Master of Science

A Fractional Order Fuzzy PID Controller for An Electro-Hydraulic Rotary Actuator

The Graduate School of the University of Ulsan

Department of Mechanical Engineering

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A Fractional Order Fuzzy PID Controller for An Electro-Hydraulic Rotary Actuator

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Nomenclature

EHA	Electro Hydraulic Actuator
EHRA	Electro-Hydraulic Rotary Actuator
VHSs	Valve-controlled Hydraulic System
CPID	Conventional Proportional Integral Derivative
FOPID	Fractional Order Proportional Integral Derivative
FPID	Fuzzy Proportional Integral Derivative
FOFPID	Fractional Order Fuzzy Proportional Integral Derivative
FOMCON	Fractional Order Modelling and Control
MF	Membership Function
AMESim	Advanced Modelling Environment Simulation software

Abstract

Nowadays, hydraulic systems play an important role in modern industry for the reason that hydraulic systems have many advantages over other technologies with electric motors, as they possess high durability and the ability to produce large forces at high speed. However, most previous hydraulic actuators contain a valve-controlled hydraulic system (VHSs) that utilizes an open loop control which is results in low efficiency and loss the energy. To solve these disadvantages of conventional hydraulic system, electro-hydraulic actuator (EHA) systems have been developed and widely used.

The electro-hydraulic rotary actuator (EHRA) is known as one kind of EHA system where the hydraulic rotary is the end-effector of system. The EHRA has inherited the advantages of EHA system. Hence it is popularly implemented in the systems wherever high torque is required. However, the main disadvantages of EHRA system are complicated dynamic, non-linearity and large uncertainties in hydraulic systems due to unstableness of some hydraulic parameter such as bulk modulus, the compressibility of oil or viscosity of oil which lead to the control problems become a difficulty.

This thesis presents a fractional order fuzzy PID (FOFPID) controller for position control of an EHRA system. The proposed control is a combination between the fractional order PID controller and a fuzzy logic rule. Fractional order PID controller with two extra parameters λ, μ changes the integer order of integral and derivative functions in conventional PID to non-integer order. Therefore, they help to improve tracking performance. Three fuzzy rules are designed to adjust the parameters of FOPID. The K_p, K_i, K_d gains are adjusted during the operation process. To verify the performance of the proposed system and controller, some simulations and experiments are carried out in different conditions.

The results demonstrate that the proposed fractional order fuzzy PID controller achieves the better performance with high accuracy in various working conditions supporting strongly the applicability of the proposed control method in modern hydraulic applications.

Chapter 1

INTRODUCTION

1.1 Overview

Considering the development of industry, automation of working cycles and supporting users have become a popular trend of research. Among them, hydraulic system is one of the good choices for modern industries heavy-duty construction machines to precision machine tools because of their advantages such as durability, high power weight ratios, controllability, accuracy, reliability, compactness, cost [1][2]. Hydraulic systems use the fluid power that converted from the electrical and mechanical power to do the requirement motions through the hydraulic actuators. Hydraulic actuators can provide a constant force regardless of changes in speed, lift and hold heavy loads without braking, can move heavy objects or apply torque without any cumbersome gears, pulleys or levers. Many hydraulic actuators can use an energy source which supplied by a main pump connected with an electric motor. The power source can be placed remotely and supply the power to hydraulic actuators through hydraulic pipelines. For the most part, hydraulic systems are simple, safe and economical because they use fewer moving parts compared to mechanical and electrical systems, which makes them easier to maintain. Another advantage of hydraulic systems is that they do not cause sparks. Hence, they are safe to use in chemical plants and mines. From the advantages given above, the hydraulic systems are applied in many application fields such as for primary flight controls, landing gear systems in aeronautic field, for heavy vehicles and construction machines in land transportation field. However, most previous hydraulic actuators contain a valve-controlled hydraulic system (VHSs) that utilizes an open loop control. Then, the actuator operation is depended on the state of the control valve. Thus, it usually results in low efficiency due to leakage via bypass valves of hydraulic pump and the energy is transferred into heat due to throttle losses at the control valves, maintenance load, heavyweight, and limited installation space. To solve these disadvantages of conventional hydraulic system and satisfy new requirements, many alternative structures of electro-hydraulic actuator (EHA) have been developed and used.

An electro-hydraulic actuator (EHA) system has been proposed by engineers at Moog, Inc. [3]. In general, the EHA consists of an electric motor, a bi-directional pump, valves, a reservoir, and an actuator such as a cylinder or a rotary actuator. Since the end-effector is directly driven by the operation of the supplying pump. Due to its efficiency, EHAs have a wide range of applications for which the high accuracy and fast response of the force or pressure control are exceedingly necessary. Subsequently, the EHA systems provide a sleeker, cleaner and more energy efficient way to produce high force than VHSs, due to its higher stiffness. As a result, many testing hybrid systems using EHAs were fabricated for doing research on controlling the force or pressure with better performances [4-14].

Electro-hydraulic rotary actuator (EHRA) is known as one kind of EHA system where rotary actuator (RA) is the end-effector of system. The EHRA has inherited the advantages of EHA system. Hence it is popularly implemented in the systems such as for tool machines, robotics, handling containers, swing, pipe and plate bending machines, wherever big loads and high torques are required. Commonly, RA is designed based on three functional principles: helical, rack and pinion, and vane principle. Among these types, vane-type actuator is capable of providing the maximum amount of output torque from the smallest possible envelope size. Thus, it has a wide variety of industrial applications. However, due to existing a large number of nonlinearities and uncertainties in RAs, the precision control is a complex and difficult problem which attracts many efforts from researchers. The EHRA is known as complicated combinations of various fluid dynamics and hardware design while the uncertainties are come from, for example, friction between the vane and body, and the effective bulk modulus due to the variation of the temperature and pressure of the working fluid.

The conventional PID (proportional-integral-derivative) control algorithm remains the most popular approach for industrial process control due to its features such as simple structure, clear functionality, and ease of implementation. However, the conventional PID (CPID) controller has some weakness such as error computation: setpoint is often given as a step function, it amounts to asking the control signal to make a sudden jump, noise degradation in the derivative control, oversimplification, and complications brought by the integral control [15-25]. Therefore, CPID controller cannot provide effective control to a highly nonlinear and coupled system which possesses uncertain behaviour especially in EHRA system. For the purpose of achieving more favourable dynamic performance and robustness of control system, a fractional order PID (FOPID) controller is proposed by Podlubny [26]. The control action for fractional order controller is based on the doctrine of fractional order calculus, which includes the (integral order) and (differential order) parameters. However, with additional extra parameters, FOPID becomes more complex to determine the values of them. To overcome this problem, some intelligent techniques such as fuzzy logic, neural network, and neuro-fuzzy, are combined with FOPID controller. Mean-while, fuzzy control, an intelligent control method which imitates the human logical thinking and is independent of the accurate mathematical model of the controlled object, has turned out to be the most successful and popular used in previous research. In Mishra [27], a fractional order fuzzy PID (FOFPID) controller is used to control the binary distillation column system. In Sharma [28], two-layered fractional order fuzzy logic controllers are applied to the robotic manipulator with variable payload. Kumar [29] presents a nonlinear adaptive fractional order fuzzy proportional integral derivative controller to control a nonlinear, coupled, multi-input and multi-output and uncertain system i.e. a 2-link planar rigid robotic manipulator with payload. The performance comparison shows that this controller can obtain a better performance. Hence, the combination between the FOPID controller and a fuzzy logic control (FLC) is a potential method that can be applied for the EHRA system.

1.2 Research Objective

This paper presents a fractional order fuzzy PID (FOFPID) controller for position control of an EHRA system. The proposed control is a combination between the fractional order PID controller and a fuzzy logic rule. Different from the previous study, three fuzzy rules are designed to adjust the parameters of FOPID. Therefore, k_p, k_i, k_d gains are changed during the operation process. To check the performance of the proposed system and controller, the dynamic of system has been built by AMESim and the FOFPID controller is programmed in Matlab/Simulink. The co-simulation between the dynamic model and FOFPID controller is carried out in different working conditions such as changed references, operation frequencies, and variant loads. The results demonstrate that the proposed fractional order fuzzy PID controller achieves the better performance with high accuracy in various working conditions supporting strongly the applicability of the proposed control method in modern hydraulic applications.

Based on the above analyses, an EHRA system containing a Fractional Order Fuzzy PID controller is proposed and discussed. The first examination of the proposed system is to control the equivalent position of the RA robustly to the nonlinear disturbances and uncertainties without the oscillation. The major contributions of this study are: 1) The detail description, the dynamical mechanism model of the EHRA are introduced and built in AMESim software. 2) In the positioning strategy, a robust controller is developed via the combination between the fractional order PID controller and a fuzzy logic rule. The fractional order PID controller is embedded to improve the tracking performance of EHRA system. Meanwhile, the previous studies just proposed a fuzzy rule with two input error and derivative error and one output value [27-35], this thesis proposed 3 fuzzy rules to adjust the value of gains K_p, K_i, K_d during the operation process. 3) Several experiments and simulations of EHRA with proportional integral derivative (PID) and fuzzy PID (FOPID); Fractional order PID (FOPID) and fractional order fuzzy PID (FOFPID) in different working conditions such as changed references, operation frequencies, and variant loads to illustrate the theoretical results and show the efficiency of the proposed controller.

1.3 Thesis Outline

This thesis begins with a general introduction and literature in chapter 1. In chapter 2, the proposed system hardware is presented. The structure and working principle of the EHRA system are introduced. In addition, the mathematical model of the EHRA is expounded which includes the hydraulic actuator dynamics. The hybrid control system design for position control is shown in chapter 3. Simulations and Experimental results for the position performance of the EHRA system and some case studies can be found in chapter 4. Finally, some conclusions and discussions for future works are provided in chapter 5.

Chapter 2

PROPOSED ELECTRO-HYDRAULIC ROTARY ACTUATOR SYSTEM

2.1 A review of an EHRA system

2.1.1 Operating fundamental

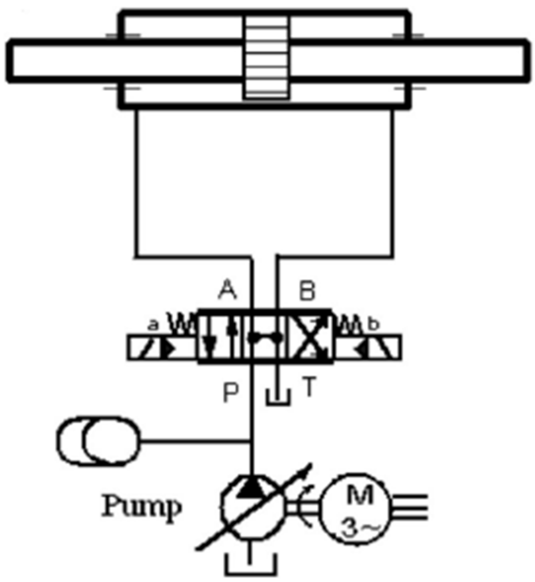


Figure 1 *Valve-controlled Hydraulic System*

Based on the circuit types as shown in Fig. 1, a conventional hydraulic system can be classified as Valve-controlled Hydraulic System (VHSs). VHSs systems have been widely applied in conventional equipment and machines implement due to its low cost and simple structure. However, it faces an obvious drawback, the enormous energy loss, i.e. throttling loss at the control valves. Study on the energy analysis of fluid power system has shown that 35% of the input energy of a VHSs is consumed in control valves. Such poor energy efficiency will lead to the high engine installed power and will generate great amounts of heat during the operation of the equipment. And overheat is also a significant reason that causes breakdown of the machines. To decrease the temperature of the system, an additional cooling system is required but it will also further increase the cost and the installed power of the equipment. And as an open circuit, VHSs usually needs large amount of hydraulic oil for the operation of the circuit, which to some extent increases the cost of the system and raises pollution problem when disposing of the oil. Control valves still place important roles in controlling the flow direction of the oil in the chambers of the cylinder, so the energy efficiency of the system is directly affected by the efficiency of the valves. Some schemes still need throttling valves in the main power lines. And due to the throttling losses in the control valves and pressure losses, the energy efficiency of the open circuit system is degraded. Although the development of load sensing technique effectively reduces the throttling loss of the control valve, eliminating such loss in VHSs is impossible.

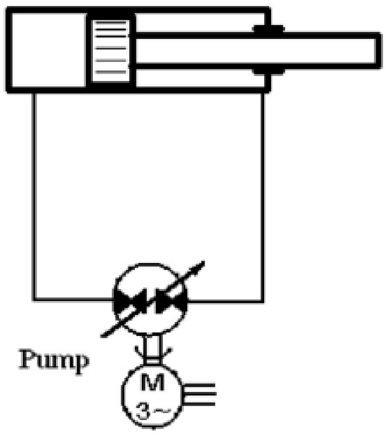


Figure 2 *Electro-hydraulic cylinder actuator*

In order to eliminate the throttling loss and reduce the overall energy consumption of hydraulic system, electro-hydraulic actuators (EHA) were proposed. As can be seen from Fig. 2, EHA system, different from VHSs, is a closed-control structure, in which both chambers of the cylinder are connected with the pump. The primary power sources of these systems are usually electric machines. Compared with the VHSs and open circuit pump controlled system, EHA system, which does not need control valves, suffers no throttling loss in the main power lines and needs less hydraulic fluid, i.e. oil, in the operation. And based on the EHA system, energy recovery solutions for kinetic energy and potential energy are possible. Hence the installed engine power, overall equipment energy consumption, the generated heat will be greatly reduced. Investigation on the energy efficiency comparison of the two kinds of hydraulic systems showed that the energy efficiency can be improved more than 40% by using advanced EHA system. One more advantage of utilizing EHA system is that the power can be delivered by wire instead of by steel pipes. Hence the EHA system is obviously a better solution to realize the green in fluid power system, i.e. low noise, high efficiency, less pollution, making it the developing trend of electro-hydraulic control technology.

Although the energy efficiency can be improved enormously with a direct pump controlled technology, limited by the dynamic performance of the pump, such principle could just be used in the EHA system with large power in the early time. It is not until the 1980s that control of the cylinder closed loop with pump became possible. By using pump displacement and valve stroke double closed loop controller, the dynamic response performance of axle and radial piston pump reach almost the same level as normal proportional valves. With higher supply pressure their frequency response in small signal range reaches more than 50 Hz, which is sufficient for today's most industrial and mobile applications. Based on the type of the actuators, the technology falls into two kinds: straight motion with actuator is single rod cylinder or double rod cylinder, rotation motion with actuator is hydraulic motor as shown in Figure 2 or hydraulic rotary as shown in Figure 3.

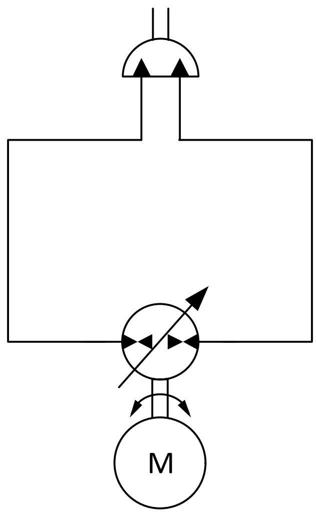


Figure 3 *Electro-Hydraulic Rotary Actuator (EHRA)*

2.1.2 Background of EHA systems.

Many researchers focused on the development and proposed the hydraulic circuits for EHA system. Hahmann and Spockhoff introduced the circuit principle of EHA system into the control of hydraulic cylinder in their dissertations and researched the static and dynamic performance of the EHA system [36,37]. The circuit is shown in Figure 4. This hydraulic system uses an oil exchanging valve with constant back pressure to exchange heat; only one chamber of the hydraulic cylinder is preloaded. Therefore, the natural frequency of the system is low, which negatively affects the dynamic performance of the system.

