(s)>e_2(s)$, however, this is impossible. Suppose $0<q_2(s)<1$. Then equality must hold simultaneously in (14) and (15). By (16) and assumption on effort, this case is also impossible. Suppose $q_2(s) = 0$. Then $p(s)e_2(s)f'(.) < w_2-R_2$. So, this case can hold. Q.E.D.

This result is similar with that of Weiss(1980). Some groups are excluded from employment even if the other groups are enjoying full employment. Groups will be hired in order of effort level or efficiency unit of labor. The following figure represents the layoff path.

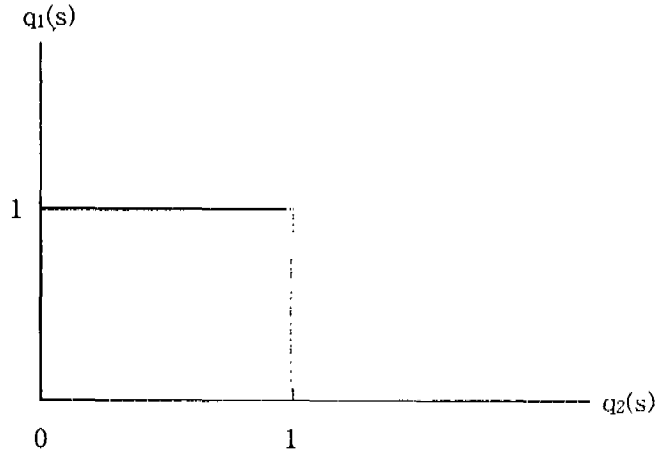

Figure 2: Layoff Path

The feasibility of work sharing can not eliminate the existence of layoffs since it is impossible for different groups to cooperate and share their working hours. In this model, layoffs or involuntary unemployment can exist. Moreover, by proposition 3 and 4, we can confirm our previous discussion about procyclical productivity. If both groups are fully employed productivity will increase with p(s) because of increased efforts. Even in times with unemployment, full employment of high effort group and their increasing effort with p(s) can compensate the decrease of productivity from partial unemployment of low effort group.

IV. Self-Enforceability

In the basic model, wage level is independent of effort level. This result is compatible with the assumption of Shapiro and Stiglitz(1984) that only contracts with the wage independent of performance are enforceable. In their paper the only incentive for good performance is the threat of dismissal and that is an effective threat only if keeping a job gives a worker strictly greater utility than losing it. In Shapiro and Stiglitz(1984), this can occur only if there is involuntary unemployment, otherwise a dismissed worker could get another job straightaway and suffer no loss of utility. In this section, we will construct a self-enforceable contract when the third party can not verify worker's effort. In order to do this, we need to consider a multi-period contract. Assume all workers are homogeneous and production function satisfies constant return to scale. Both firm and worker are infinitely lived. The possible legally enforceable contract is wage payments which depends on history of wage and employment. Denote the wage

paid to the worker in period t by wt and the effort level by et. Define a T-period contract d={wt,et}t=1. Let $\beta<1$ denote common discount factor. Then the present value of discounted profits of the firm from this contract is given by

(17) $P_1(d,T)=\sum_{t=1}^{\infty}\beta^{t-1}\{s_tf(e_t)-w_t\}+\beta^tP_t$,

where s_t is firm-specific shock in period t and P_t is the firm's present value of profits from T on if it does not employ this worker. Similarly, the worker's utility can be defined by

(18) $U_1(d,T)=\sum_{t=1}^{\infty}\beta^{t-1}\{w_t-v(e_t)\}+\beta^tU_T$,[9]

where UT is the lifetime utility of the worker from T on if not employed. Since our main concern is with the effect of information not verifiable by the third party, we assume for simplicity that information is symmetric between firm and worker. In addition, we assume that firm observes the effort level in period t before paying the wage in period t. Then the self-enforceable contract $d^*=\{wt^*,et^*\}t=1\infty$ is a subset of solutions for the following bargaining problem

Max $U_1(d,T)+P_1(d,T)$

subject to

(19) $P_1(d,T)\geq P_1$,

(20) $U_1(d,T)\geq U_1$.

Proposition 5: A contract, d^*, is self-enforceable if it satisfies (19), (20) and

(21) $\beta\{U_{t+1}(d^*,T)-U_{t-1}+P_{t+1}(d^*,T)-P_{t-1}\}>v(e_t^*)$.

proof: Condition (19) and (20) are necessary conditions for both agents to enter the contract. To show that (21) is also a necessary condition, let w_t be the lowest wage the firm can pay in period t which depends on the previous work history and so is legally enforceable. To prevent the worker from quitting,

(22) $U_t(d^*,T)\geq w_t+\beta U_{t-1}$ for $1<t\leq T$.

Note that $e_t=0$ if the worker wants to quit at the end of period t. Similarly, to prevent the firm from abrogating the contract,

(23) $P_t(d^*,T)>s_tf(e^*)-w_t+\beta P_{t-1}$ for $1<t<T$.

By adding (22) and (23),

(24) $U_t(d^*,T)+P_t(d^*,T)>s_tf(e^*)+\beta(U_{t-1}+P_{t-1})$.

From (17) and (18),

$U_t(d,T)=w_t-v(e_t)+\beta U_{t-1}(d,T)$, $\forall t$

$P_t(d,T)=s_tf(e_t)-w_t+\beta P_{t-1}(d,T)$, $\forall t$.

The substitution of the above two equations into (24) gives the expression (21).

9 For simplicity, workers are assumed to be risk neutral in wage and u(w)=w.

Q.E.D.

This proposition implies that the present value of surplus to both agents can not be less than worker's disutility of effort and d^* is a subgame perfect Nash equilibrium.

V. Concluding Remarks

The model presented in this paper is plausible to explain several economic questions which can not be explained by the previous models. It shows that productivity can move procyclically over business cycle while the previous implicit contract and efficiency wage models incur countercyclical movement which is opposite to the empirical results. It also shows layoffs are possible even when there is work sharing, and layoffs occur in order of effort level or efficiency unit of labor. Moreover, this layoff pattern strengthens our arguments about procyclical productivity movement. If effort level increases by seniority this results implies that layoffs occur by seniority rule. This result also can be applied to explain high unemployment rate in young, woman, or minor ethnical worker groups. Finally, the basic model was extended to derive self-enforceable contract when the contract is not legally enforceable.

There are two possible extensions of this work. The first is to consider the response of effort level to the changes of wage level in the context of contract. In this case wage level seems to be higher and procyclical, but relative volatility of employment are not clear. The second is to consider asymmetric information problem. If firm has relative informational advantage in state of nature but has higher observation cost in worker's effort, we need to consider a contract with two-sided private informations.

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