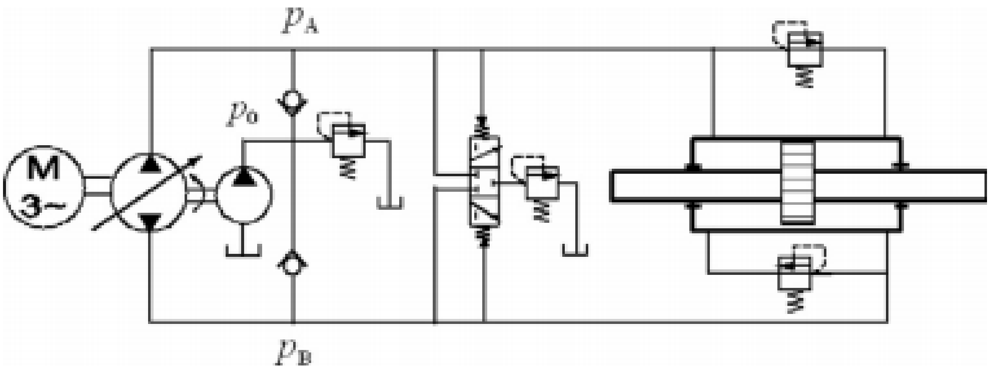


Figure 4 *EHA circuit with one chamber preloaded*

Berbuer proposed a new cylinder preload principle which consists of a constant pressure source and orifices, as shown in Figure 5. With this new principle, both chambers of the cylinder are preloaded with hydraulic pressure, the natural frequency and load rigidity of EHA system are increased making the system possess a similar characteristic as the valve control system [38].

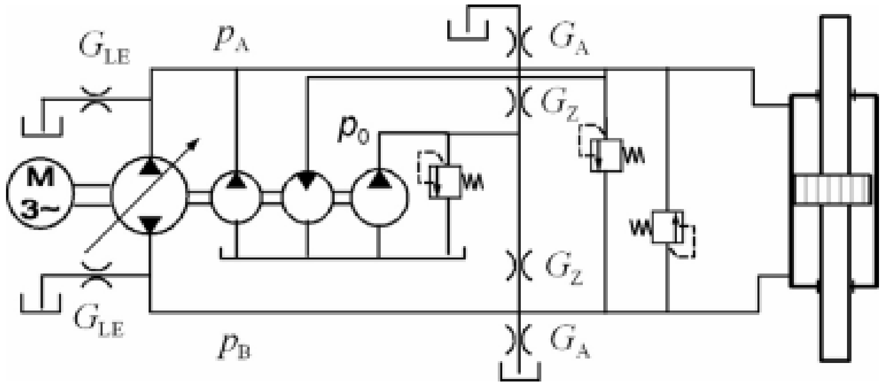


Figure 5 *EHA circuit with multi supplied pump*

But in this circuit one pump and one motor are required to exchange heat. Figure 6 shows the pressure gain curves of the above two systems. It is clear that only one chamber's pressure changes with the swing angle of the pump when adopting the traditional constant pressure control. But by adopting the two-chamber preload principle, the pressures in both chambers vary with the swing angle of the pump in the opposite direction at the same time. The pressures of both chambers are equal when no load is acted. And this balance pressure can be controlled by adjusting the value of the orifices. When the cylinder is accelerating or there is load force acted, pressures of both chambers change at the same time.

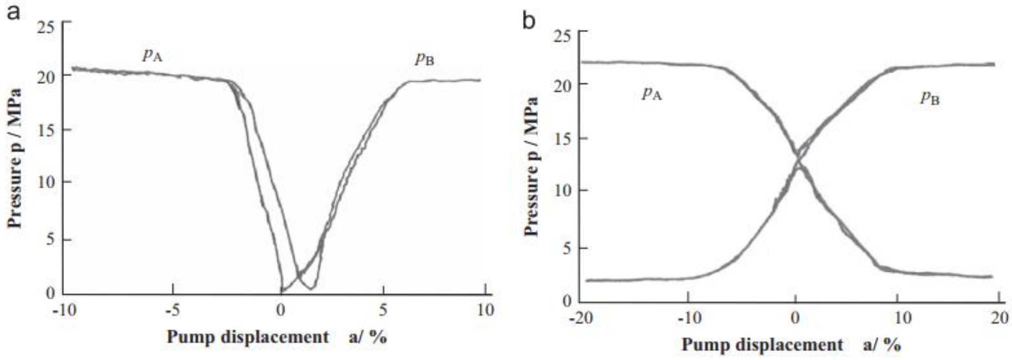


Figure 6 *Pressure gain curves of EHA system*

German Company Parker Hannifin applied for the patent of a new pump controlled double rods cylinder system for the manipulator system of airplanes [39]. Servo motor is adopted to drive a constant pump. The system scheme is shown in Figure 7. The characteristic of this system is that two chambers of the cylinder are preloaded through two check valves by a constant low-pressure oil tank or a low-pressure accumulator, and the leakage oil of the hydraulic pump is also led to the oil tank. The speed and the moving direction of the cylinder are controlled by the servo motor driven constant pump.

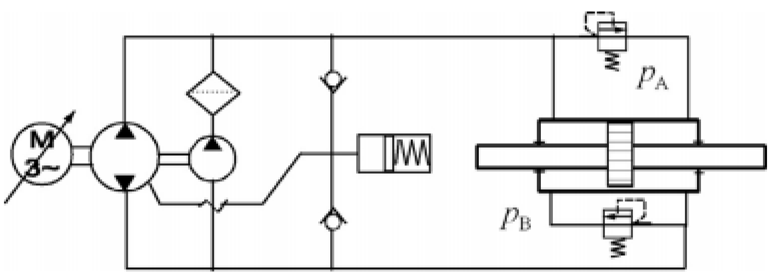


Figure *7 Speed variable motor of EHA system*

The system was implemented on an A340 airbus and conventional steel pipes were replaced by the novel Power-by-Wire actuator system, which reduced the weight of the plane at about 700 kg.

Safety and reliability of the plane were also improved [40]. The EHA system with double rods cylinder has also been applied in the flight control [41–44]. Kang et al. proposed a hydraulic power regulator for flight control system on the basis of a modified direct pump controlled double rods cylinder system [45]. A hydraulic lock was introduced to improve the stiffness and energy efficiency of the system. But on the other hand, the existence of proportional valve in the power line degrades the energy efficiency to some extent. Double rods cylinder as a symmetric actuator is simple to control. But its application is limited by the small output force and the install space.

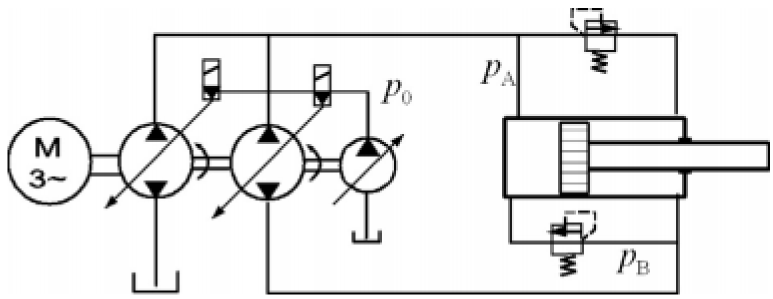


Figure 8 *EHA circuit controlled by two proportional pump*

Lodewyks. R, in IFAS RWTH Germany, put forward the circuit principles that use a hydraulic transformer or two coaxial-driven proportional pumps to compensate the asymmetrical flow in differential cylinder as shown in Figure 8. Feuser from Rexroth, adopting the constant chamber pressure preload principle, researched the static and dynamic performance of this circuit. This technology has been successfully used in the control of presser [46]. Ivantysynova applied the direct pump control technology in construction machines. And their research revealed that compared with the VHSs system, adopting the pump control technology can not only simplify the circuit, reduce the weight, improve the capability and energy efficiency, but also can make the control process easier. The research work showed a good future for the pump control technology.

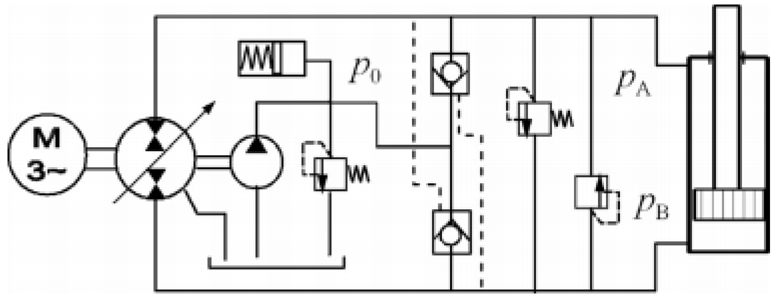


Figure 9 *EHA system with check valve balancing the flow*

In other studies, Japanese companies Yuken and Nachi developed the circuit principle shown in Figure 9 into an integrated unit and put them into practice use. The product has been used in 6-DOF motion simulator, presser, and other fields. A similar system was proposed by the American company Vickers. They applied for the patent of the closed loop control of the differential cylinder. which uses an AC servo motor to drive the constant displacement pump and uses the hydraulic controlled check valves to balance the flow. This technology was later used in the plastic injection modeling machine [47]. Jiang et al. researched the system of ship steering controlled by the speed-variable pump [48].

Recent researches have demonstrated that the closed EHA system directly driven by servo motor features many merits, such as wide speed range, high accuracy, high energy efficiency, high power density, etc.

2.1.3 Proposed EHRA hardware setup

In the previous researches, most of them used the variable pumps combined with constant speed electric motor. Though these structures have many advantages and applied in many fields, they still face limitations such as high cost, complexity, and difficult control. In order to overcome the drawback, this thesis proposed electro-hydraulic rotary actuator (EHRA) system which is a kind of EHA system with rotary actuator. The structure employed a fixed gear pump combined with a DC servo motor to supply the fluid to the actuator. The RA is employed as the end-effector of this system which has capable of handling large torques, highly precise positioning. As a result, the position angle of RA is controlled directly by the speed of DC servo motor and thus it improves the efficiency and accuracy of system. Due to the characteristic of EHRA system, two counter balance valves are used to keep the load motionless when the system does not work. The pressure of system which is influenced by an acting external force is maintained by two relief valves. The loss oil phenomenon is solved by using two check valves in both two port of the pump. Consequently, this system ensures safety during the working process, reduces the cost and achieves the high efficiency and accuracy.

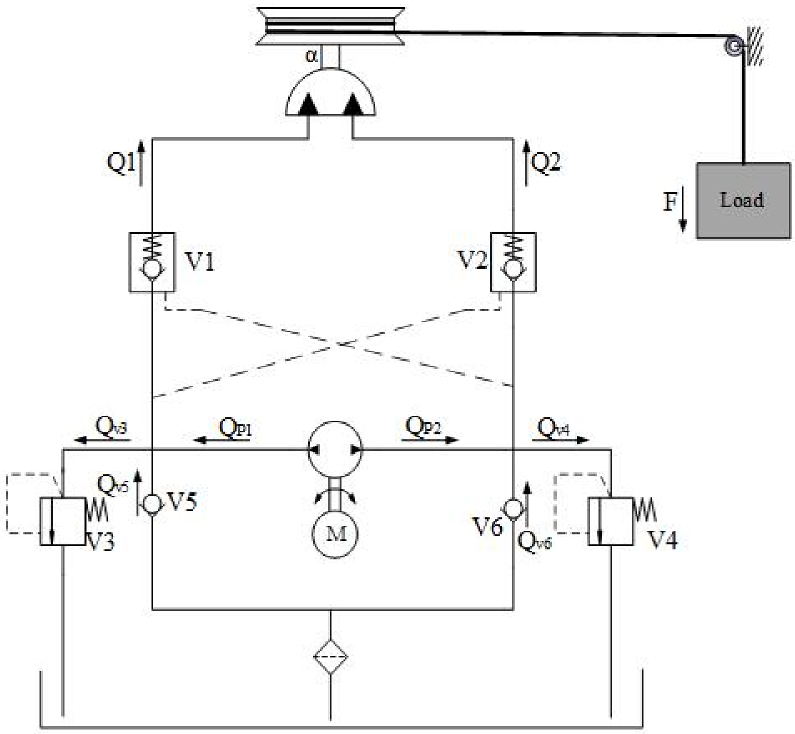


Figure 10 *Electro-hydraulic Rotary Actuator Circuit*

2.2 System mathematical modeling

The considered an EHRA in this thesis includes with a gear pump supplement valves and a hydraulic rotary actuator. Using the second Newton's law and principles of the hydraulic system, the dynamics of an EHRA can be described by the following state space.

$$J\ddot{\alpha} = (P_1 - P_2)D_R - Fr \quad (1)$$

Here, α is the degree of rotation, J is the moment of inertia, P_1 and P_2 are the pressure values of two chambers, respectively, DR is the displacement of the rotary actuator, F is an external loaded force, r is the radius of the pulley.

Based on the configuration of EHRA system, flow rates into two chambers are calculated as:

$$\begin{cases} Q_1 = Q_{P1} + Q_{v5} - Q_{v3} \\ Q_2 = Q_{p2} + Q_{v6} - Q_{v4} \end{cases} \tag{2}$$

Where $Q_{P1} = -Q_{P2} = Q_{pump}$ is the pump flow rate.

$$Q_{pump} = D\omega - k_{leakage}(P_1 - P_2) \tag{3}$$

Here, kleakage is the leakage constant, D is displacement of the pump, and ω is the speed of the DC pump system. The terms: Q_{v3}, Q_{v4}, Q_{v5}, and Q_{v6} are flow rates through valves V_3, V_4, V_5 and V_6 , respectively.

Assume there is no external leakage, the dynamics of oil flow can be calculated as:

$$\begin{cases} \dot{P}_1 = \dfrac{\beta}{V_{01} + A\alpha}(D\omega - k_{leakage}(P_1 - P_2) + Q_{v5} - A\dot{\alpha}) \\ \dot{P}_2 = \dfrac{\beta}{V_{02} - A\alpha}(-D\omega + k_{leakage}(P_1 - P_2) + Q_{v6} + A\dot{\alpha}) \end{cases} \tag{4}$$

V_{01} and V_{02} are original total volumes of two chambers, respectively, Q_1 represents the supply flow rate to the chamber 1, and Q_2 represents the supply flow rate to the chamber 2.

The derivative of (1)

$$J\dddot{\alpha} = (\dot{P}_1 - \dot{P}_2)D_R - \dot{F}r \tag{5}$$

From Eqs. (2), (5), they can be given as follows:

$$\begin{aligned} \dddot{\alpha} = {} & \omega\frac{DD_R}{J}\left(\frac{\beta}{V_{01} + A\alpha} + \frac{\beta}{V_{02} - A\alpha}\right) \\ & + \frac{\beta D}{(V_{01} + A\alpha)J}(-k_{leakage}(P_1 - P_2) + Q_{v5} - A\dot{\alpha}) \\ & - \frac{\beta D}{(V_{02} - A\alpha)J}(-k_{leakage}(P_1 - P_2) + Q_{v5} + A\dot{\alpha}) - \frac{\dot{F}r}{J} \end{aligned} \tag{6}$$

For simplicity, the last equation in Eq. (6) can be rewritten as

$$\dddot{\alpha} = \omega g(t) + f(t) \tag{7}$$

where

$$g(t) = \frac{DD_R}{J}\left(\frac{\beta}{V_{01} + A\alpha} + \frac{\beta}{V_{02} - A\alpha}\right) \tag{8}$$

$$f(t) = \frac{\beta D}{(V_{01} + A\alpha)J}(-k_{leakage}(P_1 - P_2) + Q_{v5} - A\dot{\alpha})$$
$$-\frac{\beta D}{(V_{02} - A\alpha)J}\left(k_{leakage}(P_1 - P_2) + Q_{v5} + A\dot{\alpha}\right) - \frac{\dot{F}r}{J} \tag{9}$$

The state variables of the system are defined as

$$x = \begin{bmatrix} x1 & x2 & x3 \end{bmatrix}^T = \begin{bmatrix} \alpha & \dot{\alpha} & \ddot{\alpha} \end{bmatrix}^T \tag{10}$$

Then, the simplified mathematical model of EHRA system can be described by employing (1) -(9)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{(P_1 - P_2)D_{\mathrm{R}} - F_{\mathrm{r}}}{J} \\ \dot{x}_3 = \omega g(t) + f(t) \end{cases} \tag{11}$$

Consequently, system states are adjusted by the speed of the bi-directional pump that is driven by a DC motor. Given a bounded periodical trajectory. The development of a position controller will be described in the following section.

Chapter 3

CONTROL DESIGN

3.1 Overview of the hybrid controller

For position control purpose, the main objective of the proposed controller is to generate the electric signal to the DC motor, u, to achieve the desired outputs which are the position of the rotary, α . First of all, a fractional order proportional-integral-derivative (FOPID) control is applied to the position control problem of the hybrid actuator due to its clear functionality and ease of implementation. The control system can operate in the working environment without noises. However, it generates extra parameters which include the λ (integral order) and μ (differential order) parameters. Thus, the proposed controller becomes more complex to determine the values of them. Hence, fuzzy technique is proposed to combine with the FOPID controller in order to train the P, I, and D gains with respect to the system control error. Fractional order fuzzy PID (FOFPID) control technique has been applied to some previous applications [27-29] but they only proposed a tuning rule for all the parameter. This method face with some limitations such as unstable, delay signal or low performance due to the different characteristics of each gain in control technique. Therefore, this thesis based on the theory of some previous researches applied to fuzzy PID (FPID) designed three tuning rule for each gains K_p, K_i, K_d of FOPID controller. The overall scheme of the proposed controller is presented in Figure 11

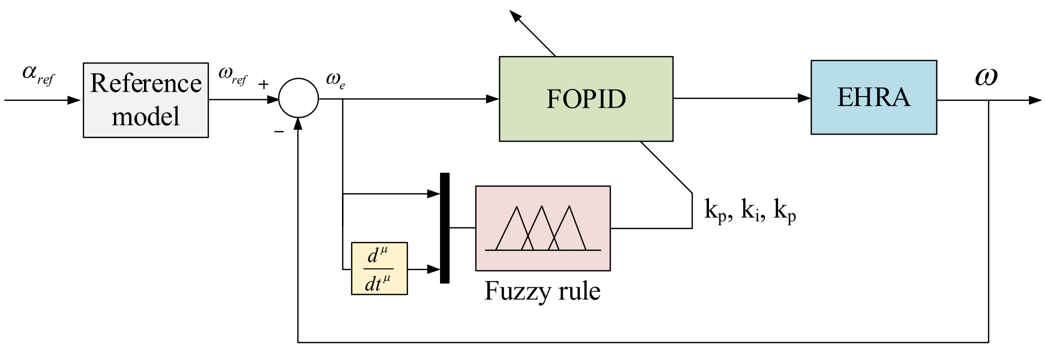


Figure 11 *Structure of the FOFPID controller*

3.2 Mathematical Background

Fractional calculus is a generalization of integration and differentiation to non-integer order operator ${}_aD_t^{\alpha}$ where a and t denote the limits of the operation and α denotes the fractional order such that

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \Re(\alpha) > 0 \\ 1 & \Re(\alpha) = 0 \\ \int_a^t (dt)^{-\alpha} & \Re(\alpha) < 0 \end{cases} \tag{12} $$

where generally it is assumed that $\alpha \in \mathbb{R}$, but it may also be a complex number. One of the reasons why fractional calculus is not yet found in elementary texts is a certain degree of controversy found in the theory [1]. This is why there is not a fractional order differ-integral operator. Rather there are multiple definitions which may be useful in a specific situation. Further, several commonly used definitions of fractional-order operators are presented.

3.2.1 Definitions

We first define the fractional differ-integral operator according to Riemann-Liouville, which is the most widely used definition in fractional calculus [5].

Riemann-Liouville definition:

$$ {}_aD_t^{\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{m}-\alpha)} \left(\frac{d}{dt} \right)^m \int_a^t \frac{f(\tau)}{(\mathrm{t}-\tau)^{\alpha-m+1}} d\tau \tag{13} $$

where $m-1 < \alpha < m, m \in \mathbb{N}, \alpha \in \mathbb{R}^+$ and $\Gamma(.)$ is Euler's gamma function.

Caputo definition:

$$ {}_0D_t^{\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{m}-\alpha)} \int_0^t \frac{f^{(\mathrm{m})}(\tau)}{(\mathrm{t}-\tau)^{\alpha-m+1}} d\tau \tag{14} $$

where $m-1 < \alpha < m, m \in \mathbb{N}$

Another definition is the Grünwald-Letnikov one. This definition can be especially useful due to its importance in applications.

Grünwald-Letnikov

$$ {}_aD_t^\alpha f(\mathrm{t}) = \lim_{h\to 0}\frac{1}{h^\alpha}\sum_{j=0}^{\left[\frac{t-a}{h}\right]}(-1)^{\mathrm{j}}\pi\binom{\alpha}{j}f(\mathrm{t-jh}) \tag{15} $$

where *[.]* means the integer part.

3.2.2 Properties

Fractional-order differentiation has the following properties

- If *f(t)* is an analytic function, then the fractional-order differentiation ${}_0D_t^\alpha f(\mathrm{t})$ is also analytic with respect to *t.*
- If $\alpha = n$ and $n \in Z^+$, then the operator ${}_0D_t^\alpha f(\mathrm{t})$ can be understood as the usual operator d^n / dt^n.
- Operator of order $\alpha = 0$ is the identity operator: ${}_0D_t^\alpha f(\mathrm{t}) = f(\mathrm{t})$.
- Fractional-order differentiation is linear; if *a, b* are constants, then

$$ {}_aD_t^\alpha\left[af(\mathrm{t})+\mathrm{bg(t)}\right] = a\,{}_aD_t^\alpha f(\mathrm{t}) + b\,{}_0D_t^\alpha\,\mathrm{g(t)} \tag{16} $$

- For the fractional-order operators with $\Re(\alpha)>0, \Re(\beta)>0$, and under reasonable constraints on the function *f(t)* it holds the additive law of exponents:

$$ {}_0D_t^\alpha\left[{}_0D_t^\beta f(\mathrm{t})\right] = {}_0D_t^\beta\left[{}_0D_t^\alpha f(\mathrm{t})\right] = {}_0D_t^{\alpha+\beta} f(\mathrm{t}) \tag{17} $$

- The fractional-order derivative commutes with integer-order derivative

$$ \frac{d^n}{dt^n}\left({}_aD_t^\alpha f(\mathrm{t})\right) = {}_aD_t^\alpha\left(\frac{d^n f(\mathrm{t})}{dt^n}\right) = {}_aD_t^{\alpha+n} f(\mathrm{t}) \tag{18} $$

Under the condition *t=a* we have $f^{(\mathrm{k})}(\mathrm{a}) = 0, \left(k = 0,1,2,...,n-1\right)$.

3.2.3 Laplace Transform

The Laplace integral transform in an essential tool in dynamic system and control engineering. A function F(s) of the complex variable s is called the Laplace transform of the original function f(t) and defined as

$$F(\mathrm{s}) = L\left[f(\mathrm{t})\right] = \int_0^{\infty} e^{-st} f(\mathrm{t})\,\mathrm{dt} \tag{19}$$

The original function *f(t)* can be recovered from the Laplace transform *F(s)* by applying the reverse Laplace transform defined as

$$f(\mathrm{t}) = L^{-1}\left[\mathrm{F(s)}\right] = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} e^{-st}\,\mathrm{F(s)}\,\mathrm{ds} \tag{20}$$

where c is greater than the real part of all the poles of function *F(s)*.

Laplace transform of the Riemann-Liouville fractional operator

$$L\left[D^{\alpha} f(\mathrm{t})\right] = s^{\alpha} F(\mathrm{s}) - \sum_{k=0}^{m-1} s^{k}\left[D^{\alpha-k-1} f(\mathrm{t})\right]_{t=0} \tag{21}$$

where $(m-1 \leq \alpha < m)$.

Laplace transform of the Caputo fractional operator

$$L\left[D^{\alpha} f(\mathrm{t})\right] = s^{\alpha} F(\mathrm{s}) - \sum_{k=0}^{m-1} s^{\alpha-k-1} f^{(\mathrm{k})}(0) \tag{22}$$

where $(m-1 \leq \alpha < m)$.

Laplace transform of the Grünwald-Letnikov fractional operator

$$L\left[D^{\alpha} f(\mathrm{t})\right] = s^{\alpha} F(\mathrm{s}) \tag{23}$$

3.2.4 Fractional-order Models

A fractional-order continuous-time dynamic system can be expressed by a fractional differential equation of the following form

$$\begin{aligned} &H\left(D^{\alpha_0\alpha_1\ldots\alpha_n}\right) y(\mathrm{t}) = \mathrm{G}\left(D^{\beta_0\beta_1\ldots\beta_n}\right) u(\mathrm{t}) \\ &H\left(D^{\alpha_0\alpha_1\ldots\alpha_n}\right) = \sum_{k=0}^{n} a_k D^{\alpha_k}, \\ &G\left(D^{\beta_0\beta_1\ldots\beta_n}\right) = \sum_{k=0}^{n} b_k D^{\beta_k}, \end{aligned} \tag{24}$$

where a_k, $b_k \in R$. In explicit form:

$$a_n D^{\alpha_n} y(\mathrm{t}) + a_{n-1} D^{\alpha_{n-1}} y(\mathrm{t}) + \ldots + a_0 D^{\alpha_0} y(\mathrm{t}) = \\ b_m D^{\beta_m}\, \mathrm{u(t)} + b_{m-1} D^{\beta_{m-1}}\, \mathrm{u(t)} + \ldots + b_0 D^{\beta_0}\, \mathrm{u(t)} \tag{25}$$

The system is said to be of *commensurate-order* if in (25) all the orders of derivation are integer multiples of a base order γ such that $\alpha_k, \beta_k = k\gamma,\ \gamma \in R^+$. The system can then be expressed as

$$\sum_{k=0}^{n} a_k D^{k\gamma} y(\mathrm{t}) = \sum_{k=0}^{m} b_k D^{k\gamma}\, \mathrm{u(t)} \tag{26}$$

If in (26) the order is $\gamma = 1/q,\ q \in Z^+$, the system will be of rational order. The diagram with linear time-invariant (LTI) system classification is given in Figure 12.

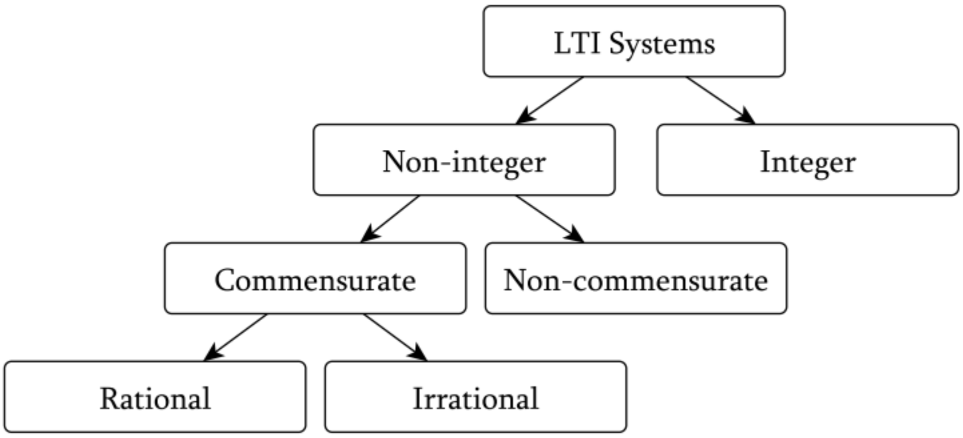


***Figure 12** Classification of LTI systems*

Applying the Laplace transform to (25) with zero initial conditions the input-output representation of the fractional-order system can be obtained in the form of a transfer function:

$$G(\mathrm{s}) = \frac{Y(\mathrm{s})}{U(\mathrm{s})} = \frac{b_m s^{\beta m} + b_{m-1} s^{\beta m-1} + \ldots + b_0 s^{\beta_0}}{a_n s^{\beta n} + a_{n-1} s^{\alpha n-1} + \ldots + a_0 s^{\alpha_0}} \tag{27}$$

In the case of a system with commensurate order γ, the continuous-time transfer function is given by

$$G(\mathrm{s}) = \frac{\sum_{k=0}^{m} b_k (\mathrm{s}^{\gamma})^{\mathrm{k}}}{\sum_{k=0}^{n} a_k (\mathrm{s}^{\gamma})^{\mathrm{k}}} \tag{28}$$

Taking $\lambda = s^{\gamma}$ the function (26) can be viewed as a pseudo-rational function *H(λ)*:

$$H(\lambda) = \frac{\sum_{k=0}^{m} b_k \lambda^k}{\sum_{k=0}^{n} a_k \lambda^k} \quad (29)$$

Based on the concept of the pseudo-rational function, a state-space representation can be established in the form:

$$\begin{aligned} D^{\gamma} x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (30)$$

The state-space model allows representation of multiple input, multiple output (MIMO) fractional-order systems. The following equation may be used to convert the state space representation to a transfer function:

$$G(s) = C\left(s^{\gamma} I - A\right)^{-1} B + D \quad (31)$$

where I is the identity matrix.

3.2.5 Fractional System Analysis

Stability

In order to determine stability of a fractional system given by (2.14) consider the following theorem.

Matignon's stability theorem: The fractional transfer function $G(s) = Z(s)/P(s)$ is stable if and only if the following condition is satisfied in σ-plane:

$$\left|\arg(\sigma)\right| > q\frac{\pi}{2}, \forall \sigma \in C, P(\sigma) = 0 \quad (32)$$

where $\sigma := s^q$. When $\sigma = 0$ is a single root of $P(s)$, the system cannot be stable. For $q=1$, this is the classical theorem of pole location in the complex plane: no pole is in the closed right plane of the first Riemann sheet.

In general, for a commensurate-order fractional-order system in the form

$$D^q \omega = f(\omega) \quad (33)$$

where $0 < q < 1$ and $\omega \in R^n$ the equilibrium points are calculated by solving

$$f(\omega) = 0 \tag{34}$$

The equilibrium points are asymptotically stable if all the eigenvalues λ_k of the Jacobian matrix

$J = \dfrac{\partial f}{\partial \omega}$, evaluated at the equilibrium, satisfy the condition

$$\left|\arg\left(eig\left(J\right)\right)\right| = \left|\arg\left(\lambda_k\right)\right| > q\frac{\pi}{2}, k = 1, 2, ..., \mathrm{n} \tag{35}$$

Alternatively, the stability condition can also be evaluated from the state-space representation of the system (30):

$$\left|\arg\left(eig\left(A\right)\right)\right| > q\frac{\pi}{2} \tag{36}$$

where $0 < q < 1$ and *eig(A)* represents the eigenvalues of the state-space matrix A. Stability regions of a fractional-order system are shown in Figure 13.

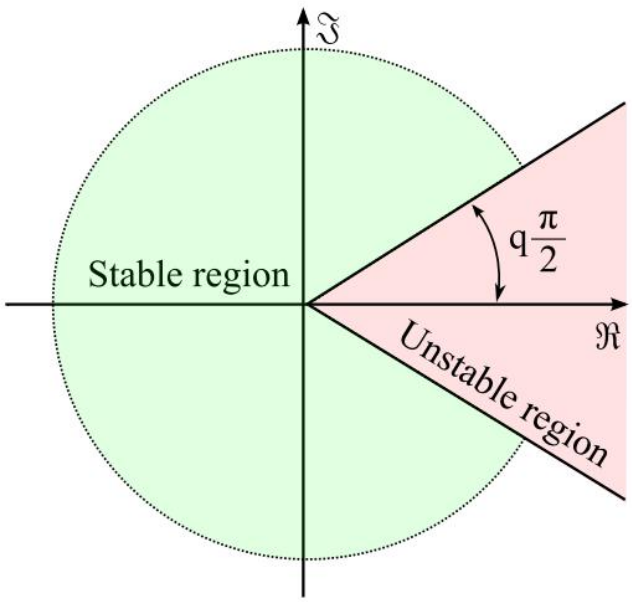


***Figure 13** LTI fractional-order system stability region for 0 < q < 1*

3.2.6 Time domain analysis

In the time domain, it is desired to obtain a transient response of a fractional-order dynamic system. One solution would be to use the inverse Laplace transform and the Mittag-Leffler function proposed by Podlubny in [26]. However, this solution method may be time consuming and tedious. Another solution involves numerical computation of fractional-order derivatives which is carried out by means of a revised Grünwald-Letnikov definition rewritten as

$$ {}_aD_t^{\alpha} f(\mathrm{t}) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} \omega_j^{(\alpha)} f(t-jh) \tag{37} $$

where h is the computation step-size and $\omega_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}$ can be evaluated recursively from

$$ \omega_0^{(\alpha)} = 1, \omega_j^{(\alpha)} = \left(1 - \frac{\alpha+1}{j}\right)\omega_{j-1}^{(\alpha)}, j = 1,2,... \tag{38} $$

To obtain a numerical solution for the equation in (25) the signal *u(t)* should be obtained first, using the algorithm in (37), where

$$ u(\mathrm{t}) = \mathrm{b_m}\, D^{\beta m}\mathrm{u(t)} + \mathrm{b_{m\text{-}1}} D^{\beta m-1}\mathrm{u(t)} + ...+ \mathrm{b_0} D^{\beta_0}\ \mathrm{u(t)}. \tag{39} $$

The time response of the system can then be obtained using the following equation:

$$ y(\mathrm{t}) = \frac{1}{\sum_{i=0}^{n} \frac{\alpha_i}{h^{\alpha_i}}} \left[u(\mathrm{t}) - \sum_{i=0}^{n} \frac{a_i}{h^{\alpha_i}} \sum_{j=1}^{\left[\frac{t-a}{h}\right]} \omega_j^{(\alpha)} y(\mathrm{t-jh}) \right] \tag{40} $$

3.2.7 Frequency domain analysis

Frequency-domain response may be obtained by substituting $s = j\omega$ in (27). The complex response for a frequency $\omega \in (0; \infty)$ can then be computed as follows:

$$ R(\omega) = \frac{P(j\omega)}{Q(j\omega)} = \frac{b_m(\mathrm{j}\omega)^{\beta_m} + b_{m-1}(\mathrm{j}\omega)^{\beta_{m-1}} + ... + b_0(\mathrm{j}\omega)^{\beta_0}}{a_n(\mathrm{j}\omega)^{\alpha_n} + a_{n-1}(\mathrm{j}\omega)^{\alpha_{n-1}} + ... + a_0(\mathrm{j}\omega)^{\alpha_0}} \tag{41} $$

where j is the imaginary unit.

3.2.8 Approximation of Fractional Operators

Due to availability of well-established tools for integer-order LTI systems analysis, the possibility of approximating the fractional model by an integer-order one is highly desirable. The Oustaloup recursive filter gives a very good approximation of fractional operators in a specified frequency range and is widely used in fractional calculus. For a frequency range (ω_b, ω_h) and of order N the filter for an operator s^{γ}, $0 < \gamma < 1$, is given by

$$G_f(\mathrm{s}) = \mathrm{K} \prod_{k=-N}^{N} \frac{s+\omega_k'}{s+\omega_k} \tag{42}$$

where

$$\omega'_{\mathrm{k}} = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}}, \omega_{\mathrm{k}} = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}}, \mathrm{K} = \omega_h^{\gamma} \tag{43}$$

Given these methods of approximating the fractional operator, a general method of approximation a fractional-order model by an integer-order one is now possible. Recall the property in (2.6). Thus, for fractional orders $\alpha \geq 1$ it holds

$$s^{\alpha} = s^{n} s^{\gamma} \tag{44}$$

where $n = \alpha - \gamma$ denotes the integer part of α and sγ is obtained by the Oustaloup approximation by using either (42) with the latter being preferred in most cases.

3.3 Fractional order Fuzzy PID controller

The notion of a fractional PID controller was introduced by Podlubny in [18]. This generalized controller is called the $PI^{\lambda}D^{\mu}$ controller and has an integrator with an order λ and a differentiator of order μ. In the same paper, Podlubny [26] demonstrated, that the fractional-order controller had a better response than an integer-order one when used in a control loop with a fractional-order plant. In more recent researches it has been confirmed that the fractional controller outperforms the integer-order PID controller. The control action of the $PI^{\lambda}D^{\mu}$ controller can be expressed as follows:

$$u(\mathrm{t}) = \mathrm{K_p e(t)} + \mathrm{K_i}\, D^{-\lambda} e(\mathrm{t}) + \mathrm{K_d}\, D^{\mu} e(\mathrm{t}) \tag{45}$$

where *e(t)* is the error signal. From Eq. (11), *e(t)* can be estimated as follow:

$$\mathrm{e}(t) = x_{ref} - x_1(t)$$

k_p, k_i, k_d : the proportional gain, integral gain, and the derivative gain, respectively.

Obviously, when taking $\lambda = \mu = 1$ the result is the classical integer-order PID controller. With more freedom in tuning the controller, the four-point PID diagram can now be seen as a PID controller plane, which is conveyed in Figure 14.

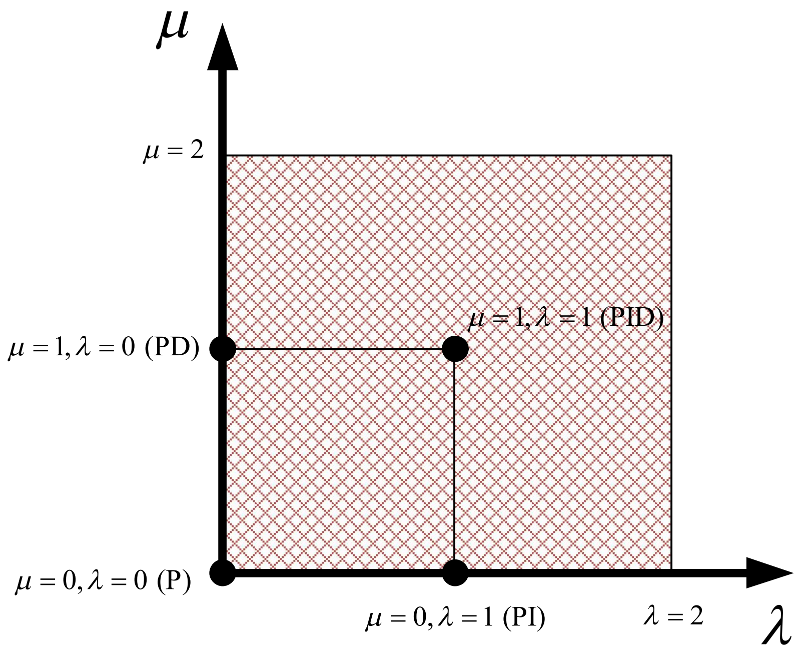


Figure 14 *Fractional order PID controller converge*

But the FOPID controllers do not yield reasonable performance over a wide range of operating conditions because of the fixed gains used. That is the reason why another control technique needs to be used to tune the parameters of the FOPID controller. And fuzzy logic is one of the effective solutions. From (45), three coefficients K_p, K_i, and K_d need to be turned by using the fuzzy technique. Details of the suggested fuzzy FOPID scheme are shown in Figure 15.

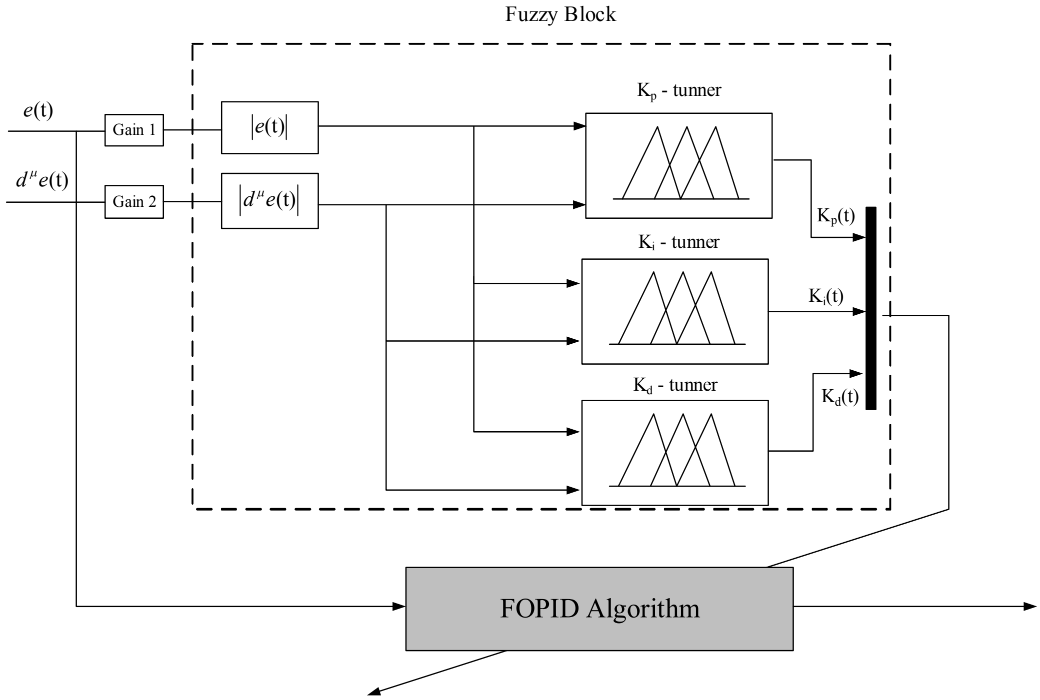


Figure 15 *The configuration of the fuzzy FOPID control block*

Through fuzzy logic knowledge, the fuzzy turners, which tune FOPID parameters (K_p, K_i, K_d) can be established using the following equation:

$$K_a = K_{a0} + U_a \Delta K_a, U_a \in [0.1] \tag{46}$$

where a is p, i, or d, U_a the parameter obtained from the output of the tuning fuzzy controllers; $\Delta K_a = K_{a1} - K_{a0}$: the allowable deviation of K_a

K_{a0}, K_{a1}: the minimum and maximum values of K_a determined as from the experiments, respectively.

Using the Eq. (46) and the scheme depicted in Figure 15, the three coefficients, K_p, K_i, and K_d were tuned using the three independent fuzzy tuners. Consequently, the three separate fuzzy P, I and D controllers were combined to form the overall fuzzy FOPID controller.

Three are two inputs to the fuzzy controllers: absolute error $|e(t)|$ and the absolute fractional order derivative of error $|de(t)|$. The ranges of these inputs are from 0 to 1, which were obtained from the absolute values of the system error and its derivative through scale factors chosen from specifications of the nonlinear system. For each input variable, triangle membership functions were requested for use. Because all of the MFs are triangle shapes, we can express these MFs as follow:

$$f_{ji}(\mathrm{x}) = \begin{cases} 1 + \dfrac{(\mathrm{x} - \mathrm{a}_{\mathrm{ji}})}{\bar{b}_{ji}} \, if \, (-\bar{\mathrm{b}}_{\mathrm{ji}}) \le (\mathrm{x} - \mathrm{a}_{\mathrm{ji}}) \le 0 \\ 1 - \dfrac{(\mathrm{x} - \mathrm{a}_{\mathrm{ji}})}{\overset{+}{b}_{ji}} \, if \, 0 \le (\mathrm{x} - \mathrm{a}_{\mathrm{ji}}) \le (\overset{+}{\mathrm{b}}_{\mathrm{ji}}), j = 1, 2, ..., \mathrm{N} \\ 0 \qquad\qquad otherwise \end{cases} \tag{47}$$

where x is the input; a_{ji}, $\bar{b}_{ji}$ and $\overset{+}{b}_{ji}$ are the centroid, left half-width, and right half-width of the j^{th} triangle membership function of the j^{th} input, respectively, and N is the numbers of triangles.

Each of the fuzzy P, I, and D controllers had one output, which were U_p, U_i, and U_d, respectively. In practice, fuzzy control is applied using local inferences. This means that each rule is inferred and the results of the inferences of the individual rules are then aggregated. The most common inference methods, max-min, max-product and sum-product method, a function where the aggregation operator is denoted by either the maximum value or the sum and the fuzzy implication operator is denoted by either the maximum or the product. In particular, the max-min calculus of fuzzy relations offers a computationally neat and expressive setting for constraint propagation. Subsequently, a defuzzification method is needed in order to obtain a crisp output from the aggregated fuzzy result. Popular defuzzification methods are maximum matching and centroid defuzzification. Centroid defuzzification is widely used for fuzzy control problems where a crisp output is needed, and maximum matching is often used for pattern matching problems where it is necessary to know the output class. Therefore, in this research, the fuzzy reasoning results of the outputs were gained using an aggregation operation of fuzzy sets of inputs and designed fuzzy rules, where the max-min aggregation method and the centroid defuzzification method were used. In the proposed fuzzy controller, we were able to compute the control output, U_p, U_i, and U_d, using a pair of inputs:

$$U_a = \frac{\sum_{k=1}^{M} mf(\mathrm{w_k}) w_k}{\sum_{k=1}^{M} mf(\mathrm{w_k})}, \tag{48}$$

where a is p, i, or d, w_k is the weight of the control output (the centroid of the k^{th} output fuzzy MF); M is the number of fuzzy output sets; $mf(w_k)$ is the fuzzy output function given by:

$$mf(\mathrm{w}) = \sum_{i,j} mf_{ij}(\mathrm{w}) \tag{49}$$

where $mf_{ij}(w)$ is defined as the consequent fuzzy output function when the first and second input are in the i and j class, respectively.

$$mf_{ij}(\mathrm{w}) = \delta_{\mathrm{ij}} \mu_{\mathrm{ij}} \tag{50}$$

where δ_{ij} is an activated factor which is active when the input $|e(\mathrm{t})|$ is in class i, and the input $|de(\mathrm{t})|$ is in class j, μ_{ij} is the height of the consequent fuzzy function obtained from the input class i and j

$$\mu_{ij} = \min\left[f_{i1}\left(|e(\mathrm{t})|\right), f_{j2}\left(|de(\mathrm{t})|\right) \right] \tag{51}$$

The output U_a of the tuning fuzzy controller contains single output values. They were initially set at the same intervals. Generally, the fuzzy rules are dependent on the system plant to be controlled and the type of the controller.

Chapter 4

SIMULATIONS AND EXPERIMENTAL RESULTS

In order to implement the efficient, proposed FOFPID controller, we created an application of it so as to control the position of an electro-hydraulic rotary actuator (EHRA) system. We then used both simulations and experiments in order to verify our results.

4.1 Position controller design

The detailed FOPID scheme is clearly described in Section 3.3. There were two inputs to the fuzzy controllers: absolute error $|e(\text{t})|$ and an absolute derivative of error $|de(\text{t})|$. The ranges of these inputs were from 0 to 1, which were obtained from the absolute values of the system error and its derivative through the scale factors.

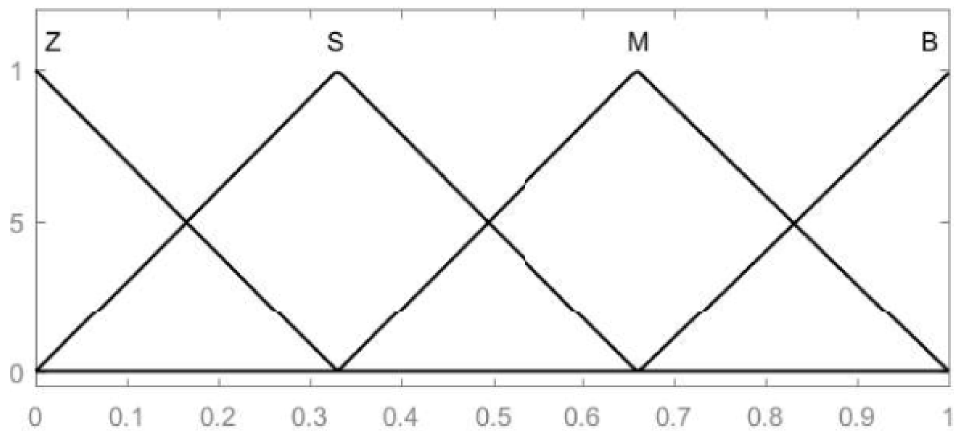
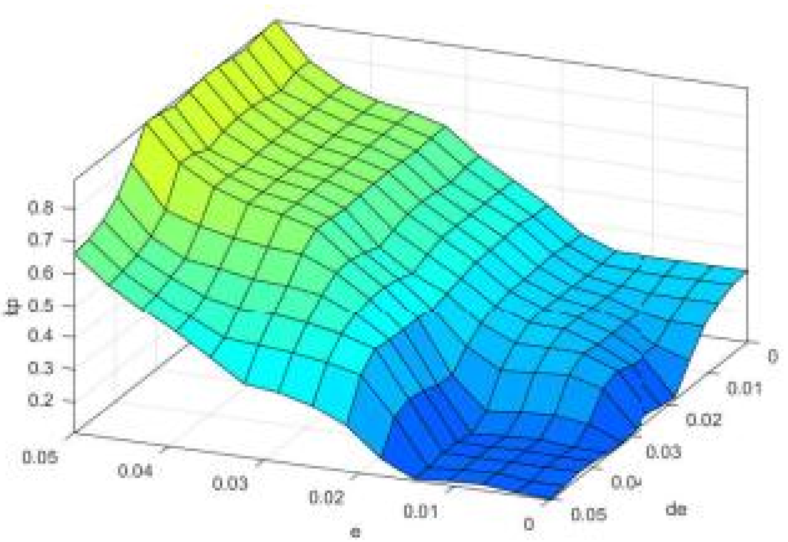


***Figure 16** Initial membership function of the inputs and output fuzzy*

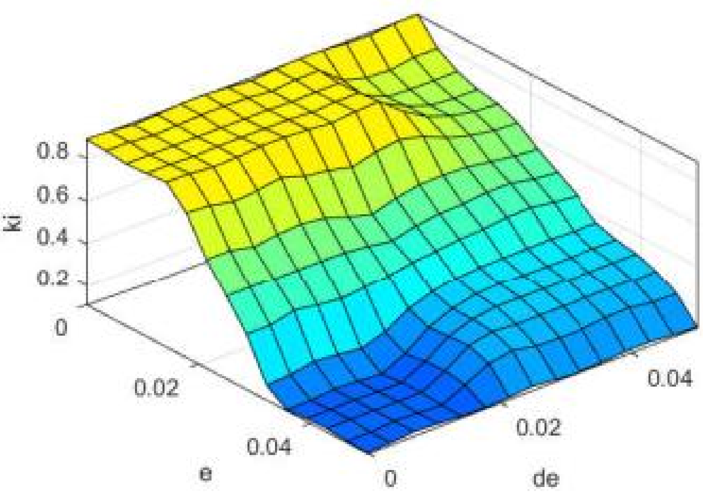
Based on design experience, we used four triangle MFs for smooth tuning the P, I, and D parameter, due to the fact that they do not require much calculation time. Here, “Z”, “M”, “B”, were “Zero”, “Small”, “Medium” and “Big”, respectively. The centroids of the MFs were initially set at identical intervals and shape sizes.

__Table 1__ Rules table of fuzzy fractional PID controller

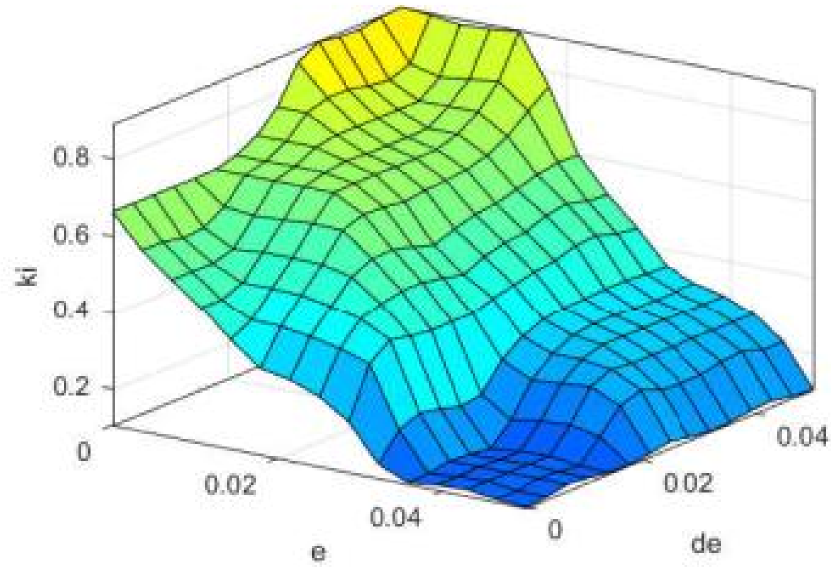
(Up, Ui, Up)		$\lvert de(\mathrm{t}) \rvert$			
		Z	S	M	B
$\lvert e(\mathrm{t}) \rvert$	Z	(S,B,M)	(S,B,S)	(M,Z,Z)	(B,Z,Z)
	S	(Z,B,M)	(S,B,M)	(M,Z,Z)	(B,Z,Z)
	M	(Z,B,B)	(Z,B,M)	(M,S,S)	(B,Z,Z)
	B	(Z,B,B)	(Z,M,B)	(S,S,S)	(M,Z,Z)



(a) k_p fuzzy tunner



(b) k_i fuzzy tunner



(c) k_d fuzzy tunner

Figure 17 *Rule surface view of fuzzy P, I, D tuner.*

From the above fuzzy sets of the input and output variables, the fuzzy rules for the online tuning fuzzy FOPID that were applied to the EHRA system are described in Table 1. From the output of the three separate fuzzy P, I and D controllers, the control signal applied to the system so as to control the servo motor was computed as described in section 2.

4.2 Simulations

4.2.1 Simulation setup

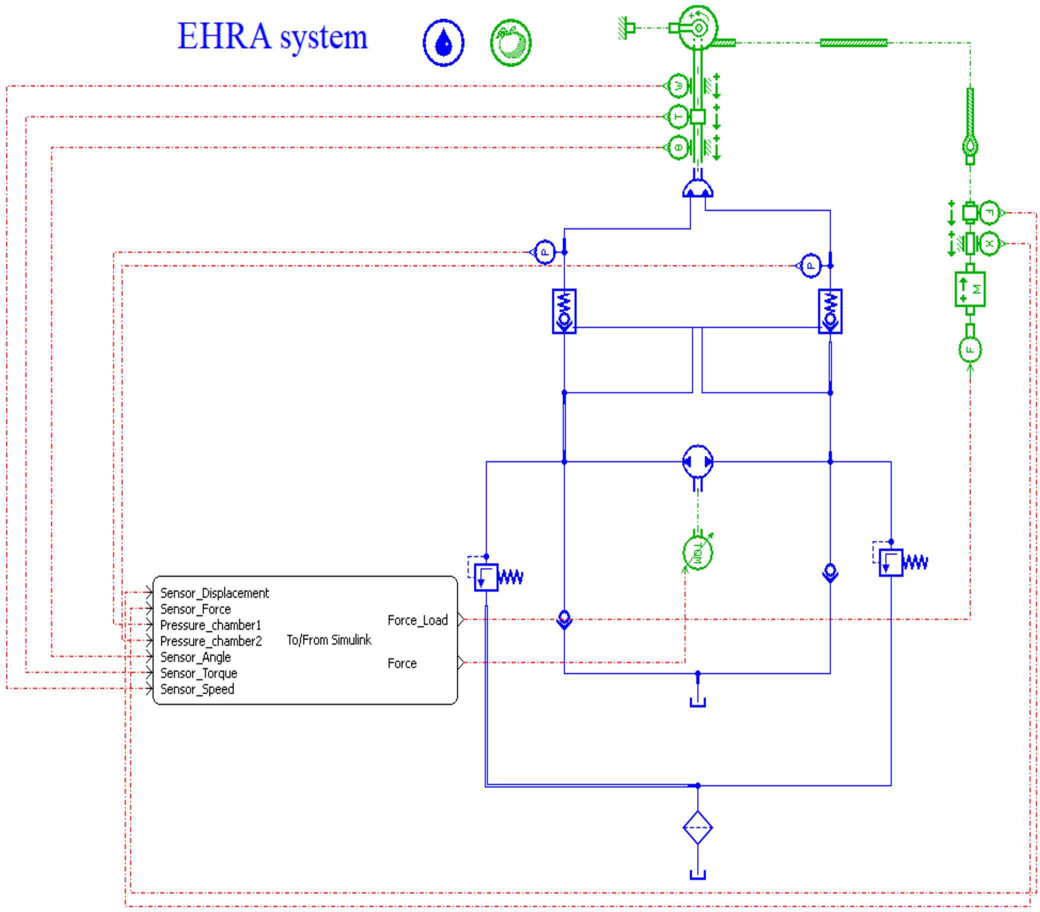


Figure 18 *AMESim model of EHRA*

In this section, we describe the simulations for the hybrid electro-hydraulic rotary actuator that were carried out so as to prove the effectiveness of the designed controller using a co-simulation between AMESim and Matlab/Simulink. AMESim (Advanced Modelling Environment Simulation software) allows the simulation of actuator dynamics, including the electric motor, and hydraulic systems using several libraries. Figure 18 shows the AMESim model of the hybrid actuator. The hybrid actuator was a combination of an AC servo motor, a bidirectional pump, a reservoir, and a hydraulic control circuit. With hydraulic components are displayed in blue blocks, the mechanical parts are illustrated in green blocks and the control signal is expressed in the red lines. The black box makes an environment to connect with Matlab/Simulink. In addition, a mass was used as a loading environment and some sensors were used to obtain the feedback force signal. The setting parameters for the hybrid system model were obtained from the real testing system as shown in Table 2. The rotary actuator was controlled by the motor and the bidirectional pump so as to have the desired performance. The AMESim model contained two inputs (control signal to control the motor and load force) and seven outputs (feedback position, torque, speed, pressure signals from the sensors) to communicate with the suggested control system built in Simulink, consequently forming a closed loop feedback control.

Table 2 *Setting parameters for the EHRA simulation system*

Components	Parameters	Specification
Hydraulic Pump	Displacement	0.97(cc/rev)
	Rate rotation speed	3000 rpm
	Relief pressure	120 bar
Hydraulic Rotary Actuator	Displacement Rotation Torque output	27.54 100 deg. 120 Nm
Hydraulic oil	Effective bulk modulus	$1.5x10^{9}$
	Specific gravity	0.87
Viscous Friction Coefficient		30 Nm/(rev/min)

4.2.2 Simulation results

Some simulations were carried out for checking the control performances of each controller used for the EHRA system in different working conditions. The parameters of the controllers are chosen as follow: IOPID: k_p=1000, k_i=1, k_d=600, FOPID: k_p=1000, k_i=1, k_d=600, $\lambda = 0.1$. The Fuzzy PID (FPID) and Fuzzy FOPID have used the same fuzzy rules and membership function set in all cases.

4.2.2.1. Case study 1

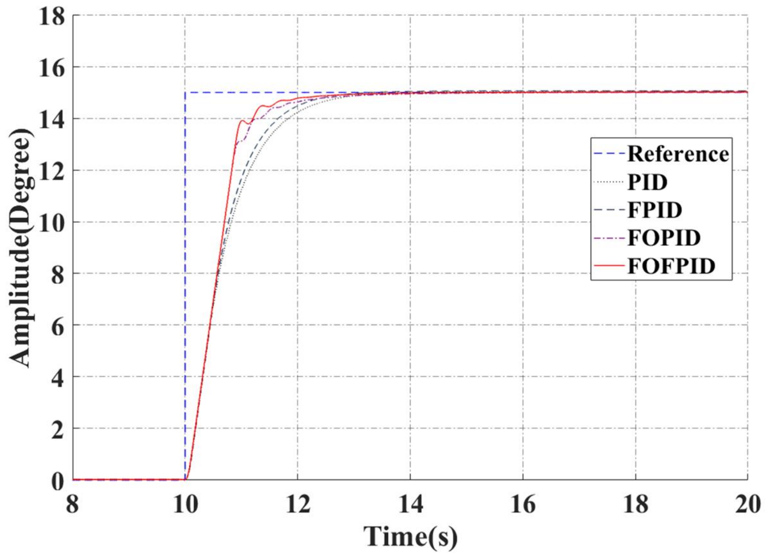


Figure 19 *Output performance of conventional PID, Fuzzy PID, FOPID and Fuzzy FOPID with a step signal and the payload is 150N*

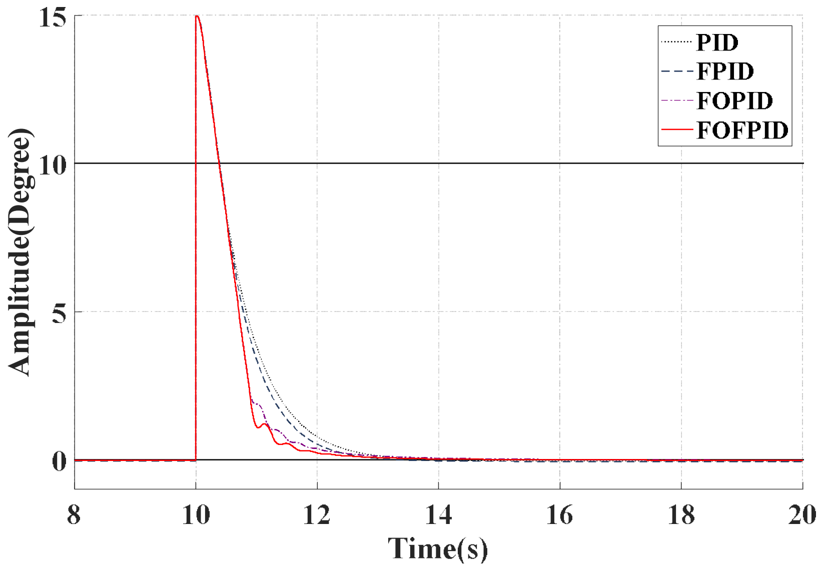


Figure 20 *Error effort of conventional PID, Fuzzy PID, FOPID and Fuzzy FOPID with a step signal and the payload is 150N*

Figure 19 shows the simulated step responses with 150N load. The performance of FPID was a little better than that using the conventional PID controller. However, when applied the FOPID and FOFPID controller, the control quality became much better. The rising time, the settling time and overshoot are reduced, especially in the FOFPID controller. The steady state error is about 0.1% of the desired position control. The details can be seen in the Figure 20.

4.2.2.2. Case study 2

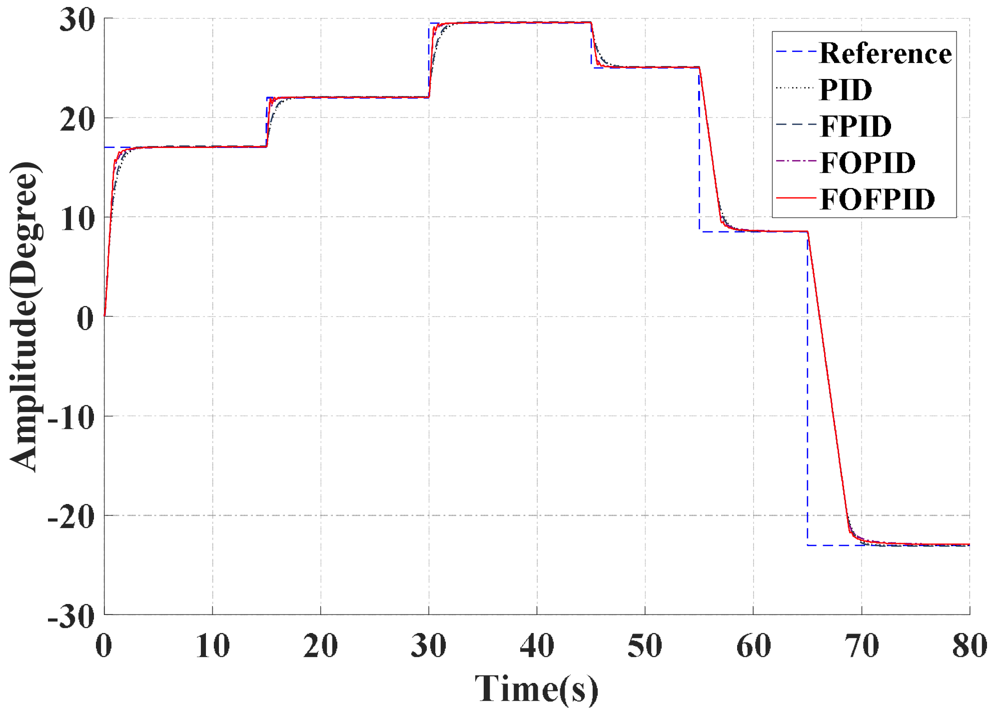
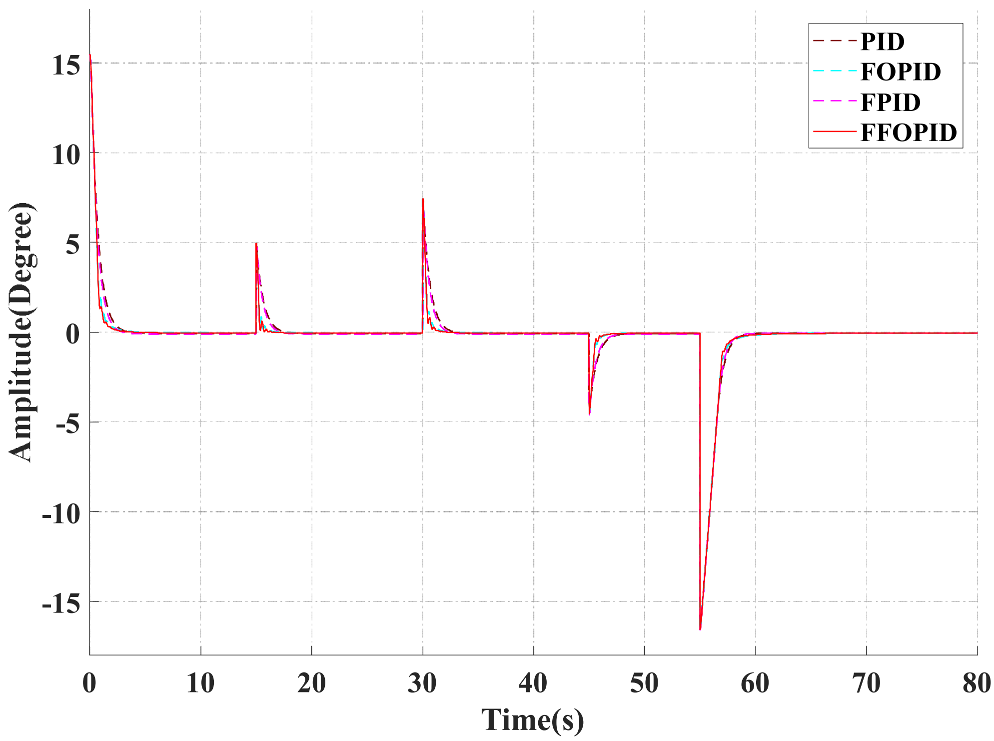
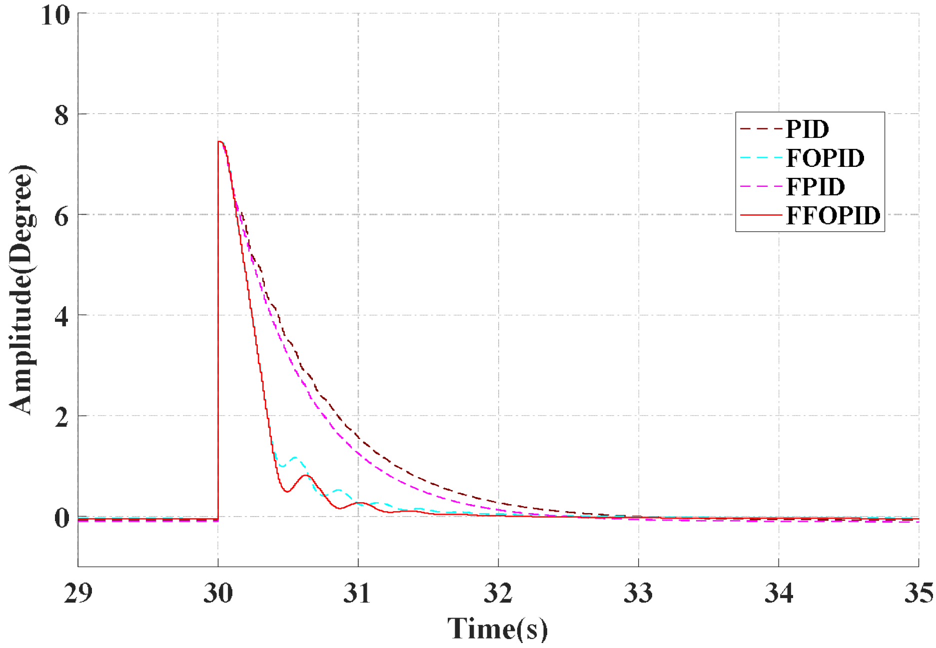


Figure 21 *Output performance of conventional PID, Fuzzy PID, FOPID and Fuzzy FOPID with a step signal and the payload is 500N*



(a) Error effort of controllers in 80 seconds



(b) Zoom of error effort from 29s to 35s

Figure 22 Error effort of conventional PID, Fuzzy PID, FOPID and Fuzzy FOPID with a step signal and the payload is 500N

Figure 21 and Figure 22 (a), (b) show the simulated multi step responses and error. As can be observed, the performances of both controllers were almost the same in the case of big amplitudes. However, with the small step values, the FOFPID has the fastest response and best quality when compared with the other controllers. The payload force was also changed in this case, but the state error was kept in the range $\pm 0.1\%$ of reference signals.

4.2.2.3. Case study 3

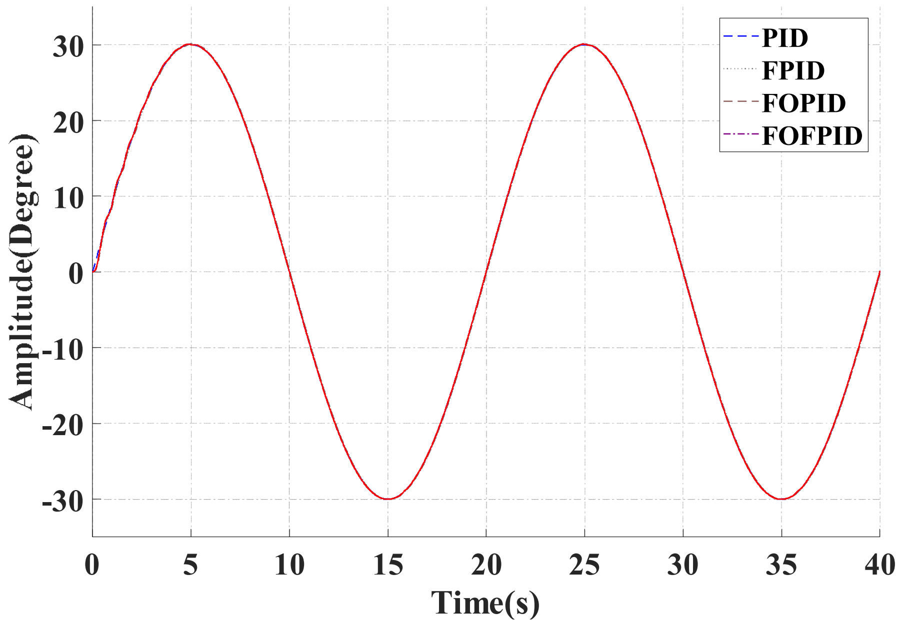


Figure 23 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a sinusoidal signal of 0.05Hz and the payload is 50N*

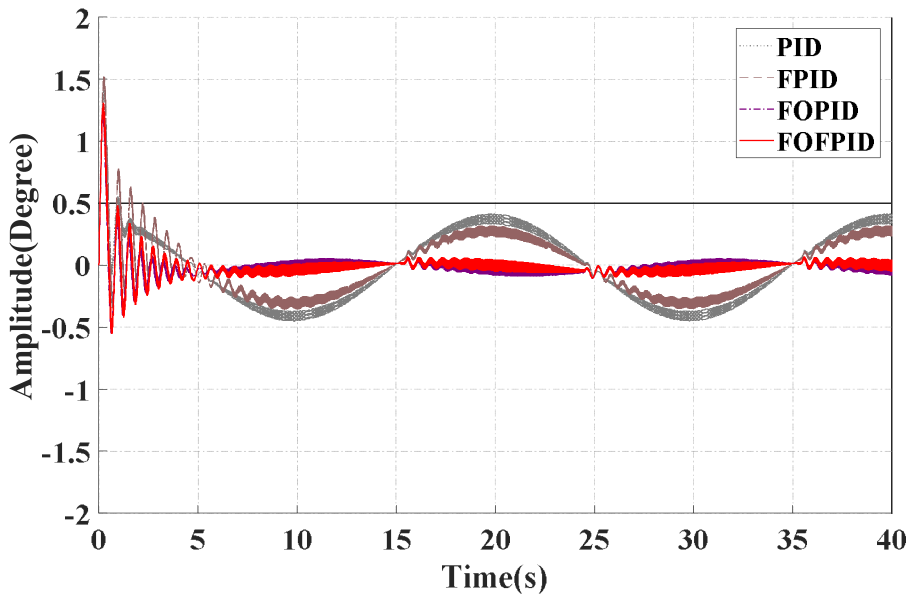


Figure 24 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a sinusoidal signal of 0.05Hz and the payload is 50N*

In the next simulations, the sinusoidal reference signal is used to check the quality of the proposed controller. First, the frequency of input sinusoidal signal is 0.05Hz and the payload is 50N. Figure 23 shows the tracking performance of all controllers. The result in Figure 24 indicated that the proposed FOFPID controller has less error than the other controllers. The error of FOFPID is in the range from -0.04 to 0.04 degree while PID is from -0.4 to 0.4 degree, FPID is from -0.2 to 0.2 and FOPID is from -0.06 to 0.06 degree.

4.2.2.4. Case study 4

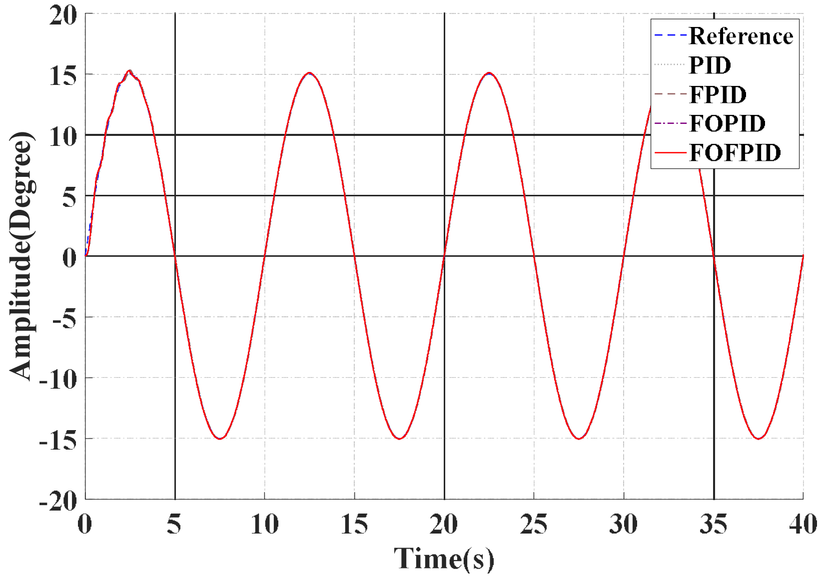


Figure 25 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a sinusoidal signal of 0.1Hz and the payload is 50N*

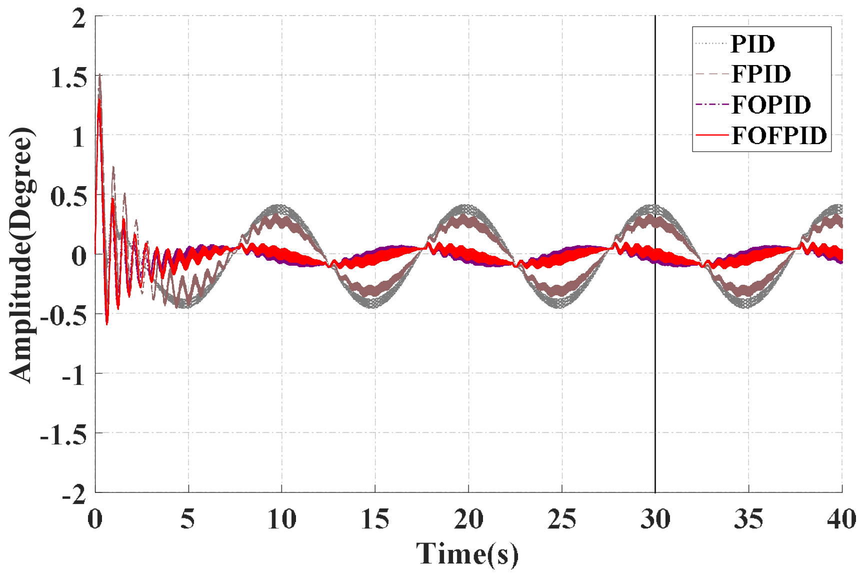


Figure 26 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a sinusoidal signal of 0.1Hz and the payload is 50N*

In the final simulation, the frequency of the sinusoidal signal is changed from 0.05Hz to 0.1Hz. Figure 25 illustrates the tracking performance of PID, fuzzy PID, FOPID and Fuzzy FOPID controllers. The detail of quality can be seen in Figure 26. The frequency of the reference signal influences on the accuracy of EHRA system. However, the proposed control was still the best quality with the smallest state error in this case. It is clear that a good position regulation is realized when using the fractional order fuzzy PID to design a position controller.

4.3 Experiments

4.3.1 Experiment setup

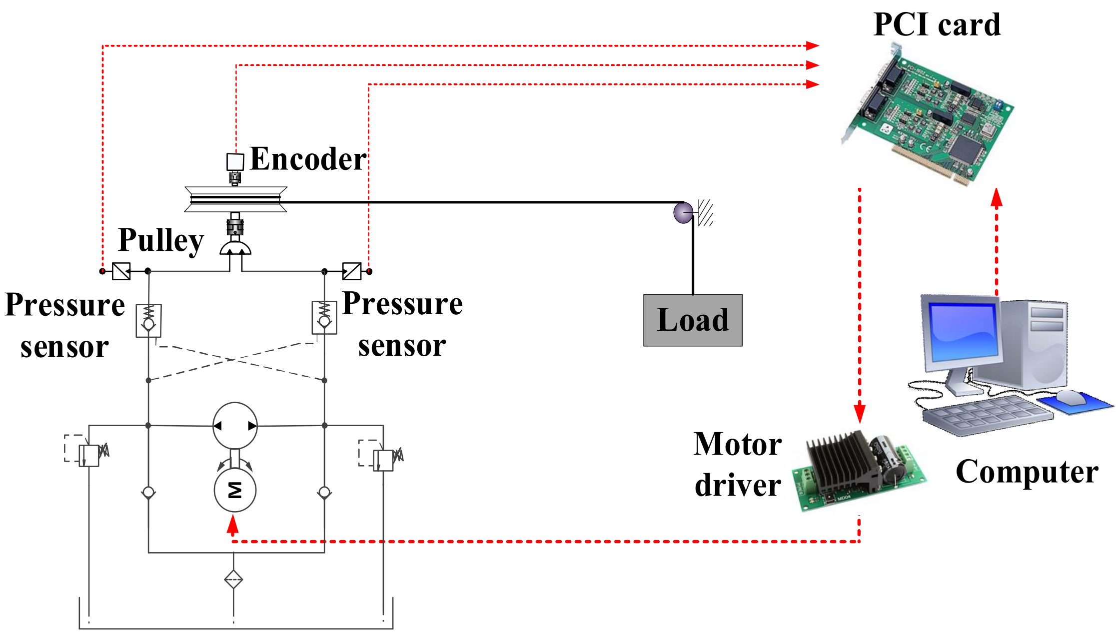


Figure 27 *Structure of experiment system*

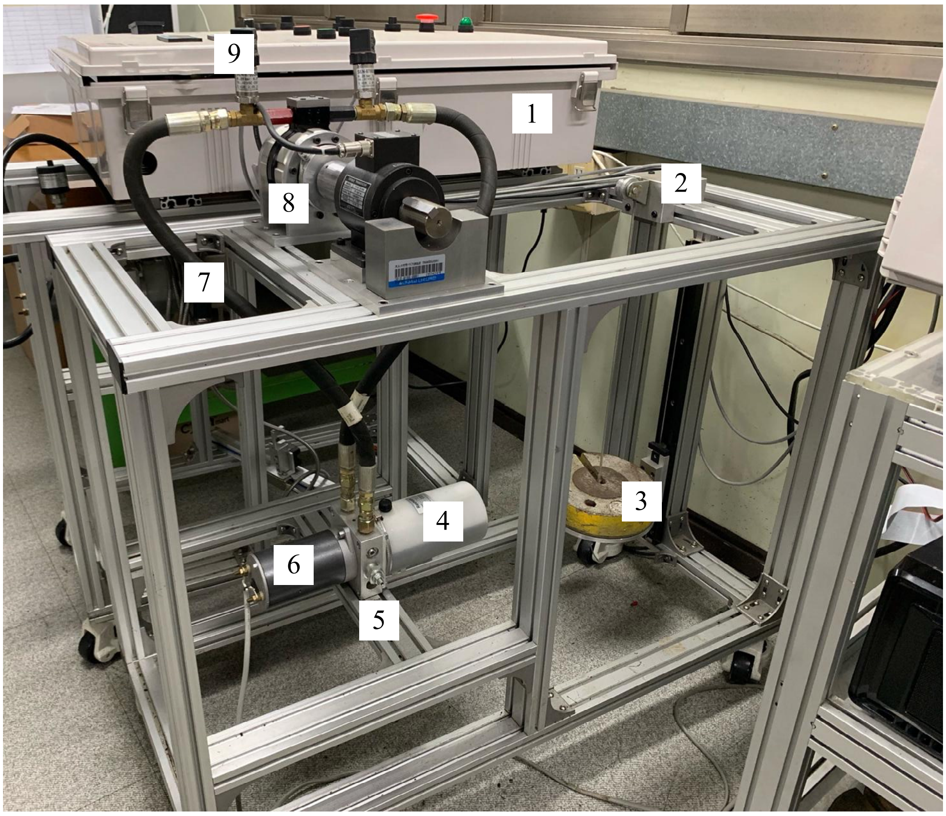


Figure 28 *Experimental apparatus with: 1- Control box, 2- Pully and cable, 3- Attached weight, 4- Oil tank, 5- Pump and valves, 6- Electric DC motor, 7- Pipeline, 8- Rotary actuator, 9- Pressure sensor*

Table 3 *Parameters for the EHRA test bench.*

Components	Parameters	Specification
Hydraulic Pump Rexroth W130049469	Displacement	0.97 [cc/rev]
	Rated rotation speed	3000 [rpm]
	Relief pressure	120 [bar]
Hydraulic Rotary KNR	Displacement	27.54
	Rotation angle	100
	Torque output	120 Nm
Hydraulic oil	Effective bulk modulus	$1.5 x 10^9$ [Pa]
	Specific gravity	0.87
Encoder	Model	E40H8
	Resolution	8192 [p/rev]
Pressure Sensor	Model	SEN-8700/2A095
	Capacity	160 [bar]
	Rated Output	16 [bar/V]

The schematic diagram of the whole pump controlled EHRA system is displayed in Figure 27. The electro-hydraulic system is chosen from the Bosch Rexroth company, which included a bi-directional gear pump, supplement valves system. The rotary actuator with a limited 100 degrees rotation is selected from KNR. The whole system was driven by a 24 V–20 A DC motor. In this configuration, the position of the load is controlled by the speed of the DC motor which speed is adjusted directly. One rotary encoder and two pressure transducers were installed to the system to measure the rotary actuator position, and the pressure in two chambers of main EHRA, respectively. The load simulator part is designed as a gravity loading system which can adjust the loading force easily by changing the attached mass. This is a simple method to simulate the variation of working condition for the EHRA system. The designed controller was implemented on the computer within real-time Window Target Toolbox of MATLAB 2013b under a sampling time of 0.01 seconds. The detailed specifications of the system components are summarized in Table 1, and the real apparatus is shown as Figure 28.

4.3.2 Experiment results

4.3.2.1. Case study with pulse signal

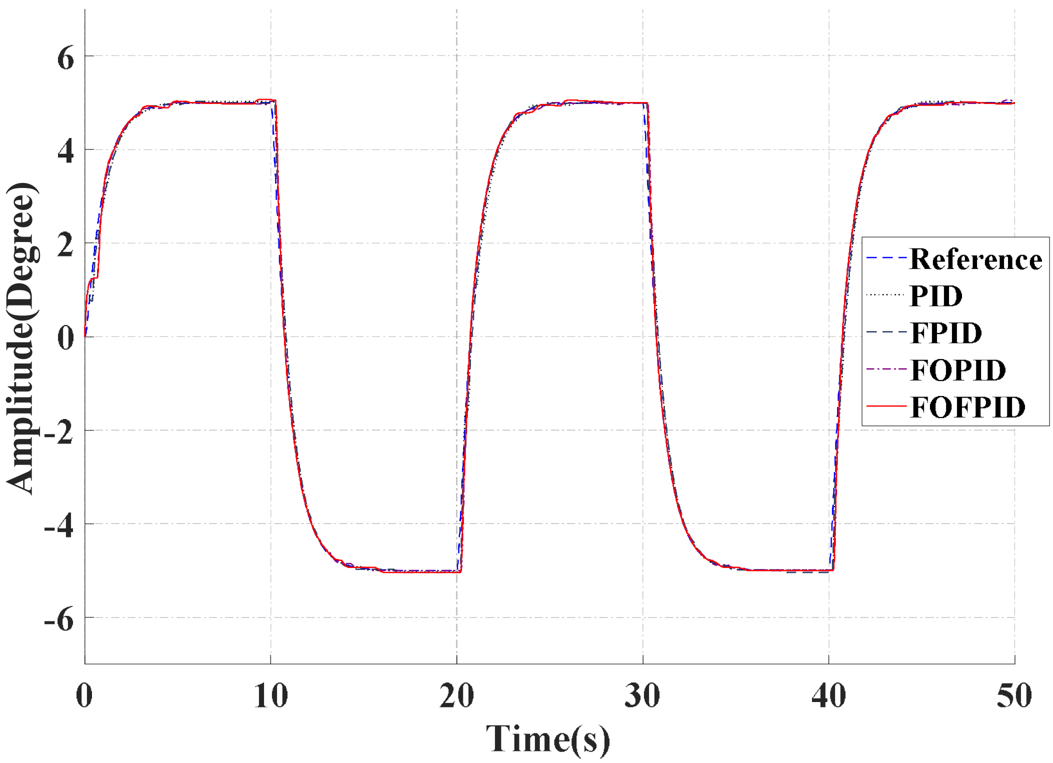


Figure 29 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a pulse signal (cutoff frequency 9Hz and the payload is 50N)*

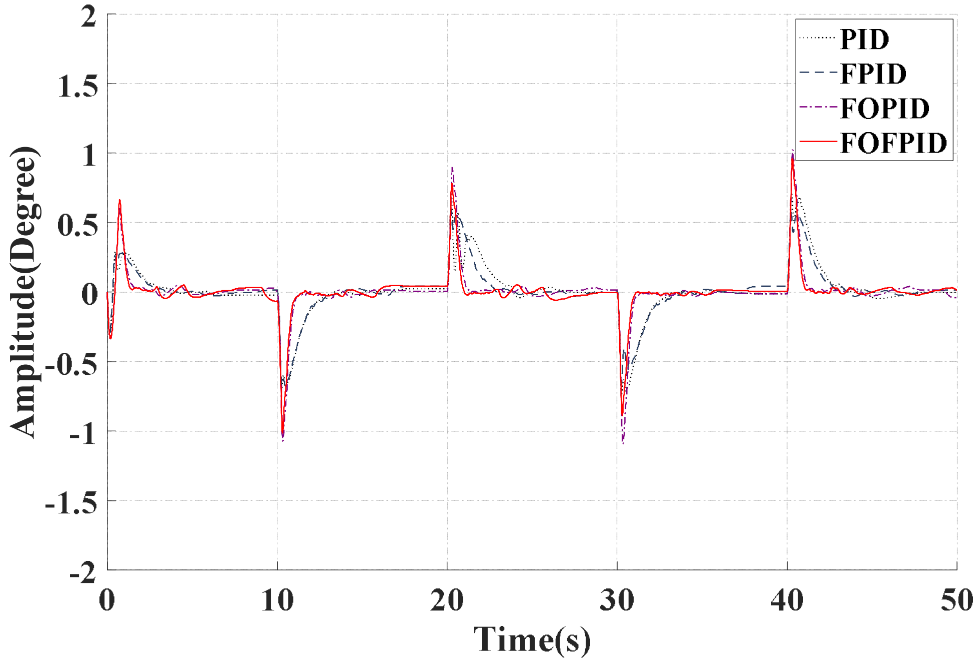


Figure 30 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a pulse signal (cutoff frequency 9Hz and the payload is 50N)*

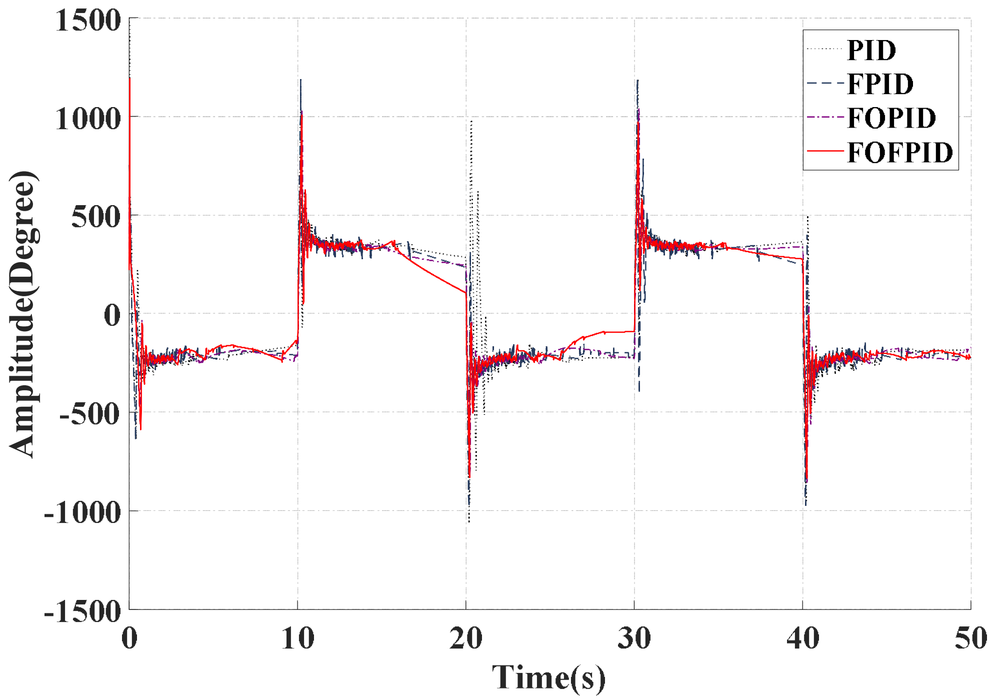


Figure 31 *Control signal of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID*

In this case, the pulse reference signal is used with amplitude [-5 5], period 10 second, the load 5kg and cutoff frequency 9 Hz. Figure 29 and Figure 30 depict the comparisons of position response using 4 controllers. From the results, we can realize that the FOFPID controller provides the best response. The overshoot is a little bit bigger than FPID and PID controllers. But the rising time and settling time is very short. The steady-state error is kept in the range of [-1 1]. The control signal in comparison shows in Figure 31.

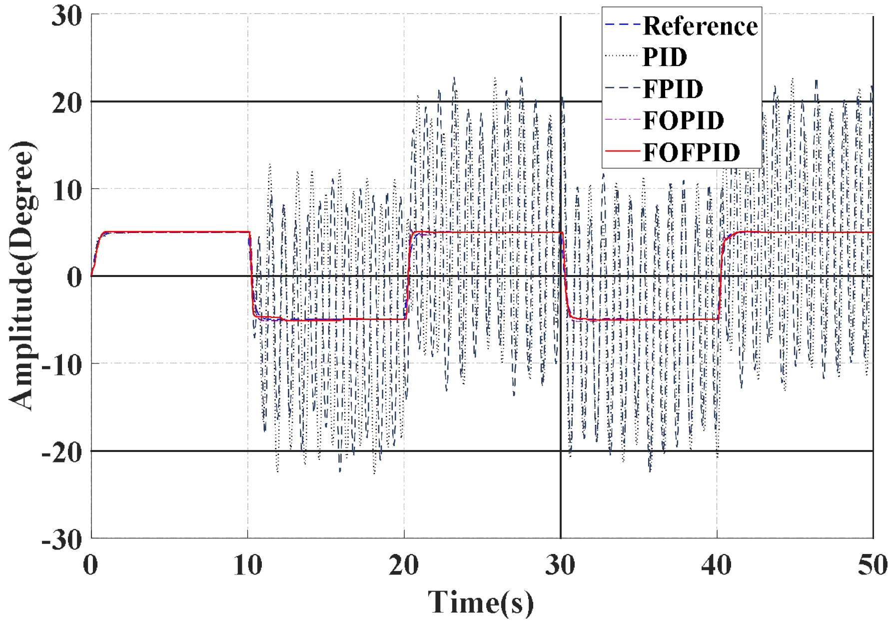


Figure 32 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a pulse signal (cutoff frequency 2Hz and the payload is 50N)*

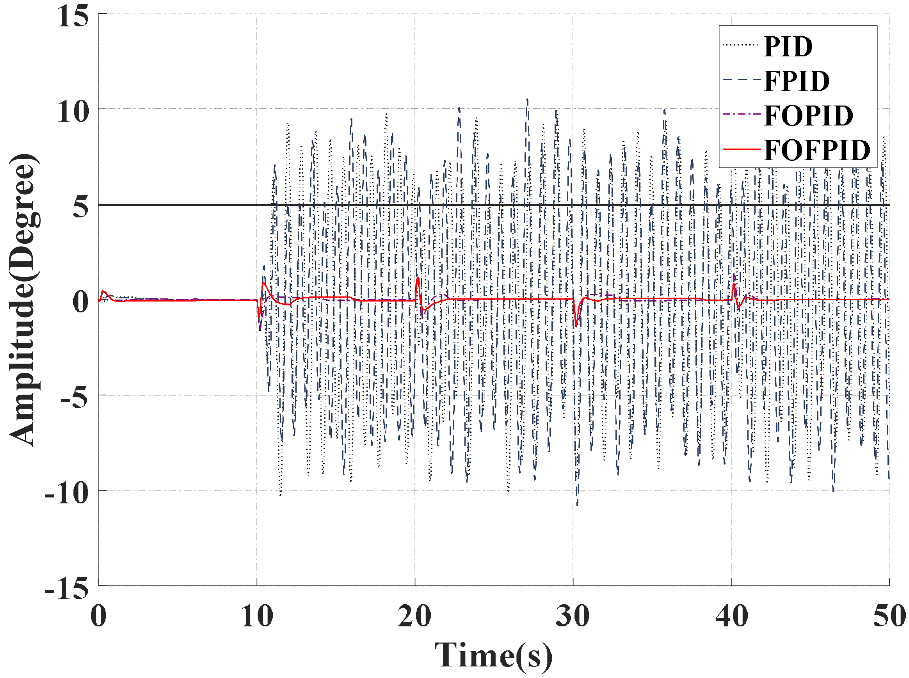


Figure 33 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a pulse signal (cutoff frequency 2Hz and the payload is 50N)*

In order to investigate a more challenging working conditions with pulse reference signal, the cutoff frequency is changed to 2 Hz. The performances of controllers are shown in Figure 32. With the PID and FPID controllers, they cannot handle the system follow the reference signal from the 10s. Meanwhile, the proposed controller still achieved an idea result with the fastest rising time and settling time. The steady-state error is also guaranteed and kept in the range from [-1 1] degrees as can be seen in Figure 33. Base on these pulse reference signal experiments, the effectiveness of the proposed controller over the conventional PID, fuzzy PID and fractional order controllers is strongly confirmed.

4.3.2.2. Case study with multistep signal

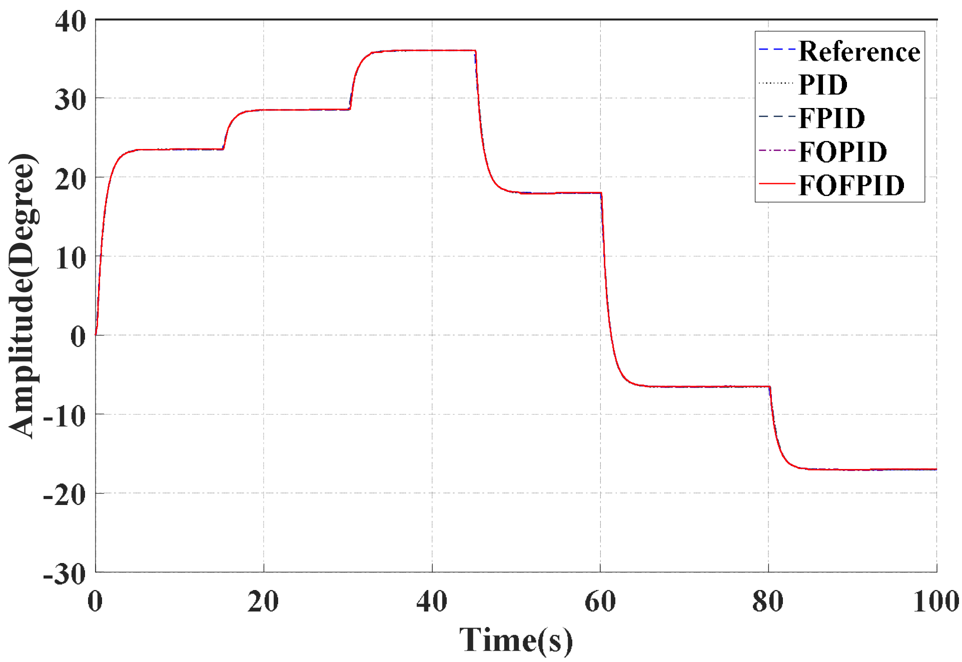


Figure 34 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a multi step signal (cutoff frequency 9Hz and the payload is 50N)*

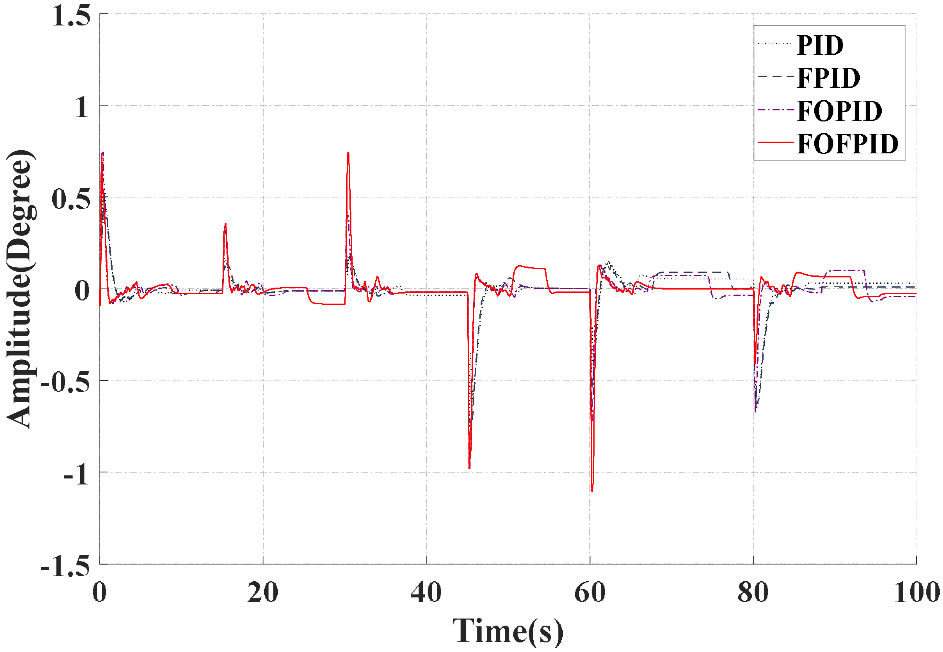


Figure 35 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a pulse signal (cutoff frequency 9Hz and the payload is 50N)*

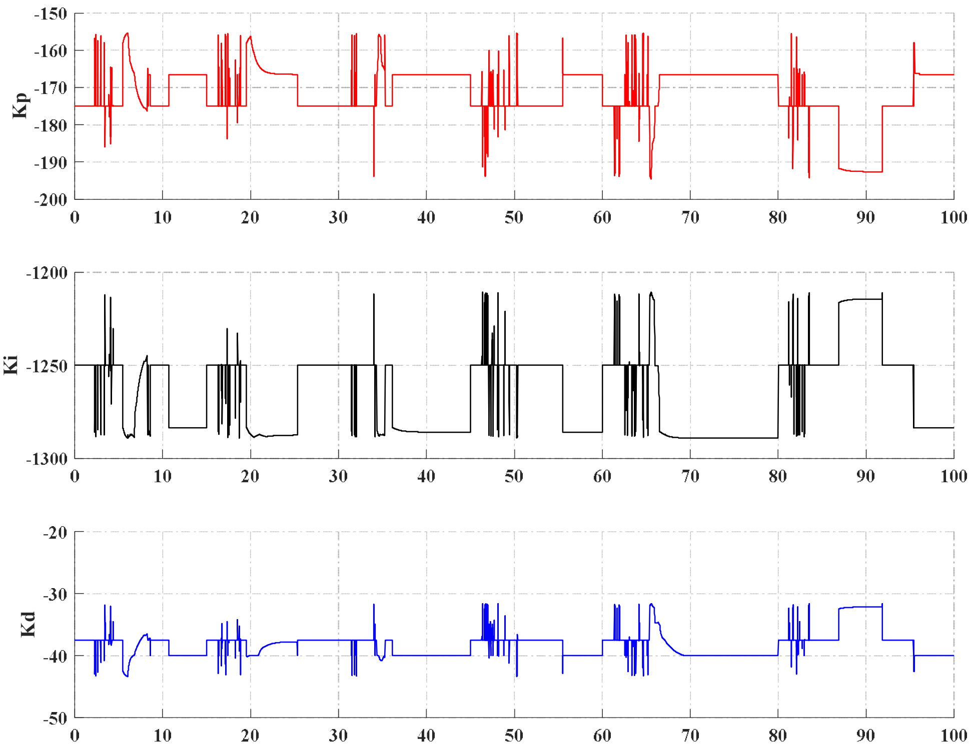


Figure 36 *Tuning parameters of K_p, K_i, and K_d.*

To further investigate the transient response, the steady-state behavior with non-periodic signals, the multistep reference with maximum amplitude of 35 degrees is used while the load is set to 5kg.

With the same controllers, the system response and tracking error are presented in Figure 34 and Figure 35 respectively. The proposed FOFPID provides a fast response with high accuracy of the steady-state control error (within 0.3 degrees). The values of K_p, K_i, K_d are adjusted by fuzzy rule can be seen in Figure 36.

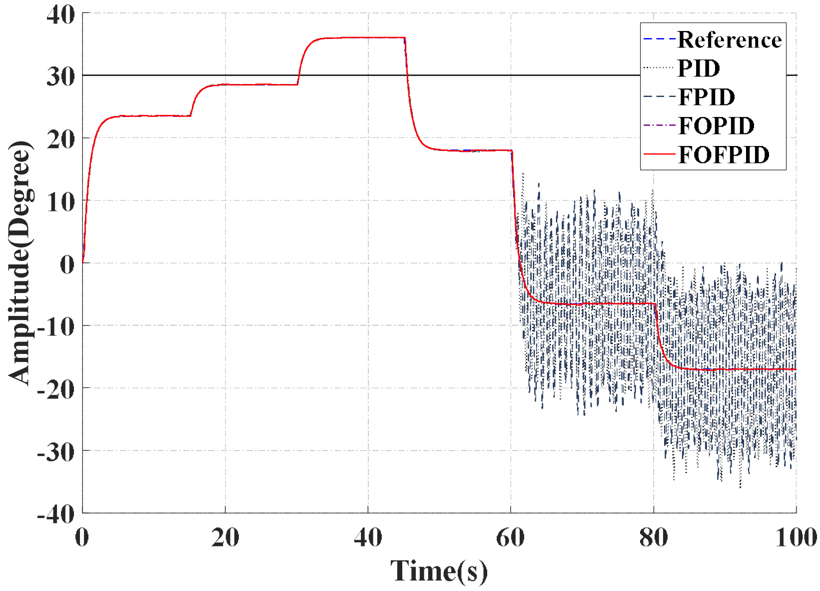


Figure 37 *Output performances of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a multi step signal (cutoff frequency 9Hz and the payload is 150N)*

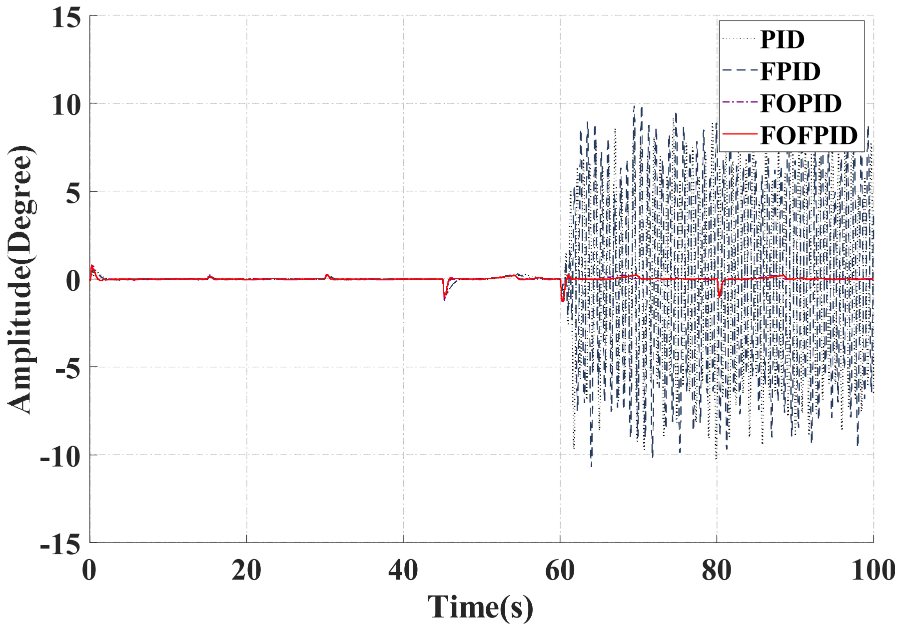


Figure 38 *Error effort of conventional PID, fuzzy PID, FOPID and Fuzzy FOPID with a multi step signal (cutoff frequency 9Hz and the payload is 150N)*

In the final case, the experiment is carried out to deal with big external force condition, the load is changed to 15 kg. The performances of controllers are shown in Figure 37 presented that conventional controllers cannot control the system when the rotary reverts its direction to lift the load. Meanwhile, the proposed FOFPID controller can maintain robust performance with the transient error is kept in the range of [-1 1] and steady-state error is in the range of [-0.3 0.3] degrees as can be seen in Figure 38. It is clear that a good position regulation is realized when using fractional order and intelligent technique to design the robust position fractional order fuzzy PID controller.

Chapter 5

CONCLUSIONS AND FUTURE WORKS

In this thesis, the design of a control system was presented for tracking tasks in an electro-hydraulic rotary actuator system using the Fuzzy FOPID controllers with the computed position control technique. An AMESim/Matlab co-simulation model was employed to identify the dynamic model of the EHRA and validate the proposed control. The objective system is subjected to some tests which are the different payloads and variant reference signals to evaluate the robust performance of FOFPID controllers with the computed position control technique.

Simulations and experiments were carried out to evaluate the effectiveness of the suggested control strategy applied to the hydraulic systems. The evaluation results showed that the proposed controller could achieve a good tracking position with respect to the design requirements. Consequently, the proposed controller has strong control ability not only for applications of EHRA system but also for other control systems. In the future work, some additional fuzzy rule will be designed to adjust the integration and differentiation orders of fractional order PID controller. Some technique will be combined to improve the robustness and performance of proposed controllers.

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